Technological Diversification*

Miklós Koren

Department of Economics, Harvard University

Institute of Economics, Hungarian Academy of Sciences

Silvana Tenreyro

Department of Economics, London School of Economics

Abstract

Why is GDP so much more volatile in poor countries than in rich ones? To answer this question, we propose a theory of technological diversification. Production makes use of different input varieties, which are subject to imperfectly correlated shocks. As in endogenous growth models, technological progress increases the number of varieties, raising average productivity. The new insight is that an expansion in the number of varieties also lowers the volatility of output. This is because additional varieties provide diversification benefits against variety-specific shocks. In the model, technological complexity evolves endogenously in response to profit incentives. Complexity (and hence output stability) is positively related with the development of the country, the comparative advantage of the sector, and the sector's skill and technology intensity. Using sector-level data for a broad sample of countries, we provide extensive empirical evidence confirming the cross-country and cross-sectoral predictions of the model.

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1 Introduction

Economies at early stages of the development process are often shaken by abrupt changes in growth rates. In his influential paper, Lucas (1988) brings attention to this fact, noting that "within the advanced countries, growth rates tend to be very stable over long periods of time," whereas within poor countries "there are many examples of sudden, large changes in growth rates, both up and down." This negative relationship between the volatility of growth rates and the level of development is illustrated in Figure 1, which plots the standard deviation of annual growth rates against the level of real GDP per capita for a large cross section of countries.

In an attempt to understand the sources of volatility, Koren and Tenreyro (2004) quantify the contribution of various factors at different stages of development, finding that the high volatility in poor countries is due to 1) higher levels of sectoral concentration, 2) higher levels of sectoral risk (that is, poor countries not only specialize in few sectors, but those sectors also tend to bear particularly high risk), and 3) higher country-specific macroeconomic risk. A volatility accounting exercise carried out by these authors indicates that approximately 50 percent of the differences in volatility between rich and poor countries can be accounted for by differences in the sectoral composition of the economy (higher concentration and sectoral risk), whereas the other 50 percent is due to country-specific risk. These characteristics of the development process, as we later explain, are inconsistent with previous theoretical explanations of the dynamics of volatility and development. The purpose of this paper is to provide an alternative theory that is in line with the empirical evidence.

To that end, we develop an endogenous growth model of technological diversification. The key idea of the model is that firms using a larger variety of inputs can mitigate the impact of shocks affecting the productivity of individual inputs. This takes place through two channels. First, with a larger variety of inputs, each individual input matters less in production, and productivity becomes less volatile by the law of large numbers. Second, whenever a shock hits a particular input, firms can adjust the use of the other inputs to partially offset the shock. This second channel operates even if production exhibits an extreme form of complementarity (as in Kremer (1993)'s O-ring technology). Both channels make the productivity of firms using more sophisticated technologies less volatile.

The idea can be illustrated with an example from agriculture: Growing wheat with only land and labor as inputs renders the yield vulnerable to idiosyncratic shocks (for example, weather shocks such as a severe drought). In contrast, using land and labor together with artificial irrigation, fertilizers, pesticides, etc., makes wheat-growing not only more productive but also less risky, because farmers have more options to react to external shocks. Figure 2 provides a graphical illustration of this example. It displays the volatility of wheat yield (calculated as the standard deviation of percentage deviations from the country's average yield) of the 20 biggest wheat producers against their level of GDP per capita.[1] Yield volatility falls sharply with development. This remains true if we control for differences in climate across countries, including the volatility of rainfall and temperature (see Table 1).

The shocks affecting individual inputs or individual production techniques may come from various sources. Another example of such a shock could be a sudden change in the price of a major input of a production technique. Countries with a diverse set of available techniques can cope better with the shock. For instance, the types of power plants that countries rely on to generate electricity vary with development. Small and less-developed countries have only a few plants very highly concentrated on one particular technique of electricity production (employing either traditional thermal or hydroelectric plants). Developed countries, on the other hand, have access to nuclear and renewable-resource plants and are typically more diversified. Firms in these countries will react differently to oil price shocks. Table 2 reports how the electricity production of countries responds to oil price changes. The electricity production of less-developed and small countries concentrated on few types of power plants is significantly more sensitive to oil price shocks than that of countries with a diverse set of plants. More specifically, while the electricity production of countries concentrated on a single energy source drops by about 1 percent after a 30 percent oil price hike, there is no such drop for diversified countries. Firms in countries with diverse sources of electricity can mitigate the negative impact of an oil price shock by substituting away from oil. The share of oil in total energy consumption falls by 0.3 percent after a 30 percent oil price hike, whereas no substitution takes place in concentrated countries.

We next turn to the questions of what determines technological diversification and why poorer countries specialize in less sophisticated sectors. We extend the model to allow for international mobility of goods and for cross-country differences in endowments. Much as in models of endogenous growth and directed technical change, the technological complexity of a sector in a given country evolves endogenously in response to the incentives of the creators and users of new technologies. In particular, more input varieties will be directed towards sectors in which the country has a comparative advantage, making them more complex and less volatile. The stage of development of the country will also matter, because inventing and/or using the new inputs is subject to increasing returns to scale. Countries accumulate new inputs as they develop, which brings about a gradual decline in their volatility. The speed of development, and hence the speed with which volatility declines, may be influenced by the initial level of volatility. If investment risk is harmful for growth, which is the case for a range of plausible parameter values in our model, then poor and volatile countries will

[1] Note that agricultural technology varies substantially with development. For example, of the top 20 wheat producers, India uses 2.3 tractors per 1,000 acres of arable land; this number is 128.8 for Germany. Fertilizer use also varies hugely. India uses 21.9 tons of nitrogenous fertilizers per acre; Germany uses 183.8 tons. We take the level of development as an overall indicator of agricultural sophistication.

develop slower and will remain highly volatile for long periods.[2]

The model delivers clear-cut predictions about the relationship among technological diversification, volatility, and productivity. Using sector-level data for a broad sample of countries, we provide empirical support for these predictions. First, any given sector is less volatile in developed countries. This result holds if we control for the quality of institutions which may facilitate a smoother response to external shocks, such as financial development and the flexibility of the labor market. Second, within a given country, large, skill intensive sectors using complex technologies are less volatile. This is consistent with our model which says that new inputs/technologies will be directed towards such sectors, thus reducing volatility. These two mechanisms lead to a decline in aggregate volatility as a country develops: The economy experiences less volatility in each sector and resources move towards relatively safer sectors.

The link between volatility and development has been studied before by Acemoglu and Zilibotti (1997), Greenwood and Jovanovic (1990), Saint-Paul (1992), and Obstfeld (1994), who describe the technology choice as a portfolio decision: In order to reap the benefits of high productivity and high growth, an economy has to bear more risk. The risk tolerance typically relates to the level of development and the financial structure of the economy. Acemoglu and Zilibotti (1997)'s model also features increasing returns to scale: Early in the development process diversification opportunities are limited, owing to the scarcity of capital and the indivisibility of investment projects. This feature can explain the high levels of sectoral concentration observed in poor countries. However, all these models predict that at early stages of development countries will tend to specialize in safer (even if less productive) sectors as a way of seeking insurance. This prediction is not borne out by the data: Koren and Tenreyro (2004) document that poor countries are highly concentrated in sectors that bear particularly high volatility. In addition, these authors find that most developing countries are inside the "mean-variance frontier," being highly prone to specialize in high-variance, low-mean sectors. These findings contradict the predictions of the portfolio-based models and suggest that important constraints must be at play, preventing developing countries from investing in safer and, at the same time, more productive assets.[3]

Our model departs from the portfolio view of the world that features a necessary trade-off between volatility and performance at the sector level. It can then naturally accommodate the fact that poor countries tend to exhibit high sectoral concentration and also that the high concentration falls mainly on high-risk sectors. In addition, unlike in previous contributions,

[2]See Angeletos and Calvet (2001) and Angeletos (2004) for a discussion of how volatility affects investment. Note, however, that in these papers there is no explanation for why volatility is higher in the first place. See also Ramey and Ramey (1995) on the empirical evidence.

[3]Kalemli-Ozcan, Sørensen and Yosha (2003) and Imbs and Wacziarg (2003) document that, for highly developed countries, industrial specialization tends to increase with development. However, as we later show, this does not result in higher aggregate volatility because these sectors tend to be technologically diversified and are hence more stable than the rest of the economy. The fact that the higher specialization of rich countries does not increase their aggregate risk has also been shown by Koren and Tenreyro (2004).

the volatility of individual sectors in our model is endogenous: It depends on the level of development and the comparative advantage of the country.[4]

Our paper is related to previous work by Kraay and Ventura (2001). As in their paper, the open-economy version of our model features the prediction that rich countries have a comparative advantage in less-volatile sectors. The difference lies in the way this result is achieved. In Kraay and Ventura (2001), high-skill sectors, which are prevalent in developed countries, enjoy less-elastic product demand. Markups can then serve as a buffer against productivity shocks, reducing the volatility of high-skill sectors. For example, a drop in output of a differentiated product makes that product more expensive in the world market. This terms of trade improvement partly offsets the original shock. On the other hand, no such "terms-of-trade insurance" is taking place for homogenous products that poor countries specialize in.

There are, however, empirical objections to the mechanism proposed by Kraay and Ventura (2001) and its implications. The model predicts a negative relationship between productivity shocks and terms-of-trade fluctuations (particularly negative for developed countries). That is, negative productivity shocks should be associated with an improvement in the terms of trade. In the data, however, the relationship between fluctuations in labor productivity and the terms of trade is somewhat positive, and there is no difference between rich and poor countries in terms of this relationship.[5]

Finally, our model builds on a vast literature on endogenous growth models in which the development of new varieties of goods enhances productivity. (See for example, Romer (1990) and Grossman and Helpman (1991).) The contribution of our paper is to provide a unified framework for the explanation of development and volatility. We provide sectoral evidence for a broad cross-section of countries that confirms the predictions of the model.

The remainder of the paper is organized as follows. In Section 2 we present the model. In Section 3 we discuss the empirical implications and offer novel evidence in support. We summarize and conclude in Section 4.

2 A Model of Technological Diversification

2.1 Technological diversification, productivity, and volatility

In this section, we introduce a production process that features technological diversification: Input varieties contribute not only to higher productivity but also, because inputs are subject to imperfectly correlated shocks, to lower volatility.

[4]As Koren and Tenreyro (2004) have shown, differences in the sectoral composition of developed and less-developed countries account for about 50 percent of the difference in volatility.

[5]It is possible that other factors are at play, blurring the predicted relationship; at this point, nonetheless, we can say that the extent of countercyclicality in the terms of trade is not the *prima facie* mechanism behind the negative relationship between development and volatility.

Output Y is produced using a composite of "machine varieties" with a constant-elasticity-of-substitution (CES) technology,

$$Y = \left[\sum_{i=1}^{n} X_i^{\sigma} \right]^{1/\sigma}, \tag{1}$$

where X_i is capital services from capital variety i , n denotes the number of working machines and $1/(1-\sigma) \in (0, \infty)$ is the elasticity of substitution across varieties.[6]

Machines can fail randomly, in which case they irreversibly cease to contribute to production. We assume that failure occurs independently across machines and time periods with probability $\gamma\, \mathrm{d}t$. That is, the lifetime of a machine is exponentially distributed with parameter γ. For our argument we need only that failures are imperfectly correlated. We take the extreme assumption of independence for expositional clarity. The assumption that random failures turn the machine completely useless makes the model more tractable, since we need only keep track of the number of working machines. However, technological diversification would still take place with less terminal shocks: Appendix B considers an example where there is only a partial drop in productivity after a machine failure.

In an open-economy context, changes in the world price of the input or sudden disruptions of trade could also be an important source of input-specific shocks. Technological diversification can mitigate the impact of such shocks.

Using machines in production involves increasing returns to scale: Machines are indivisible. This means that anyone operating a machine has to buy one unit of the machine beforehand. This minimum scale requirement limits the scope of diversification across machine varieties.[7]

Since we are interested in the inner workings of a sector and how technology choice affects volatility, we posit increasing returns at the input level. Indivisibility and minimum scale requirements are inherent characteristics of many an input used in technologically advanced sectors. Note that increasing returns are also a feature of the *use* of the machines, not only their invention or production. That is, we assume that machines can be produced and bought in any quantity but only a full unit is productive.

The setup of a machine requires κ units of the final good. Once the machine is set up, the owner gains monopoly power over its services. This monopoly lasts until the machine (exogenously) becomes obsolete, that is, the lifetime of the "patent" is the same as the lifetime of the machine and is exponentially distributed with parameter γ.

[6]As usual in endogenous growth models, we assume that $\sigma > 0$, that is, machines are gross substitutes. Appendix B considers an example when this is not the case. Introducing additional (scarce) factors of production would not change our qualitative results, it would just make the returns to variety more decreasing.

[7]Note that there is no incentive to install two or more units of a single machine variety, both because the production function features a "love of variety" and because machines are subject to idiosyncratic shocks. A similar assumption is made by Acemoglu and Zilibotti (1997) who work with minimum scale requirements at the industry level.

We assume that the machine can be used with different intensities by employing "operators." Machine i can provide twice as much service if operated by twice as many workers.[8] Producing a unit of capital service requires one unit of labor (by appropriate definition of labor units).

Formally, the services of machine i at time t are:

$$X_{it} = \begin{cases} l_{it}, & \text{if } K_{i0} = \kappa \text{ and } t < T_i; \\ 0, & \text{otherwise;} \end{cases} \tag{2}$$

where K_{i0} is the amount of capital devoted to machine variety i, l_{it} is the number of operators, and T_i is the exponentially distributed lifetime of the machine.

Consider the output of a firm, using n types of machine services, with X_i units of each,

$$Y = n^{1/\sigma} X_i. \tag{3}$$

As is apparent from (3), productivity is increasing in the number of varieties holding the amount of each individual variety fixed. This is the usual "love of variety" effect of many endogenous growth models (Romer 1990, Grossman and Helpman 1991). The effect is stronger the lower is σ, that is, the less substitutable machines are. Intuitively, if machines are highly substitutable, any additional variety is less needed.

As X_i also denotes the number of operators working on machine i, the overall number of machine operators working at the firm is $L = nX_i$. Hence (3) can be rewritten as

$$Y = n^{1/\sigma - 1} L. \tag{4}$$

Productivity is also increasing in the number of machines if we hold the total number of operators (L) constant (since $\sigma < 1$). The dependence is weaker than in (3), because any new machine requires operators taken away from old machines.

This implies that we have two alternative definitions of productivity, one holding the operators *per machine* constant, the other holding the *total* number of operators constant. We think both measures are useful, since the adjustment across different machine varieties can take place relatively fast within the firm (in particular, no hiring or firing of workers or capital installation is needed).[9]

Given that the number of machines is a random variable (individual machines fail at random and there is a finite number of machines), productivity will be random, too. What

[8] This is a way of capturing endogenous capacity utilization which is recently emphasized in business cycle studies. Allowing for capacity constraints or decreasing returns to capacity utilization would not alter our setting qualitatively. First, capacity constraints would not bind in equilibrium. Economic growth takes place via the expansion of machine varieties while the services of an individual variety shrink. Second, investors will be interested in the *total*, not the *marginal* profit when deciding whether to build a machine. This will remain positive even with decreasing returns to scale. Moreover, if the cost function were isoelastic, the share of profit in total revenue would be constant, just as in the present formulation.

[9] The effectiveness of this margin depends on how quickly and how efficiently machine operators can switch between different machines. Our assumption that any worker can operate any machine captures the

happens to output when a machine fails? First, the number of machines becomes $n-1$, making output lower for any given X_i. However, the demand for the services of an individual machine will also change. Again, it will be important to distinguish between the two measures of productivity. First, if we hold the number of operators per machine constant, productivity drops from $n^{1/\sigma}$ to $(n-1)^{1/\sigma}$. However, if we allow operators to be reallocated evenly among the remaining machines, the drop in output will be smaller, because we have allowed the firm to adjust the capacity utilization of the remaining machines in response to the shock. Productivity drops from $n^{1/\sigma-1}$ to $(n-1)^{1/\sigma-1}$, so the proportional drop is smaller.

The variance of productivity changes declines with the number of machines for both productivity measures. In the first case, this is just an application of the law of large numbers: Since output is an average (more precisely, a CES aggregate) of imperfectly correlated individual machine services, proportional changes in productivity become less and less volatile as the number of machines increases. The second case displays an additional effect: The more machine varieties the firm uses, the better they can respond to shocks by employing the remaining inputs at different intensities. Qualitatively, both effects imply that volatility declines with "technological complexity" (n). However, important differences arise when individual inputs are complements rather than substitutes of each other (such as in the O-ring production function of Kremer (1993)). Then the law of large numbers does not apply, because aggregate productivity is no longer an average of individual productivities. Still, as we demonstrate in Appendix B, volatility will fall with technological complexity if we allow for the second margin, variable capacity utilization.

To derive the variance of productivity formally, let a denote the log of productivity when the number of operators *per machine* (X_i) is held constant.

$$a = y - x_i = \frac{1}{\sigma}\ln n.$$

(Lower-case letters denote logarithms.) On the other hand, if we hold the total number of operators constant at L, we have $\tilde{a}$, the log of productivity, allowing for *variable capacity utilization* (VCU).

$$\tilde{a} = y - l = \phi \ln n_i,$$

where we have introduced the notation $\phi = 1/\sigma - 1$.

Our measures of volatility will be the variance of the changes in these two TFP variables:

$$\mathrm{Vol} = \mathrm{Var}(\mathrm{d}y|n, X_i) = \mathrm{Var}(\mathrm{d}a|n) = \left(\frac{1}{\sigma}\right)^2 \mathrm{Var}(\mathrm{dln}\, n|n),$$

$$\mathrm{Vol}_{\mathrm{VCU}} = \mathrm{Var}(\mathrm{d}y|n, L) = \mathrm{Var}(\mathrm{d}\tilde{a}|n) = \phi^2\, \mathrm{Var}(\mathrm{dln}\, n|n).$$

extreme case when such a switch is immediate and fully efficient. In reality, of course, we would see less than perfect flexibility. However, as the skills needed to work with advanced technology are very diverse (for example, Autor, Levy and Murnane (2003) document that computerization increased the demand for non-routine cognitive tasks), we believe that such adjustment is important in practice.

Let us for the moment assume away growth in the number of machines and study what happens to existing machines over time. (We introduce growth in the next section). The number of machines, in the absence of investment, changes because machines break at random.[10] Given that machine lifetimes are exponentially distributed with parameter γ and lifetimes are independently distributed, the first failure comes after an exponentially distributed time with parameter $n_t\gamma$.

$$\begin{aligned} T_k &\sim \exp(\gamma), \\ T_{\min} &= \min_{k=1,\ldots,n} \{T_k\} \sim \exp(n_t\gamma). \end{aligned}$$

The first failure reduces the number of machines by one, so

$$n_{t+h} = \begin{cases} n_t & \text{if } h < T_{\min}, \\ n_t - 1 & \text{otherwise.} \end{cases}$$

The probability of no machine failure over a period of length h is $1 - e^{-n_t\gamma h}$. The expected change in the number of machines is

$$\mathrm{E}(n_{t+h} - n_t | n_t) = -e^{-n_t\gamma h},$$

whereas the variance of the change is

$$\mathrm{Var}(n_{t+h} - n_t | n_t) = e^{-n_t\gamma h}\left(1 - e^{-n_t\gamma h}\right).$$

If we take $h \to 0$, we get the instantaneous mean and variance of the number of machines,

$$\begin{aligned} \mathrm{E}(\mathrm{d}n_t | n_t) &= -\gamma n_t \, \mathrm{d}t, \\ \mathrm{Var}(\mathrm{d}n_t | n_t) &= \gamma n_{it} \, \mathrm{d}t. \end{aligned}$$

More formally, in the absence of investment, the number of machines follows a continuous-time, discrete-space Markov process known as a "pure death" process with death rate γn_t. Such a process can be well approximated by an Itô process if n_t is large. (See Appendix A.) That is, for large n_t, the changes in the number of machines in a dt period of time will be approximately normal. This is just a version of the central limit theorem for discrete-space Markov processes. In the next section we allow for growth in the number of machines. The economy will then exhibit long-run growth in n_t, implying that the approximation will get better and better over time.

[10] Investment in new machines also changes the number of machines, but only gradually, because only a finite flow of investment will be allocated to new machines at any point in time. That is, investment does not contribute to the *volatility* of the number of machines. Formally, investment follows a bounded variation process. Even if we allowed for "jumps" in the number of new machines (say, investors could borrow a whole machine abroad), we are interested in the productivity shock before such an investment response takes place. Note that we are also assuming away integer problems of investment here. See Appendix A for a formal treatment of the number machines as a discrete-state Markov process.

Consequently, abstracting from growth, we can express the evolution of n_t using the following stochastic differential equation,

$$\mathrm{d}n_t = -\gamma n_t\,\mathrm{d}t + \sqrt{\gamma n_t}\,\mathrm{d}z,$$

where $\mathrm{d}z$ is the increment of a standard Wiener process. Given this approximation for $\mathrm{d}n_t$, we can use Itô's lemma to write $\mathrm{d}\ln n_t$ as follows:

$$\mathrm{d}\ln n_t = -\gamma(1 + 0.5/n_t)\,\mathrm{d}t + \sqrt{\gamma/n_t}\,\mathrm{d}z.$$

What is important here is that the volatility of the log number of machines declines with the existing number of machines. Even though as n_t gets big, the first failure gets more and more likely, the proportional (that is, log) drop in the number of machines it induces is less and less important. As is standard in statements of the law of large numbers, the second effect outweighs the first one. In other words, diversification across several machines makes log productivity less volatile.

Given that n_t measures the number of inputs subject to different shocks, we take it as an index of *technological complexity*. It is clear from (3) and the discussion above that technological complexity both increases average productivity and reduces the volatility of productivity. In the next section, we endogenize the investment in new machines, and consequently, the resulting level of technological complexity.

2.2 Endogenous technological complexity

What determines the level of technological complexity in the long run? In this section we endogenize the decision to invest in machines. Much as in models of endogenous growth (Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992), machine owners will be attracted by greater profit opportunities.

We first look at a one-sector economy to bring out the relationship between volatility and development clearly. In Section 2.3, we introduce multiple sectors and investigate how the relative complexity of sectors evolve endogenously. As we have documented in Koren and Tenreyro (2004), intrinsic volatility differences across sectors together with countries' different patterns of specialization are responsible for an important portion of the difference in output volatility between rich and poor countries. As in other multi-sector models of endogenous technology, we will have directed technical change (Acemoglu 2002, Caselli and Coleman 2000). Profits per machine variety will depend on the size of the sector (number of available operators), its relative wage, the degree of competition (number of existing machines), and trade openness.

Technology will be the same as in (1), which results in the following aggregate production function for the final good (4):

$$Y = n^{\phi}L.$$

The economic environment is characterized as follows. The final good sector is perfectly competitive, that is, firms take output and input prices as given. In contrast, machine providers act as monopolistic competitors, that is, they are price setters for their own machine but take the overall price of the composite machine varieties as given.[11]

There is a continuum of symmetric consumers/investors with unit mass. Each consumer has access to the well-diversified mutual fund of all machines. They can trade the share of this mutual fund freely, instantly, and in any quantity (even shorting is allowed). This will ensure that the mutual fund is priced by the consumer's stochastic discount factor. In other words, we assume no frictions in the domestic financial market. Note that there is no positive supply of a riskless asset in the economy, in other words, every production technology is risky.[12] Each consumer supplies L units of labor inelastically in the labor market. Consumers decide how much to consume and how much to invest in the mutual fund of machines, taking the rate of return and wage rate as given.

Time is continuous and consumers maximize lifetime expected utility over consumption with constant relative risk aversion, θ,

$$\mathcal{U}_t = \mathrm{E}_t \int_t^\infty e^{-\rho(s-t)} \frac{C_s^{1-\theta}}{1-\theta}\,\mathrm{d}s, \tag{5}$$

subject to a standard intertemporal budget constraint,

$$\mathrm{d}a_i = [(\mu_P + D/P)a_i + wL - C_i]\,\mathrm{d}t + \sigma_P a_i\,\mathrm{d}z.$$

The change in the asset holding of consumer i, $\mathrm{d}a_i$, comes from capital gains (μ_P) and dividend yield (D/P) on the asset and from labor income (wL) minus consumption (C_i). As we later show, stock prices follow a diffusion process, so asset holdings of consumer i will also follow a diffusion with instantaneous volatility $\sigma_P a_i$. The mean and variance of the rate of return on machines generally depends on the state of the economy, that is, the number of machines.

To derive the equilibrium of the economy, let us first consider the pricing decisions of firms. The demand for machine variety i is

$$X_i = \frac{\chi_i^{-1/(1-\sigma)}}{\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}} Y,$$

which is decreasing in the variety's own price (χ_i), increasing in competitors' prices ($\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}$) and the final good output (Y). Taking the price index $\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}$ and final demand Y as given, the machine owner faces a constant elasticity demand curve with elasticity $1/(1-\sigma)$.

[11] Note that this is a valid assumption even if there is a finite number of different machine varieties. First, the market share of each machine owner falls at the rate $1/n$, whereas the standard deviation of output is of order $1/\sqrt{n}$. That is, even if n is large enough to make monopolistic competition a realistic assumption, we still have positive aggregate volatility. Second, volatility falls with the number of *independent* machine varieties, which may be smaller than the number of machine owners if some of the machines are subject to common shocks or if there are interactions across machine-operating firms.

[12] Alternatively, the rate of return on a riskless asset (for example, storage) is so low that investors do not demand a positive amount.

She will hence follow a constant markup rule when pricing its services. The optimal monopoly price of each capital service will be

$$\chi_i = w/\sigma,$$

where w is the wage rate. Final good prices are, in turn, determined from the price of the services of an individual machine and the number of machines.

Profit maximization implies that price equals marginal cost in the final good sector. We normalize the final good price to unity:

$$1 = \left(\sum_{i=1}^{n} \chi_i^{\sigma/(\sigma-1)} \right)^{1-1/\sigma} = n^{-\phi} w.$$

This implies that wages increase in productivity:

$$w = n^{\phi}.$$

Labor market equilibrium requires that the number of operators exhausts the (exogenous) labor supply,

$$\sum_{i=1}^{n} X_i = nX_i = L.$$

The markup rule also implies that profits are a constant, $(1/\sigma - 1) = \phi$, share of wages. Total wages are wL, hence the wage costs of a single machine operating firm are wL/n (by symmetry), implying that profits are

$$\pi_i = \phi w L/n. \tag{6}$$

The owner of a machine uses this profit flow to calculate the lifetime cash-flow of the machine. Investors take the number of installed capital varieties, the wage rate, capital prices, and the return on equity as given.

We assume free entry into the machine market. This means that any investor can buy κ units of final good and install a new machine variety. As long as there is positive entry, this pins down the value of a machine at $V = \kappa$.[13] We further assume that the sunk cost required to install a machine is falling with the level of technological progress, n, because of spillovers or learning-by-doing externalities. In particular, it falls at a rate $\phi - 1$ to ensure balanced growth. Expanding variety growth models usually make a similar assumption to ensure long-run growth (Grossman and Helpman 1991, Chapter 3). Alternatively, one could set $\phi = 1$ by restricting the elasticity of substitution across varieties to be 2. Barro and Sala-i-Martin (1995, Chapter 6) put restrictions on the elasticity of substitution across input varieties. Either assumption delivers balanced growth and qualitatively similar results.

[13] If $V < \kappa$, no new machines are built and the growth rate is zero. We will later verify that this is not an equilibrium.

Also, we assume that fixed costs are proportional to the overall size of the economy, L. This assumption ensures that the growth rate is not dependent on country size. (See Jones (1995) on the "scale effect" of endogenous growth models.) Recall that κ measures the unit of a machine *variety* that is subject to variety specific shocks. Arguably, bigger countries use more capital of each variety and therefore require a bigger investment. Our main results are not sensitive to this assumption. The only result that would change without this assumption is that large countries would develop faster.

Formally,

$$\kappa = \kappa_0 L n^{\phi-1}. \tag{7}$$

The dividend yield on a machine is

$$\frac{\pi_i}{V} = \phi \frac{wL}{n\kappa} = \frac{\phi}{\kappa_0}, \tag{8}$$

where we have used $w = n^{\phi}$ and $\kappa = \kappa_0 L n^{\phi-1}$. The dividend yield is higher the higher the profit rate and the lower the fixed cost of installing a machine. The assumption of falling fixed costs ensured that the dividend yield does not vanish as n increases. If the dividend yield tended to zero as n became large, we would obtain a steady-state distribution of n instead of an ever-growing economy.

Note that even if the dividend yield on a machine is constant, the rate of return is random, because there are random capital losses due to machine failures. This results in an average depreciation rate (and hence capital loss) $\gamma\Delta t$ over a period Δt, but this capital loss is random even as we take $\Delta t \to 0$. We next turn to characterizing the stochastic process driving the value of machines.

Let P_{ts} denote the period-s value of all the machines installed at time t. At the time of installation, $P_{tt} = n_t V$. However, after an exponential lifetime the first machine fails and P_{ts} becomes $(n_t - 1)V$. The first failure occurs with probability $1 - e^{-\gamma n h}$ over an interval h. Taking $h \to 0$, we get the following mean and variance for the change in the value of the machines

$$\mathrm{E}(\mathrm{d}P_t) = -\gamma n_t V \,\mathrm{d}t = -\gamma P_t \,\mathrm{d}t,$$
$$\mathrm{Var}(\mathrm{d}P_t) = \gamma n_t V^2 \,\mathrm{d}t = \gamma P_t^2 / n \,\mathrm{d}t.$$

We use the same arguments put forward before for the number of machines and approximate the value of machines by a diffusion process:

$$\mathrm{d}P/P = -\gamma \,\mathrm{d}t + \sqrt{\gamma/n}\,\mathrm{d}z. \tag{9}$$

Once we know the rate-of-return process for stocks (which, recall, are the only form of investment), we can use the Euler equation to determine the optimal consumption/saving policy.

The continuous-time equivalent of the fundamental asset pricing equation, $p = \mathrm{E}(Mx)$, is

$$\mathrm{E}\left[M\frac{D}{P}\,\mathrm{d}t + \mathrm{d}(MP)\right] = 0,$$

where M is marginal utility, $M_t = e^{-\rho t}C_t^{-\theta}$, D is the dividend on an asset, and P is its price. This holds whenever investors can freely trade the asset (as is assumed here).

Because of complete domestic financial markets, both marginal utility and the stock price follow a geometric diffusion process driven by the same shock $\mathrm{d}z$. The asset pricing equation then simplifies to

$$\mu_M + D/P + \mu_P + \sigma_M\sigma_P = 0,$$

where μ_M is the proportional drift and σ_M^2 is the proportional variance of marginal utility (for example, Cox and Huang (1989)). The expected change in the discounted asset price (inclusive of dividends) can be zero only if the sum of all drift terms is zero.

Applying Itô's lemma for marginal utility, $M_t = e^{-\rho t}C_t^{-\theta}$,

$$\mathrm{d}M/M = \left[-\rho - \theta\mu_c + \frac{\theta(\theta+1)}{2}\sigma_c^2\right]\mathrm{d}t - \theta\sigma_c\,\mathrm{d}z,$$

where μ_c and σ_c^2 refer to the proportional drift and diffusion of C, respectively. Marginal utility declines with impatience and with the mean consumption growth rate and increases with consumption volatility (that is, the convexity of marginal utility gives rise to precautionary saving motives). Substituting this formula in the asset pricing equation results in the following continuous-time Euler equation

$$\mu_c(n) = \frac{1}{\theta}[r(n) - \rho] - \sigma_c(n)\sigma_P(n) + \frac{\theta+1}{2}\sigma_c^2(n), \tag{10}$$

where $r(n)$ refers to the sum of dividend yield and mean price decrease of stocks.

The mean growth rate of consumption depends positively on the mean rate of return, with a coefficient equal to the elasticity of intertemporal substitution, $1/\theta$. At the same time, because the consumer's portfolio is risky, its covariance with consumption will make saving less attractive and will hence result in lower consumption growth. Given that we have complete markets, in other words, there is only one source of uncertainty in the economy, the instantaneous correlation between stock prices and consumption is 1, so the covariance is $\sigma_c\sigma_P$. Finally, since future consumption is risky, prudent consumers have precautionary savings depending on the volatility of consumption and the degree of prudence, $(\theta+1)/2$.

Recall that the mean return on the portfolio of machines is

$$r(n) = \frac{\phi}{\kappa_0} - \gamma,$$

and the volatility of stock price is

$$\sigma_P^2 = \gamma/n.$$

In general, consumption also depends on the state of the economy, n. Let $C = v(n)$ be the policy function that describes the optimal amount of consumption given the number of machines in the economy. Then by Itô's lemma,

$$\mathrm{d}C = \left[v'(n)n\mu_n + \frac{1}{2}v''(n)n^2\sigma_n^2\right]\mathrm{d}t + v'(n)n\sigma_n\,\mathrm{d}z,$$

and the Euler equation becomes

$$\mu_n\frac{v'n}{v} + \frac{1}{2}\sigma_n^2\frac{v''n^2}{v} = \frac{1}{\theta}(r-\rho) - \sigma_n^2\frac{v'n}{v} + \frac{\theta+1}{2}\sigma_n^2\left(\frac{v'n}{v}\right)^2, \tag{11}$$

where we have omitted the argument n from $v(n)$ for brevity.

The growth rate of machines, μ_n, depends on investment, which, in turn, depends on the consumption policy. By equilibrium in the final good market,

$$Y = n^\phi L = v(n) + (\mu_n+\gamma)n\kappa = v(n) + (\mu_n+\gamma)n^\phi\kappa_0 L. \tag{12}$$

Total output has to equal the value of consumption plus investment. Investment is the sum of net investment (μ_n) and the replacement of broken machines (γ).

Equations (11) and (12) together with $\sigma_n^2 = \gamma/n$ define a second-order ordinary differential equation for v, which has two linearly independent solutions. We therefore need two boundary conditions to pin down the optimal policy function, $v(n)$. One is that no consumption takes place without capital, $v(0) = 0$. The other one comes from the fact that as n becomes arbitrarily large, σ_n^2 becomes zero and the economy resembles a non-stochastic Ramsey model. Consumption growth in the non-stochastic Ramsey model is given by $\tilde{\mu}_c = (r-\rho)/\theta$, so we should have

$$\lim_{n\to\infty}\mu_n\frac{v'n}{v} = \tilde{\mu}_c = (r-\rho)/\theta.$$

To obtain an analytical solution, we put restrictions on the CRRA and the elasticity of substitution across machine varieties and assume that $\theta\phi = 1$.[14] Whether this is plausible depends on how broadly we interpret machine varieties. If these are different intermediate inputs necessary to produce a particular good, the inputs may be strong complements, in which case the elasticity is less than one. This would lead to a negative ϕ which we have ruled out (but see Appendix B for an example of such a production function). However, if we think of machine varieties as representing alternative production techniques that

[14]For other values of relative risk aversion, numerical techniques can be applied to solve (11). If $\theta < 1/\phi$, then the saving rate is increasing in n. Intuitively, low-θ consumers are less prudent, so the precautionary motive is relatively small. This means that risk aversion dominates and saving declines with volatility. In this case, poor countries develop slowly because their excess volatility discourages investment. The reverse is true for $\theta > 1/\phi$. Similarly to Angeletos (2004), we have the cutoff at an IES less than one (RRA greater than one) because capital does not exhaust all income as long as $\phi < 1$.

can highly substitute each other, then higher elasticities are more plausible. For example the elasticity of substitution across goods produced in different countries (within a narrow product category) is estimated to be around 4–7 (Hummels 2001). Estimates based on the time series of U.S. imports are usually lower, in the range of 1–2 (Gallaway, McDaniel and Rivera 2003). For an intermediate range of 3–4, the value of ϕ is 1/2–1/3, resulting in a θ of 2–3. This is plausible both as a measure of relative risk aversion and as an inverse elasticity of intertemporal substitution (Vissing-Jørgensen 2002).

Proposition 1. If $\theta\phi = 1$, the optimal policy function, $v(n)$ takes the form $v_0 n^\phi$, where v_0 is given by

$$v_0 = (1-\phi)L + \rho\kappa_0 L. \tag{13}$$

Proof. Direct substitution reveals that whenever v_0 satisfies (13), $v_0 n^\phi$ satisfies (11). For this policy function, $v'' n^2/v = \phi(\phi-1)$, $v'n/v = \phi$, and μ_n is independent of n. Since $v_0 n^\phi$ also satisfies the boundary conditions, it is a unique solution. □

Defining the value of all the machines as $K = n\kappa$, equation (13) can be rewritten in terms of aggregate variables as

$$Y - C = \phi Y - \rho K = (\phi - \rho\kappa_0)\, Y,$$

since the capital output ratio in this economy is $n\kappa_0 L n^{\phi-1}/(n^\phi L) = \kappa_0$. Investors save (and invest) a constant fraction of current output. The saving rate is increasing in the profit rate (ϕ) and decreasing in the degree of impatience (ρ) and sunk cost of investment (κ_0).

From (12), we can express the growth rate of the number of machines as

$$\mu_n = \phi/\kappa_0 - \gamma - \rho,$$

resulting in an output growth rate of

$$\phi\mu_n = \phi\left(\phi/\kappa_0 - \gamma - \rho\right).$$

This completes the characterization of the dynamic equilibrium of this economy. Countries with high profit rates and low investment costs will develop faster, implying both a faster growth of output and a faster fall of volatility. In the next section, we extend the model in two directions in order to account for the differences in specialization patterns between rich and poor countries.

2.3 A two-sector model of technological diversification

In this section, we allow for a richer characterization of the economy, by extending the model to a two-sector economy. The sectors differ in the extent of skill intensity. We introduce a multi-country setup, allowing for cross-country differences in endowments and compare the results for closed and open economies. Allowing for international trade, as we later show, can explain the observation that poor countries specialize in less sophisticated sectors. In fact, comparative advantage magnifies the differences in volatility between poor and rich countries through its effect on the patterns of specialization.

Let us assume that there are two sectors, one producing a capital good Y_k, the other producing a consumption good Y_c:

$$Y_k = \left[\sum_{i=1}^{n_k} X_i^\sigma \right]^{1/\sigma},$$
$$Y_c = \left[\sum_{i=n_k+1}^{n_k+n_c} X_i^\sigma \right]^{1/\sigma}.$$

Both sectors use the same CES technology, but they have access to a different set of machines. In particular, the number of machines in the two sectors will (endogenously) be different. The owner of each machine will decide which sector to operate in. The total number of machines $n_k + n_c$ is denoted by n.

We assume that machines in the capital good sector are operated by skilled labor, whereas those in the consumption good sector are operated by unskilled labor.[15] Note that machines are a metaphor for technology in our model so this amounts to assuming that some technologies are skilled labor intensive, whereas others are unskilled labor intensive. Autor, Katz and Krueger (1998) show that computerization has increased the demand for skilled labor. However, previous technological advances such as the industrial revolution and the introduction of the production line relied more on unskilled rather than skilled workers (James and Skinner 1985, Goldin and Katz 1998). In this paper, we think of the skilled-labor intensive sector as a technologically advanced sector, and the unskilled-labor intensive sector as a less technologically complex one.[16]

More formally,

$$X_{it} = \begin{cases} h_{it}, & \text{if } i \leq n_k,\ K_{i0} = \kappa \text{ and } t < T_i; \\ l_{it}, & \text{if } n_k < i \leq n,\ K_{i0} = \kappa \text{ and } t < T_i; \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Here h_{it} denotes the number of skilled operators and l_{it} the number of unskilled operators of variety i, and κ and T_i are defined, as previously, as the fixed cost of building a machine and the random lifetime of machine i.

Let H denote the overall stock of skilled labor in the economy and L the stock of unskilled labor. If the labor markets are in equilibrium, the production functions can be rewritten as

$$Y_k = n_k^\phi H,$$
$$Y_c = n_c^\phi L.$$

[15] Any positive difference in skill intensity is sufficient for our results; we assume this extreme difference in skill intensity for tractability.

[16] In Section 3 we discuss how we identify technological complexity in the data.

Similarly to the one-sector case, the productivity of a firm in sector i will be increasing in the number of machines. This also causes the volatility of productivity to decline with the number of machines:

$$\text{Vol}_k = \text{Var}(\text{dln}\, Y_k | n_k, H) = \phi^2\gamma/n_k, \tag{15}$$
$$\text{Vol}_c = \text{Var}(\text{dln}\, Y_c | n_c, L) = \phi^2\gamma/n_c. \tag{16}$$

Both sectors are perfectly competitive. Each producer takes the wage rate and the set of machine varieties available to the sector as given.

What determines the allocation of machines across the two sectors? Again, investors will maximize profits and move toward sectors with better profit opportunities. The price of machine service i will be marked up over the skilled wage in the capital good sector and over the unskilled wage in the unskilled sector:

$$\chi_i = \begin{cases} w_H/\sigma, & \text{if } i \leq n_k; \\ w_L/\sigma, & \text{if } n_k < i \leq n, \end{cases}$$

where w_H denotes skilled, w_L denotes unskilled wages. This makes profit per machine a constant ϕ fraction of wages per machine:

$$\pi_i = \begin{cases} \phi w_H H/n_k, & \text{if } i \leq n_k; \\ \phi w_L L/n_c, & \text{if } n_k < i \leq n. \end{cases}$$

In long-run equilibrium, the rate of return on machines in the two sectors have to be equal. Since the values of a machine are the same in each sector, so is the dividend (that is, profit per machine), implying

$$\frac{n_k}{n_c} = \frac{\omega H}{L}, \tag{17}$$

with $\omega = w_H/w_L$ denoting the skill premium.

The number of machines in a sector is proportional to the total wage bill of the sector. Whenever (17) fails to hold, that is, one of the sectors has relatively few machines, that sector has more profits per machine than the other one. Investors will then move machines across sectors (if machines are movable), or invest only in the more profitable sector until the equality is resolved.

We assume that machines are freely and instantly movable across sectors. This assumption ensures that $r_k = r_c$ at any time in any state of the world, implying that (17) holds at any point in time.[17] Since (17) would always hold in the long run, we only need this assumption to simplify the transitional dynamics: If machines can instantly adjust across sectors

[17] Formally, the present discount value of a machine is $p_k\kappa$ in both sectors. If the dividend yield in one sector is higher than in another, an investor could buy one unit of high-dividend stock and short one unit of low-dividend stock. Since the prices of the two stocks always move in parallel, this strategy presents an arbitrage opportunity with positive net dividends in all future periods and no price risk. Absence of arbitrage hence implies the equalization of dividend yields.

then the economy will immediately jump to its balanced-growth path. Because the machines are movable across sectors, the single state variable is the total number of machines, n.

Equation (17) describes a version of *directed technical change*, as in Acemoglu (2002) or Caselli and Coleman (2000): Machine varieties are directed towards the sector with a higher share in employment. On the one hand, this is a size effect: if there are many operators in a sector, it is more profitable to operate machines there and hence more machines will move towards this sector. On the other hand, there is also a relative price effect: If the skill premium is high, the relative price of the capital good is high, so profits are higher in the capital good sector.

Since the final good sectors are competitive, the relative price of the capital good will equal the relative marginal cost:

$$p_k = \omega \left(\frac{n_k}{n_c} \right)^{-\phi}. \tag{18}$$

If there are more machines allocated to the capital sector, it becomes more productive, and its relative price falls.

2.3.1 The closed economy

In a closed economy, the relative price is determined by the relative supply and demand of the two goods. Suppose that the demand for the two goods take the isoelastic form,

$$\frac{Y_k}{Y_c} = A p_k^{-\varepsilon},$$

where A is a constant and ε is the elasticity of substitution between capital and consumption goods. Then, the skill premium is endogenously determined as

$$\omega = p_k \left(\frac{n_k}{n_c} \right)^{\phi} = A^{1/\varepsilon} \left(\frac{H}{L} \right)^{-\phi/\varepsilon} \left(\frac{n_k}{n_c} \right)^{\phi(1-1/\varepsilon)}, \tag{19}$$

$$\frac{n_k}{n_c} = A^{\frac{1}{\varepsilon 1-\phi+\phi}} \left(\frac{H}{L} \right)^{\frac{\varepsilon-\phi}{\varepsilon(1-\phi)+\phi}}. \tag{20}$$

As we will show, the nominal saving rate in the economy is constant, which means that $p_k Y_k / Y_c$ is constant. This implies that the relative demand for the consumption and the capital good has unitary elasticity, $\varepsilon = 1$. Then equation (20) simplifies to

$$\frac{n_k}{n_c} = A \left(\frac{H}{L} \right)^{1-\phi}. \tag{21}$$

Since we have assumed $0 < \phi < 1$, the relative number of machines is increasing in the relative amount of skilled labor in the economy. However, a 1 percent increase in skill abundance induces a less than 1 percent increase in the relative number of machines in the capital good sector. This is because the abundant factor (more precisely, the good that uses the abundant factor intensively) becomes cheaper and hence less profitable for machine owners. This relative price effect has been pointed out by Acemoglu (2002).

The fixed cost required to build a machine is assumed to arise in capital goods. In terms of consumption goods, the fixed cost is $p_k\kappa$. The mean rate of return on machines is the same in both sectors, so let us take the capital good sector:

$$r = \frac{\pi_i}{V} - \gamma = \phi \frac{w_H H/n_k}{p_k \kappa} - \gamma = \phi \left(\frac{n_k}{n}\right)^{\phi-1} \frac{H/L}{\kappa_0} - \gamma, \tag{22}$$

where we have made use of the facts that capital good prices equal marginal costs ($w_H n_k^{-\phi}$) and that the sunk cost is falling at the rate $\phi - 1$, $\kappa = \kappa_0 L n^{\phi-1}$. Relative to (8), the important difference is that the rate of return is falling in the relative complexity of the capital good sector. The number of machines affects profitability in two ways. First, more machines make the capital sector more productive and hence more profitable (productivity is proportional to n_k^{ϕ}). Second, competition increases in the number of machines, because machines compete for a scarce supply of operators (H). This second effect lowers profits proportionally with n, so it dominates the first for $\phi < 1$.

The relative demand for capital and consumption goods will be determined by the consumption-saving decision. Since the mutual fund holds all machines in both sectors, the Euler equation of the consumer is the same as in (11), but the growth rate of machines and the return on machines may be different.

The mean growth rate of the number of machines, n is

$$\mu_n = \frac{Y_k}{n\kappa} - \gamma = \frac{H/L}{\kappa_0} \left(\frac{n_k}{n}\right)^{\phi} - \gamma. \tag{23}$$

As before, we conjecture that optimal consumption is given by $v_0 n^{\phi}$, which, by the equilibrium in the consumption good market, implies

$$\frac{n_c}{n} = \left(\frac{v_0}{L}\right)^{1/\phi}, \tag{24}$$

$$\frac{n_k}{n} = 1 - \left(\frac{v_0}{L}\right)^{1/\phi}. \tag{25}$$

The Euler equation simplifies to $\mu_n = r - \rho$. Substituting from (22) and (23), we see that $v_0 n^{\phi}$ is indeed a solution as long as

$$\frac{H/L}{\kappa_0}\left[1 - \left(\frac{v_0}{L}\right)^{1/\phi}\right]^{\phi} = \phi \frac{H/L}{\kappa_0}\left[1 - \left(\frac{v_0}{L}\right)^{1/\phi}\right]^{\phi-1} - \rho. \tag{26}$$

Note that v_0 does not depend on n, implying that the allocation of machines across sectors as well as the relative prices are independent of development. In other words, the economy exhibits balanced growth. This also means that the saving rate is constant as previously claimed.

2.3.2 A small open economy

Suppose instead that the country is a small open economy freely trading the output of the two final good sectors at an exogenously given world relative price. We assume that the individual machine varieties cannot be traded. In other words, investors can buy foreign capital goods and install them in their own country as machines, but the physical machines installed abroad cannot contribute to production.[18] This assumption ensures that countries cannot circumvent the fixed costs of machine operation by importing machine services from abroad and hence cannot fully diversify instantly. The number of machines in the country will hence be a state variable that can only be adjusted gradually. At any given point in time, the number of available machines and hence overall technological complexity is given. In the long run, investment in new machines will determine technological complexity, economic development, and volatility.

Trade is balanced at any point in time, ruling out international borrowing and lending. This also means that investment is finite (growth in the number of machines is gradual) in every instant, because the country has only finite flow output to offer in exchange for foreign capital goods. In contrast, if we allow for borrowing, investors can immediately borrow to replace a broken machine, smoothing out some of the shock to productivity. We assume away such consumption smoothing behavior because the current accounts of countries (especially those of less-developed ones) do not seem to act as buffers against productivity shocks.[19]

Let $\tilde{p}_k$ denote the world price of capital. We then have from (18)

$$\tilde{p}_k = \omega \left(\frac{n_k}{n_c} \right)^{-\phi},$$

which results in *conditional factor price equalization*,

$$\omega = \tilde{p}_k \left(\frac{n_k}{n_c} \right)^{\phi}. \tag{27}$$

Conditional on the levels of productivity in the two sectors, the world relative price of the two goods completely determines the relative wage. All else equal, a higher relative price of the capital good (high $\tilde{p}_k$) leads to a higher relative wage of the factor which is used intensively in that sector (high ω). This is the FPE part. At the same time, the more productive the capital good sector is relative to the consumption good sector, the higher the relative wage of skilled labor.

[18] If we interpret machine varieties as different techniques of production, this amounts to very costly imitation and no technology spillovers across countries. Comin and Hobijn (2004) document a relatively slow adoption of leading technologies developed elsewhere. A positive but finite cost of technology adoption could be modeled such that machine varieties already in use abroad have a lower installation cost $\tilde{\kappa} < \kappa$. A $\tilde{\kappa} > 0$ would be sufficient to deliver qualitatively similar results.

[19] Kalemli-Ozcan et al. (2003) show that the beta coefficient of consumption response to output shocks of countries is close to one.

Note that, as standard in small open-economy models with free trade, the production structure is independent of demand considerations. Relative demand for the two sectors (in our case, the consumption/investment decision) will matter only for the patterns of trade. Since $\tilde{p}_k$ is exogenously given, we can substitute (27) into (17):

$$\frac{n_k}{n_c} = \tilde{p}_k^{1/(1-\phi)} \left(\frac{H}{L}\right)^{1/(1-\phi)}. \tag{28}$$

Notice that, similarly to the closed economy case, (21), the relative number of machines in the capital sector increases in skill abundance. However, the dependence on skill abundance is stronger, $1/(1-\phi)$, because we no longer have an offsetting relative price effect. This is just the Rybczynski theorem applied to directed technical change.

The impact is also greater than in the case of pure factor price equalization. The reason for this is that machines flow towards the sector that already has a comparative advantage, making it relatively more productive. This becomes an additional source of comparative advantage. In other words, the initial comparative advantage gets magnified by directed technical change. Our model says that even small human capital differences can account for large differences in specialization patterns and, hence, in the relative volatilities of sectors.

2.4 Extension to multiple sectors

Suppose now that there are S sectors, each using the same CES technology but requiring different levels of skill for machine operation. In particular, sector s requires that each operator possesses at least h_s amount of human capital, and we order sectors such that $h_S > h_{S-1} > ... > h_1$. The output of machine i in sector s is

$$X_{is} = \begin{cases} h_s l_{it}, & \text{if } K_{i0} = \kappa, \text{ and } t < T_i; \\ 0, & \text{otherwise,} \end{cases}$$

where l_{it} is the number of workers on machine i who are "qualified" to operate the machine in the sense that they have a level of human capital higher than h_s.

There are altogether L workers in the economy, and their human capital endowment is distributed according to a cumulative distribution function $F(h)$. The number of workers capable of operating machines in sector s is hence $[1 - F(h_s)]L$. The two-sector case of Section 2.3 is a special case of this framework, with a fraction of people having high human capital (skilled workers) and the rest having low human capital (unskilled workers).

Given the number of machines in each sector, $(n_1, n_2, ..., n_S)$, labor market equilibrium requires that each worker be employed on machines that require the highest skill level that this worker can supply.[20]

[20] To prove this, suppose there exists a worker with human capital level $h_j \geq h_{s+1}$ (that is, capable of working in sector $s+1$) working in sector s. This worker is not willing to switch to sector $s+1$ because $w_{s+1} < w_s$. But all workers in sector $s+1$ are capable of operating sector s machinery, and they would earn higher wages in that sector. Hence this cannot be an equilibrium.

This implies that a fraction $1 - F(h_S)$ of workers is employed in sector S, and a fraction $F(h_{s+1}) - F(h_s)$ in sector s. The output of sector s is hence

$$Y_s = n_s^{\phi} h_s \alpha_s L,$$

where α_s is defined as the share of workers in sector s, $F(h_{s+1}) - F(h_s)$ (defined for all s with $h_0 = 0$, $h_S = \infty$). Profits per machine are a constant, $(1-\sigma)$, fraction of revenues per machine,

$$\pi_s = (1-\sigma)\tilde{p}_s n_s^{\phi-1} h_s \alpha_s L,$$

where $\tilde{p}_s$ is the price of product s determined in world markets. Directed technical change will equate per-machine profits across sectors, $\pi_s = \pi_z$, so the relative number of machines in any two sectors is given by

$$\frac{n_s}{n_z} = \left\{ \frac{\tilde{p}_s h_s \alpha_s}{\tilde{p}_z h_z \alpha_z} \right\}^{1/(1-\phi)}. \tag{29}$$

A sector will use relatively more machines if it is producing an expensive good, it is skill intensive, or has a bigger pool of workers with matching skills. Such sectors are also more productive and less volatile. In other words, given the overall number of machines, $n = n_1 + n_2 + ... + n_S$, technological complexity and productivity are increasing, while volatility is decreasing in the sector's skill intensity and its share in total employment.

The variance of sector s in country i is $\phi^2\gamma/n_{is}$, so we can write the log variance as

$$\ln \mathrm{Var}_{is} = 2\ln\phi + \ln\gamma - \ln n_{is} = \nu_i - [\ln\tilde{p}_s + \ln h_s + \ln(L_{is}/L_i)]/(1-\phi), \tag{30}$$

where ν_i is a country fixed effect.

This is a key equation for our empirical exercise. While we can measure a sector's skill intensity and its share in employment, we do not observe $\tilde{p}_s$, the price of the sector's output in world markets. Instead, we interpret it broadly as an unobserved sector-specific variable that affects the level of complexity, capturing not only variations in the value of output but also, for example, technological differences across sectors. Note that this variable is common across countries within a given sector, so we can control for it using either sector fixed effects or observing technological complexity in any given country.

3 Productivity, Volatility, and Technological Complexity: The Empirical Evidence

The model developed in the previous sections leads to a set of predictions concerning the relationships among productivity, volatility, and technological diversification. We discuss these predictions in light of the empirical evidence.

Prediction 1. GDP volatility declines with development.

This is one of the stylized facts in the literature and the main motivation of this paper. There are large cross-country differences in volatility. The standard deviation of annual GDP growth during the period 1970 through 2000 ranges from 1.4 percent to 21.8 percent (a factor of 15) across 167 countries. The most volatile decile of countries had a standard deviation of GDP growth of 12.9 percent. This is seven times as high as the volatility of the least volatile decile (1.8 percent). This cross-country variation in volatility is highly correlated with the cross-country variation in the level of development, gauged by real GDP per capita. More specifically, as shown in Table 4, the elasticity of GDP variance with respect to GDP per capita is -0.326 (with a robust standard error of 0.066).[21]

In the model, investment in new machines brings about development and a gradual decline in volatility. Countries that have few machines are both less developed and more volatile. In the multi-sector version, our model proposes two channels to explain this negative association. First, a within-sector channel, whereby a given sector exhibits higher technological complexity in more-developed countries. This, in turn, implies that a given sector is both more productive and less volatile in developed countries. Second, a compositional channel, whereby poor countries specialize in relatively less complex sectors. This implies that poor countries concentrate in sectors with (absolute) lower productivity and higher variance. In what follows, we check the empirical consistency of the predictions associated with these two channels.

Prediction 2. For any given sector, poor countries utilize less complex technologies. This implies that for any sector, *a*) poor countries are both less productive and more volatile and *b*) productivity and volatility are negatively correlated.

- For a given sector, poor countries utilize less complex technologies.

Various studies have explored the process of technology diffusion across countries. For example, Caselli and Coleman (2000), document that the adoption of computers depends heavily on the level of development of the country, and, more specifically, on the level of human capital. Caselli and Wilson (2004) show that this result extends to a broader set of high-technology equipment (where the extent of technology embodied in capital equipment is measured as the R&D content).

Our model implies that these cross-country differences in technology are also present *within* sectors. Since directed technical change equates the rates of return on machines in all sectors, poor countries will use proportionately fewer machines in all sectors, holding comparative advantage patterns constant.

The two examples mentioned in the introduction suggest important cross-country technological differences for a given sector: Developed countries tend to use more agricultural machinery, fertilizers, and pesticides in agriculture and have access to more types of power plants in the energy sector. Recent empirical studies provide additional support for this

[21] Table 3 presents the list of countries included in the computation.

observation. For example, Comin and Hobijn (2004) document how specific technological innovations have spread across countries. Many of these innovations are relevant only to certain sectors (for example, mule spindle, blast oxygen furnace, internal combustion engine, aviation). The authors show that most innovations originated in developed countries and spread gradually to less-developed countries. This implies that in any given year, in all relevant sectors, poor countries use less sophisticated production techniques than rich ones.

- For a given sector, poor countries are both less productive and more volatile.

In the context of our model, the previous finding, in turn, implies that a given sector is both less productive and more volatile in poor countries. We test this prediction using sectoral data from the United Nations Industrial Development Organization (UNIDO, 2002). The UNIDO data set covers all manufacturing at the 3-digit level of aggregation from 1963 to 1998 for a sample of 64 countries, providing information on employment and value added on an annual basis. Table 3 indicates the countries for which the data are available and Table 5 reports the index of technological diversification for each sector, with the corresponding (average) size of the sector in manufacturing. We compute the sample average of labor productivity for each country and sector. As a measure of volatility, we use the 5-year variance of labor productivity (value added per worker) growth.

To check the consistency of the prediction, we first regress the (log of) sectoral labor productivity on the level of development, proxied by the (log of) real GDP per capita of the country, controlling for sector-specific dummies. The regression yields a positive and significant coefficient: As shown in the first column of Table 6, the point estimate for the elasticity is 0.70 (with a country-clustered standard error of 0.07). This means that, on average, any given sector is significantly less productive in poor countries.

Similarly, we regress the (log of) sectoral variance on the level of development, including sector-specific dummies. We obtain a negative and significant coefficient, displayed in the second column of Table 6. The estimated elasticity is -0.30 (with a country-clustered standard error of 0.10), implying that, on average, every sector is significantly more volatile in poor countries.

- For a given sector, productivity and volatility are negatively correlated.

Because poor countries use less complex technologies for any given sector, this implies that the within-sector relationship between volatility and productivity should be negative. To check this implication, we regress the (log) level of volatility on the (log) level of labor productivity, controlling for sectoral dummies. The estimated coefficient, reported in Table 7, is negative and significant: We obtain an elasticity of -0.29 (with a country-clustered standard error of 0.10).

Prediction 3. More complex sectors are both more productive and less volatile. A mean-variance frontier might not exist.

- More complex sectors are both more productive and less volatile.

This is a direct prediction of production with "technological diversification." To test this prediction, we use the measures of labor productivity and volatility computed from the UNIDO data set we referred to before.

Central to this test is the construction of a measure of technological complexity. Following Clague (1991), we measure the technological complexity of a sector by the diversity of inputs it uses. A sector is more complex if it uses more varieties of capital goods. There are two practical shortcomings with this measure of complexity. First, there are no comprehensive data on sector-level input usages for most countries in the sample. Second, even if the data were available, the actual extent of complexity observed would respond endogenously to the level of development of the country and the relative abundance of skilled labor.

To address these issues, we use the approach followed by Clague (1991) and Rajan and Zingales (1998) and calculate the complexity measures for sectors in the U.S. There are two key assumptions for the validity of the test we will perform: First, there are technological reasons why some industries demand a relatively larger number of capital goods than others. Second, these technological differences persist across countries, leading to a positive correlation between the rankings of technological complexity in the United States and any other given country.[22] More formally, as discussed after equation (30), we treat these complexity measures as a proxy for unobserved technological complexity that is not explained by the sector's skill intensity and relative size.

To calculate our measure of technological diversity, we use the 1997 Capital Flow Tables of the Bureau of Economic Analysis. This table distinguishes 180 capital good categories (structures, equipment, and software), each usually corresponding to a 6-digit 1997 NAICS category. We then measure technological diversification as the inverse the Herfindahl index of investment expenditure shares. Table 5 reports the (log) technological diversification index for each of the sectors in our sample.

The simple correlation between (log of) labor productivity and our index of technological diversity is positive and statistically significant (without and with country-specific dummies). However, one might argue that this strong positive correlation might be driven by other determinants. For example, capital intensity is likely to be correlated with the level of technological diversification and also to influence productivity. Incidentally, our model also predicts that the skill intensity of the sector also influences the productivity of the sector. The first column in Table 8 shows the within-country regression results, after controlling for the additional potential determinants of labor productivity. We control for the share of materials in the sector, its skill and capital intensity (measured by the share of skilled or semi-skilled workers in production and the value of equipment per worker, respectively), and the relative

[22] A meaure of technological complexity based on the U.S. is a noisy measure of the complexity of a sector in other countries. As Raddatz (2003) argues, to the extent that the noise corresponds to classical measurement error, the coefficients we are interested in will be biased towards zero, against the hypothesis of our study.

size of the sector. The regression shows that technological diversification is significantly and positively correlated with the level of labor productivity. A one-standard-deviation increase in the measure of technological diversification is associated with a 3 percent increase in the level of productivity. Also in line with our predictions, skill intensity raises productivity.

The same considerations stated before lead us to include a similar set of controls in the regression of (log) variance on the extent of technological diversification. The results are summarized in the second column of Table 8. Technological diversification is significantly and negatively associated with sectoral volatility. A one-standard-deviation increase in technological diversification is associated with a 4 percent decrease in the volatility of the sector. Volatility also decreases with skill intensity and, as we later document in more detail, the size of the sector.

- There is no evidence of a mean-variance frontier.

As discussed before, portfolio-view models predict a positive correlation between mean productivity and variance. However, in the data, the simple correlation between volatility and productivity is negative (-0.10 and significantly different from zero). Controlling for sectoral size, and country- and sector-specific effects yields no positive relationship between the two variables. Using a different approach, Koren and Tenreyro (2004) also reject the notion that countries move along a mean-variance frontier in the data.

Our model is consistent and, in fact, predicts the absence of a mean-variance frontier: As countries develop, they use more sophisticated technologies, which leads both to higher productivity and lower variance.

Prediction 4. Poor countries have a comparative advantage in less complex and hence riskier sectors. Consequently, poor countries specialize in less technologically complex sectors. This also implies that poor countries specialize in more volatile sectors.

- Poor countries have a comparative advantage in less complex and hence riskier sectors. Consequently, poor countries specialize in less technologically complex sectors.

As seen in Sections 2.3 and 2.4, skill intensive sectors will endogenously become more complex. This implies that skill abundant countries have a comparative advantage in complex sectors. Note that even a small difference in skill abundance can result in a large comparative advantage because of directed technical change.

That poor countries have a comparative advantage in less complex sectors was first documented by Clague (1991). He finds that poor countries are relatively less efficient in industries with a lower index of technological complexity (where complexity is measured similarly to the method employed in the present paper).

This pattern of comparative advantage, according to the model, implies that poor countries should specialize in less complex sectors. We checked this implication, by examining the sectoral composition of the economy. Using the UNIDO data set, we regressed the (log) average sectoral shares on *a*) the index of technological diversification of the sector; *b*) the level of development, proxied by the (log) level of GDP of the country; and *c*) the interaction between sectoral variance and the level of development. According to the model, the interaction term should be positive: As countries develop, they should move to more complex sectors. The results are displayed in Table 9. The interaction term is positive and significantly different from zero, consistent with the model.

- Poor countries specialize in more volatile sectors.

To check whether the pattern of comparative advantage might also imply that poor countries specialize in relatively more volatile sectors, we regress the (log) average sectoral shares on *i*) the variance of the sector; *b*) the (log) of GDP of the country; and *c*) the interaction between sectoral variance and the level of development. As the model predicts, the regression yields a negative and significant coefficient for the interaction term, implying that more developed countries tend to specialize in lower-variance sectors. The results are displayed in Table 10, which shows the regressions without and with country-fixed effects.

Prediction 5. Larger sectors, in which the country has a comparative advantage are less volatile.

Profits for an individual machine owner are larger in large sectors (with more machine operators), *ceteris paribus*. Hence more machines will be attracted toward large sectors, making them less volatile. (See equation (30).) The size of the sector, in turn, is determined by its comparative advantage, implying that sectors with a comparative advantage are less volatile than sectors with comparative disadvantage.

Table 11 shows that sectors with a larger share of employment are less volatile even when controlling for country and industry fixed effects. This remains true of we control for other sectoral characteristics such as capital and skill intensity, and technological complexity (Table 8).

Canning, Amaral, Lee, Meyer and Stanley (1998) explored the relationship between GDP volatility and the size of the economy, finding that variance falls with size with an elasticity of about 1/6. We find very similar elasticities for the size of a *sector*. Note that if all risks are idiosyncratic to individual workers or machines, the fall in volatility should be faster, with an elasticity of 1. Canning et al. argue that interactions across economic actors magnify the aggregate importance of idiosyncratic risk. An alternative explanation for why idiosyncratic shocks are important in the aggregate is provided by Gabaix (2004). He shows that if the size distribution of firms has an infinite variance (such as, for example, a Pareto distribution), the decay of idiosyncratic risk with respect to size is slower. In our model, we can account for the slow decay of volatility with size if we assume that each machine has a random productivity drawn from a Pareto distribution. Then, we will have few very productive machines employing many operators. Idiosyncratic shocks to these machines then have a large effect on aggregate productivity.

In our model, the size of a sector is endogenously determined by comparative advantage: sectors that use the abundant factor intensively are relatively larger. Instead of identifying all the necessary factors of production for each sector, we measure the "revealed" comparative advantage (Balassa 1965) of sectors as

$$\text{RCA}_{is} = \frac{X_{is}/X_i}{X_{ws}/X_w},$$

where X_{is}/X_i is the share of sector s in total manufacturing exports of country i and X_{ws}/X_w is the same sectoral share for the world. We use exports data from the Trade and Production Database of the World Bank, which merges product-level trade flow data from UN Comtrade with sector level production data from UNIDO. World export is measured as total exports of the 64 countries for which trade data exist in all 28 manufacturing sectors.

Table 12 reports the results of regressing log variance of sectoral productivity on the log of its revealed comparative advantage. Comparative advantage is associated with significantly lower volatility even when controlling for country and sector fixed effects. A one-standard-deviation increase in revealed comparative advantage leads to 8 to 14 percent lower variance.

3.1 Robustness

In this section, we conduct a number of robustness checks for our empirical results. First, some institutions may facilitate the response to external sectoral shocks. Since rich countries have better institutions, this may contribute to lower output volatility. We therefore look at the role of financial development and labor market flexibility in reducing volatility.

Financial development makes raising capital cheaper and faster. Hence if firms are hit by liquidity shocks, they can borrow the necessary funds without significantly disrupting production. This can make the productivity of firms smoother, especially over shorter horizons. Aghion, Angeletos, Banerjee and Manova (2004) show how the liquidity needs of long-term investments make output volatile in financially underdeveloped countries. Empirically, Braun and Larrain (forthcoming) and Larrain (2004) have shown that financial development makes output less volatile, especially in highly finance dependent sectors.

Our model can easily incorporate the pattern that volatility declines with financial development. The development of new inputs requires financing, because initial development/installation costs have to be covered up front. The value of new machines will hence be higher in financially developed countries where the cost of capital is lower, making these countries less volatile. Across sectors, differences in financing needs ("external finance dependence") lead to similar predictions.

Column 2 of Table 13 reports the regression of sectoral variance on the level of GDP and the degree of financial development, gauged by private credit over GDP. We control for sector-specific fixed effects. Financial development leads to significantly lower volatility, but the effect of general economic development also remains significantly negative.

Our measure of volatility is the variance of labor productivity (value added per worker) growth. This may be higher in countries with rigid labor markets, because firms are less able to react to demand shocks. For example, if the demand for the product of a particular firm falls, optimally it would downsize its workforce. However, firing costs and regulations make this costly, so the firm retains its workers in the hope that the shock is transitory. In the data we would observe this shock as if it were a negative productivity shock; less output is produced with the same number of workers.

To see whether this measurement problem contaminates our results, we control for the costs of modifying and terminating employment contracts across countries, as compiled by Botero, Djankov, La Porta, Schleifer and Lopez-de-Silanes (2004). As Column 3 of Table 13 shows, labor market rigidities significantly increase the volatility of labor productivity within any given sector. However, this does not alter our prediction that volatility declines with development; in fact, the point estimate of the coefficient of GDP is greater in absolute value. Intuitively, some highly developed countries have rather rigid labor markets (notably European countries) but are still highly stable in terms of labor productivity.

In Column 4, we control for both financial development and labor market rigidities. The effect of overall economic development is still highly significant with a coefficient very similar to our benchmark estimates.

An alternative explanation for the decline of volatility with development is that high-income countries specialize in differentiated products, which are subject to idiosyncratic demand and supply shocks. This could result in lower volatility because idiosyncratic shocks wash out when aggregated over many products. Also, if sectors producing multiple differentiated products use a wider variety of inputs, then "output diversification" is correlated with "input diversification," which potentially biases our results on technological diversification.

To test for the presence of output diversification, the use of firm-level data would be desirable. Lacking such data, however, we can use data on the number of establishments reported by UNIDO. If products are differentiated by producer firms and these products are subject to idiosyncratic demand or supply shocks, the volatility of a sector should decline with the number of firms.

Our model also predicts that larger sectors should be less volatile. The distinction between the two theories relies in the margin through which this takes place. Output diversification takes place across firms, hence volatility declines with the number of firms (extensive margin) but not with the average size of firms (intensive margins). In our model, larger firms attract proportionately more machines, and hence both margins are equally important.

To test the relative importance of the extensive and intensive margins, we decompose the size of sector s in country i, L_{is}, into the number of firms and the average size of firms,

$$\ln L_{is} = \ln N_{is} + \ln(L_{is}/N_{is}).$$

We then regress the log variance of a sector on the log number of firms and the log of their average size, controlling for both country- and industry-specific fixed effects. The output diversification would suggest that the number of firms decreases volatility while firm size does not. Table 14 reports our results. Both the extensive and the intensive margins of sector size contribute to lower volatility, and, in line with our theory, there is no significant difference in the importance of the two. Moreover, when we only focus on "complex" sectors, where product differentiation may be more prevalent, there are still no significant differences. This suggests that "output diversification" does not significantly contribute to the decline in volatility.

4 Conclusion

This paper proposes a model in which the production process makes use of different input varieties, which are subject to imperfectly correlated shocks. As in other expanding variety growth models, technological progress takes place as an expansion in the number of input varieties, increasing productivity. The new insight we develop is that the expansion in varieties also leads to lower volatility of production via two channels. First, as each individual variety matters less and less in production, the contribution of idiosyncratic fluctuations to overall volatility declines. Second, each additional input provides a new adjustment margin in response to external shocks, making productivity less volatile.

In the model, the number of varieties evolves endogenously in response to profit incentives. The consequent change in volatility associated with changes in the number of varieties feeds back into the investment and savings decisions of producers.

The model yields empirical predictions concerning the relationships among productivity, volatility, technological complexity, and comparative advantage. We discuss these predictions in light of the existing empirical evidence and provide novel findings supporting the results.

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A Approximating the Discrete State Space Markov Process with a Diffusion

In this section we formalize the diffusion approximation for the number of machines, relying on limit theorems for birth and death processes in Stone (1963) and Iglehart (1965).

Let n follow a continuous-time birth and death process with birth rate A and death rate bn. That is, we assume that a new machine is finished after an exponential time, with arrival parameter A measuring the investment into new machines. Making the arrival of machines random takes care of potential integer problems: since the arrival of a finished machine only depends on the current intensity of investment, we do not need to track how much investors have invested since the building of the last machine. In what follows, we assume A is constant.

The instantaneous mean and variance of this process are

$$\begin{aligned} \mathrm{E}(\mathrm{d}n) &= (A - bn)\,\mathrm{d}t, \\ \mathrm{Var}(\mathrm{d}n) &= (A + bn)\,\mathrm{d}t. \end{aligned}$$

Let $\xi_A = (n - A/b)/\sqrt{A}$ denote a transformed Markov process. Obviously,

$$\begin{aligned} \mathrm{E}(\mathrm{d}\xi_A) &= -b\,\mathrm{d}t, \\ \mathrm{Var}(\mathrm{d}\xi_A) &= \left(2 + \frac{b}{\sqrt{A}}\xi_A\right)\mathrm{d}t. \end{aligned}$$

What happens as A tends to infinity? The process ξ_A weakly converges in the Markov sense to a diffusion process

$$\mathrm{d}\xi_\infty = -b\,\mathrm{d}t + \sqrt{2}\,\mathrm{d}z.$$

Proof. This requires (Stone 1963) that, in any compact interval of $\mathcal{R}$, (1) the state space of ξ_A becomes dense and (2) the instantaneous mean and variance of ξ_A converge uniformly to $-b$ and 2.

(1) Let $[I_1, I_2] \subset \mathcal{R}$ be a compact interval and $x \in [I_1, I_2]$ an arbitrary point in the interval. For each A there exists an integer k_A for which

$$(k_A - A/b)/\sqrt{A} \leq x < (k_A + 1 - A/b)/\sqrt{A}.$$

The distance of x to the closest element of the state space of ξ_A is hence smaller than $1/\sqrt{A}$. That is, for any $\varepsilon > 0$, there exists an element of the state space in the ε-neighborhood of x as long as $A > 1/\varepsilon^2$. This proves that the state space becomes dense in $[I_1, I_2]$ as $A \to \infty$.

(2) Clearly, the instantaneous mean and variance of ξ_A converge to $-b$ and 2, we just need to establish uniformity. Let $\xi_a \in [I_1, I_2]$. The for any ε there exists an $A_0 = (bI_2/\varepsilon)^2$ such that $|b\xi_a/\sqrt{a}| < \varepsilon$ as long as $a > A_0$.

□

Note that if ζ_A is a diffusion process described by

$$\mathrm{d}\zeta_A = (A - bn)\,\mathrm{d}t + \sqrt{A + bn}\,\mathrm{d}z,$$

its transformation $(\zeta_A - A/b)/\sqrt{A}$ also weakly converges to ξ_∞.

That is, for large enough A, the birth and death process will be well approximated by a diffusion process with corresponding instantaneous mean and variance.

B An Example with Fixed-Coefficients Technology

In the benchmark model we assume that $\sigma \in (0, 1)$, that is, the elasticity of substitution across machine varieties is bigger than 1 (the machines are gross substitutes). This is a standard assumption in the expanding variety literature and is needed to ensure that the varieties not yet invented (or installed) are not essential in production.

However, complementarities across different inputs (or tasks) may be an important feature of the development process. As Kremer (1993) points out, many production processes feature an "O-ring" technology: even if a single input fails, it may jeopardize the whole outcome. (Also see Young (1993) and Grossman and Maggi (2000)) on the importance of complementarities for productivity patterns.) We hence consider an example in which all the machine varieties are essential in production. We show that even in the extreme form of complementarity (O-ring), technological diversification may still take place via variable capacity utilization.

In particular, the production function takes the Leontief form:

$$Y = \min_{i=1,\ldots,n} \{x_i\}.$$

The services of individual machine varieties are produced as before, with skilled operators, $x_i = h_i$. We assume, however, that the failure of the machine does not render it completely useless (otherwise log output would become minus infinity), but, rather, makes it more expensive to operate. In particular, while good machines require 1 unit of skill labor, broken machines require $\delta > 1$ units.

Let us first focus on the case without variable capacity utilization, that is, when the number of operators per machines is constant at h. This implies that when the first machine fails, output drops from h to h/δ. (Further failures have no impact on output.) So the change in log output is

$$\Delta \ln Y = -\ln \delta.$$

Since the first failures arrives after an exponentially distributed working time with an arrival rate of γn, the instantaneous variance is

$$\mathrm{Var}(\mathrm{d}\ln Y)/\,\mathrm{d}t = (\ln \delta)^2 \gamma n.$$

This is in fact increasing in n; the more complex the technology, the more likely a machine failure is. We do not have an offsetting effect from the law of large numbers because the working machines do not substitute for the broken one.

Consider now the case with variable capacity utilization. In this case, we let firms reshuffle operators across machine varieties, only holding the total number of operators fixed at $H = nh$. If a machine "fails," the firm allocates more operators to that machine to partially offset this negative productivity shock. With free reallocation of operators, it is optimal to equalize the services of each machine variety at, say, x. This requires δx operators on the broken machine and $(n-1)x$ operators on the rest. The total number of operators is unchanged, so

$$\delta x + (n-1)x = nh.$$

The change in log output is hence

$$\Delta \ln Y = \ln n - \ln(n - 1 + \delta),$$

which is negative, that is, output still drops but it drops by less than without VCU. The firm can successfully mitigate some of the impact of the shock with VCU.

The instantaneous variance in this case is

$$\mathrm{Var}(\mathrm{d}\ln Y)/\,\mathrm{d}t = [\ln n - \ln(n-1+\delta)]^2 \gamma n.$$

The first part decreases with n. The more machine varieties there are, the more possibilities there exist to reshuffle operators without affecting output too much. The second part is increasing in n because more complex technologies fail more often. In general the effect of technological complexity on volatility is hence ambiguous. Nonetheless, as complexity increases without bound ($n \to \infty$), the first effect dominates and volatility goes to zero,

$$\lim_{n\to\infty} \mathrm{Var}(\mathrm{d}\ln Y)/\,\mathrm{d}t = 0.$$

To see this, use the intermediate value theorem to rewrite $[\ln n - \ln(n-1+\delta)]$ as $(\delta - 1)/[n + \xi_n(\delta - 1)]$, where $\xi_n \in [0,1]$. Since ξ_n is bounded, $[\ln n - \ln(n-1+\delta)]^2 = O(n^{-2})$ and $\gamma n = O(n)$.

In summary, the ability to vary capacity utilization can make more complex technologies less volatile even in the case of fully complementary inputs.

C Data Appendix

Variable Definitions

GDP per capita GDP per capita of the country in 1997, measured in 1995 international dollars. [WDI, PWT]

Population Population of the country in 1997. [WDI]

Yield volatility Variance of the log of annual wheat yield. [FAOSTAT]

Rainfall volatility Variance of cumulated log changes in precipitation. Precipitation data are recorded monthly at several meteorological stations within a country. Because many stations do not report data in all months, we take the average of year-on-year changes for all months and all stations within the country. We cumulate these changes to obtain the country's deviation from long-run precipitation trends. [Global Historical Climatology Network]

Temperature volatility Variance of cumulated changes in temperature, calculated in the same way as rainfall volatility. [Global Historical Climatology Network]

Change in oil price Two-year change in the U.S. CPI-deflated price of West Texas Intermediate oil.

Diverse powerplants dummy Takes the value of one if the concentration of powerplants by type (conventional, hydroelectric, nuclear, renewable) in the country is below the median. [International Energy Annual]

Technological Diversification The log of the inverse of the Herfindahl index of concentration of equipment purchases across different varieties of capital goods. A sector has a high Technological Diversification index if it purchased many different capital goods. [1997 Capital Flow Tables]

Average Share in Manufacturing The sector's share in manufacturing employment, averaged across the sample period, 1963–1998. [UNIDO]

Labor Productivity Value added per worker in 1995 dollars, averaged across the sample period, 1963–1998. [UNIDO]

Variance of Productivity The variance of 5-year growth of value added per worker in 1995 dollars across the sample period, 1963–1998. [UNIDO]

Skill Intensity The fraction of production workers in the 3-digit ISIC sector that are employed in skilled or semi-skilled occupations. [Occupational Employment Statistics]

Share of Materials The share of intermediate inputs in total sales. [NBER-CES Manufacturing Industry Database]

Equipment per Worker, Structure per Worker [NBER-CES Manufacturing Industry Database]

Revealed Comparative Advantage The share of sector s in country i's manufacturing export relative to the world average. [Trade and Production Database]

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1 Introduction

Economies at early stages of the development process are often shaken by abrupt changes in growth rates. In his influential paper, Lucas (1988) brings attention to this fact, noting that "within the advanced countries, growth rates tend to be very stable over long periods of time," whereas within poor countries "there are many examples of sudden, large changes in growth rates, both up and down." This negative relationship between the volatility of growth rates and the level of development is illustrated in Figure 1, which plots the standard deviation of annual growth rates against the level of real GDP per capita for a large cross section of countries.

In an attempt to understand the sources of volatility, Koren and Tenreyro (2004) quantify the contribution of various factors at different stages of development, finding that the high volatility in poor countries is due to 1) higher levels of sectoral concentration, 2) higher levels of sectoral risk (that is, poor countries not only specialize in few sectors, but those sectors also tend to bear particularly high risk), and 3) higher country-specific macroeconomic risk. A volatility accounting exercise carried out by these authors indicates that approximately 50 percent of the differences in volatility between rich and poor countries can be accounted for by differences in the sectoral composition of the economy (higher concentration and sectoral risk), whereas the other 50 percent is due to country-specific risk. These characteristics of the development process, as we later explain, are inconsistent with previous theoretical explanations of the dynamics of volatility and development. The purpose of this paper is to provide an alternative theory that is in line with the empirical evidence.

To that end, we develop an endogenous growth model of technological diversification. The key idea of the model is that firms using a larger variety of inputs can mitigate the impact of shocks affecting the productivity of individual inputs. This takes place through two channels. First, with a larger variety of inputs, each individual input matters less in production, and productivity becomes less volatile by the law of large numbers. Second, whenever a shock hits a particular input, firms can adjust the use of the other inputs to partially offset the shock. This second channel operates even if production exhibits an extreme form of complementarity (as in Kremer (1993)'s O-ring technology). Both channels make the productivity of firms using more sophisticated technologies less volatile.

The idea can be illustrated with an example from agriculture: Growing wheat with only land and labor as inputs renders the yield vulnerable to idiosyncratic shocks (for example, weather shocks such as a severe drought). In contrast, using land and labor together with artificial irrigation, fertilizers, pesticides, etc., makes wheat-growing not only more productive but also less risky, because farmers have more options to react to external shocks. Figure 2 provides a graphical illustration of this example. It displays the volatility of wheat yield (calculated as the standard deviation of percentage deviations from the country's average yield) of the 20 biggest wheat producers against their level of GDP per capita.[1] Yield volatility falls sharply with development. This remains true if we control for differences in climate across countries, including the volatility of rainfall and temperature (see Table 1).

The shocks affecting individual inputs or individual production techniques may come from various sources. Another example of such a shock could be a sudden change in the price of a major input of a production technique. Countries with a diverse set of available techniques can cope better with the shock. For instance, the types of power plants that countries rely on to generate electricity vary with development. Small and less-developed countries have only a few plants very highly concentrated on one particular technique of electricity production (employing either traditional thermal or hydroelectric plants). Developed countries, on the other hand, have access to nuclear and renewable-resource plants and are typically more diversified. Firms in these countries will react differently to oil price shocks. Table 2 reports how the electricity production of countries responds to oil price changes. The electricity production of less-developed and small countries concentrated on few types of power plants is significantly more sensitive to oil price shocks than that of countries with a diverse set of plants. More specifically, while the electricity production of countries concentrated on a single energy source drops by about 1 percent after a 30 percent oil price hike, there is no such drop for diversified countries. Firms in countries with diverse sources of electricity can mitigate the negative impact of an oil price shock by substituting away from oil. The share of oil in total energy consumption falls by 0.3 percent after a 30 percent oil price hike, whereas no substitution takes place in concentrated countries.

We next turn to the questions of what determines technological diversification and why poorer countries specialize in less sophisticated sectors. We extend the model to allow for international mobility of goods and for cross-country differences in endowments. Much as in models of endogenous growth and directed technical change, the technological complexity of a sector in a given country evolves endogenously in response to the incentives of the creators and users of new technologies. In particular, more input varieties will be directed towards sectors in which the country has a comparative advantage, making them more complex and less volatile. The stage of development of the country will also matter, because inventing and/or using the new inputs is subject to increasing returns to scale. Countries accumulate new inputs as they develop, which brings about a gradual decline in their volatility. The speed of development, and hence the speed with which volatility declines, may be influenced by the initial level of volatility. If investment risk is harmful for growth, which is the case for a range of plausible parameter values in our model, then poor and volatile countries will

[1] Note that agricultural technology varies substantially with development. For example, of the top 20 wheat producers, India uses 2.3 tractors per 1,000 acres of arable land; this number is 128.8 for Germany. Fertilizer use also varies hugely. India uses 21.9 tons of nitrogenous fertilizers per acre; Germany uses 183.8 tons. We take the level of development as an overall indicator of agricultural sophistication.

develop slower and will remain highly volatile for long periods.[2]

The model delivers clear-cut predictions about the relationship among technological diversification, volatility, and productivity. Using sector-level data for a broad sample of countries, we provide empirical support for these predictions. First, any given sector is less volatile in developed countries. This result holds if we control for the quality of institutions which may facilitate a smoother response to external shocks, such as financial development and the flexibility of the labor market. Second, within a given country, large, skill intensive sectors using complex technologies are less volatile. This is consistent with our model which says that new inputs/technologies will be directed towards such sectors, thus reducing volatility. These two mechanisms lead to a decline in aggregate volatility as a country develops: The economy experiences less volatility in each sector and resources move towards relatively safer sectors.

The link between volatility and development has been studied before by Acemoglu and Zilibotti (1997), Greenwood and Jovanovic (1990), Saint-Paul (1992), and Obstfeld (1994), who describe the technology choice as a portfolio decision: In order to reap the benefits of high productivity and high growth, an economy has to bear more risk. The risk tolerance typically relates to the level of development and the financial structure of the economy. Acemoglu and Zilibotti (1997)'s model also features increasing returns to scale: Early in the development process diversification opportunities are limited, owing to the scarcity of capital and the indivisibility of investment projects. This feature can explain the high levels of sectoral concentration observed in poor countries. However, all these models predict that at early stages of development countries will tend to specialize in safer (even if less productive) sectors as a way of seeking insurance. This prediction is not borne out by the data: Koren and Tenreyro (2004) document that poor countries are highly concentrated in sectors that bear particularly high volatility. In addition, these authors find that most developing countries are inside the "mean-variance frontier," being highly prone to specialize in high-variance, low-mean sectors. These findings contradict the predictions of the portfolio-based models and suggest that important constraints must be at play, preventing developing countries from investing in safer and, at the same time, more productive assets.[3]

Our model departs from the portfolio view of the world that features a necessary trade-off between volatility and performance at the sector level. It can then naturally accommodate the fact that poor countries tend to exhibit high sectoral concentration and also that the high concentration falls mainly on high-risk sectors. In addition, unlike in previous contributions,

[2]See Angeletos and Calvet (2001) and Angeletos (2004) for a discussion of how volatility affects investment. Note, however, that in these papers there is no explanation for why volatility is higher in the first place. See also Ramey and Ramey (1995) on the empirical evidence.

[3]Kalemli-Ozcan, Sørensen and Yosha (2003) and Imbs and Wacziarg (2003) document that, for highly developed countries, industrial specialization tends to increase with development. However, as we later show, this does not result in higher aggregate volatility because these sectors tend to be technologically diversified and are hence more stable than the rest of the economy. The fact that the higher specialization of rich countries does not increase their aggregate risk has also been shown by Koren and Tenreyro (2004).

the volatility of individual sectors in our model is endogenous: It depends on the level of development and the comparative advantage of the country.[4]

Our paper is related to previous work by Kraay and Ventura (2001). As in their paper, the open-economy version of our model features the prediction that rich countries have a comparative advantage in less-volatile sectors. The difference lies in the way this result is achieved. In Kraay and Ventura (2001), high-skill sectors, which are prevalent in developed countries, enjoy less-elastic product demand. Markups can then serve as a buffer against productivity shocks, reducing the volatility of high-skill sectors. For example, a drop in output of a differentiated product makes that product more expensive in the world market. This terms of trade improvement partly offsets the original shock. On the other hand, no such "terms-of-trade insurance" is taking place for homogenous products that poor countries specialize in.

There are, however, empirical objections to the mechanism proposed by Kraay and Ventura (2001) and its implications. The model predicts a negative relationship between productivity shocks and terms-of-trade fluctuations (particularly negative for developed countries). That is, negative productivity shocks should be associated with an improvement in the terms of trade. In the data, however, the relationship between fluctuations in labor productivity and the terms of trade is somewhat positive, and there is no difference between rich and poor countries in terms of this relationship.[5]

Finally, our model builds on a vast literature on endogenous growth models in which the development of new varieties of goods enhances productivity. (See for example, Romer (1990) and Grossman and Helpman (1991).) The contribution of our paper is to provide a unified framework for the explanation of development and volatility. We provide sectoral evidence for a broad cross-section of countries that confirms the predictions of the model.

The remainder of the paper is organized as follows. In Section 2 we present the model. In Section 3 we discuss the empirical implications and offer novel evidence in support. We summarize and conclude in Section 4.

2 A Model of Technological Diversification

2.1 Technological diversification, productivity, and volatility

In this section, we introduce a production process that features technological diversification: Input varieties contribute not only to higher productivity but also, because inputs are subject to imperfectly correlated shocks, to lower volatility.

[4]As Koren and Tenreyro (2004) have shown, differences in the sectoral composition of developed and less-developed countries account for about 50 percent of the difference in volatility.

[5]It is possible that other factors are at play, blurring the predicted relationship; at this point, nonetheless, we can say that the extent of countercyclicality in the terms of trade is not the *prima facie* mechanism behind the negative relationship between development and volatility.

Output Y is produced using a composite of "machine varieties" with a constant-elasticity-of-substitution (CES) technology,

$$Y = \left[\sum_{i=1}^{n} X_i^{\sigma} \right]^{1/\sigma}, \tag{1}$$

where X_i is capital services from capital variety i , n denotes the number of working machines and $1/(1-\sigma) \in (0, \infty)$ is the elasticity of substitution across varieties.[6]

Machines can fail randomly, in which case they irreversibly cease to contribute to production. We assume that failure occurs independently across machines and time periods with probability $\gamma\, dt$. That is, the lifetime of a machine is exponentially distributed with parameter γ. For our argument we need only that failures are imperfectly correlated. We take the extreme assumption of independence for expositional clarity. The assumption that random failures turn the machine completely useless makes the model more tractable, since we need only keep track of the number of working machines. However, technological diversification would still take place with less terminal shocks: Appendix B considers an example where there is only a partial drop in productivity after a machine failure.

In an open-economy context, changes in the world price of the input or sudden disruptions of trade could also be an important source of input-specific shocks. Technological diversification can mitigate the impact of such shocks.

Using machines in production involves increasing returns to scale: Machines are indivisible. This means that anyone operating a machine has to buy one unit of the machine beforehand. This minimum scale requirement limits the scope of diversification across machine varieties.[7]

Since we are interested in the inner workings of a sector and how technology choice affects volatility, we posit increasing returns at the input level. Indivisibility and minimum scale requirements are inherent characteristics of many an input used in technologically advanced sectors. Note that increasing returns are also a feature of the *use* of the machines, not only their invention or production. That is, we assume that machines can be produced and bought in any quantity but only a full unit is productive.

The setup of a machine requires κ units of the final good. Once the machine is set up, the owner gains monopoly power over its services. This monopoly lasts until the machine (exogenously) becomes obsolete, that is, the lifetime of the "patent" is the same as the lifetime of the machine and is exponentially distributed with parameter γ.

[6]As usual in endogenous growth models, we assume that $\sigma > 0$, that is, machines are gross substitutes. Appendix B considers an example when this is not the case. Introducing additional (scarce) factors of production would not change our qualitative results, it would just make the returns to variety more decreasing.

[7]Note that there is no incentive to install two or more units of a single machine variety, both because the production function features a "love of variety" and because machines are subject to idiosyncratic shocks. A similar assumption is made by Acemoglu and Zilibotti (1997) who work with minimum scale requirements at the industry level.

We assume that the machine can be used with different intensities by employing "operators." Machine i can provide twice as much service if operated by twice as many workers.[8] Producing a unit of capital service requires one unit of labor (by appropriate definition of labor units).

Formally, the services of machine i at time t are:

$$X_{it} = \begin{cases} l_{it}, & \text{if } K_{i0} = \kappa \text{ and } t < T_i; \\ 0, & \text{otherwise;} \end{cases} \tag{2}$$

where K_{i0} is the amount of capital devoted to machine variety i, l_{it} is the number of operators, and T_i is the exponentially distributed lifetime of the machine.

Consider the output of a firm, using n types of machine services, with X_i units of each,

$$Y = n^{1/\sigma} X_i. \tag{3}$$

As is apparent from (3), productivity is increasing in the number of varieties holding the amount of each individual variety fixed. This is the usual "love of variety" effect of many endogenous growth models (Romer 1990, Grossman and Helpman 1991). The effect is stronger the lower is σ, that is, the less substitutable machines are. Intuitively, if machines are highly substitutable, any additional variety is less needed.

As X_i also denotes the number of operators working on machine i, the overall number of machine operators working at the firm is $L = nX_i$. Hence (3) can be rewritten as

$$Y = n^{1/\sigma - 1} L. \tag{4}$$

Productivity is also increasing in the number of machines if we hold the total number of operators (L) constant (since $\sigma < 1$). The dependence is weaker than in (3), because any new machine requires operators taken away from old machines.

This implies that we have two alternative definitions of productivity, one holding the operators *per machine* constant, the other holding the *total* number of operators constant. We think both measures are useful, since the adjustment across different machine varieties can take place relatively fast within the firm (in particular, no hiring or firing of workers or capital installation is needed).[9]

Given that the number of machines is a random variable (individual machines fail at random and there is a finite number of machines), productivity will be random, too. What

[8]This is a way of capturing endogenous capacity utilization which is recently emphasized in business cycle studies. Allowing for capacity constraints or decreasing returns to capacity utilization would not alter our setting qualitatively. First, capacity constraints would not bind in equilibrium. Economic growth takes place via the expansion of machine varieties while the services of an individual variety shrink. Second, investors will be interested in the *total*, not the *marginal* profit when deciding whether to build a machine. This will remain positive even with decreasing returns to scale. Moreover, if the cost function were isoelastic, the share of profit in total revenue would be constant, just as in the present formulation.

[9]The effectiveness of this margin depends on how quickly and how efficiently machine operators can switch between different machines. Our assumption that any worker can operate any machine captures the

happens to output when a machine fails? First, the number of machines becomes $n-1$, making output lower for any given X_i. However, the demand for the services of an individual machine will also change. Again, it will be important to distinguish between the two measures of productivity. First, if we hold the number of operators per machine constant, productivity drops from $n^{1/\sigma}$ to $(n-1)^{1/\sigma}$. However, if we allow operators to be reallocated evenly among the remaining machines, the drop in output will be smaller, because we have allowed the firm to adjust the capacity utilization of the remaining machines in response to the shock. Productivity drops from $n^{1/\sigma-1}$ to $(n-1)^{1/\sigma-1}$, so the proportional drop is smaller.

The variance of productivity changes declines with the number of machines for both productivity measures. In the first case, this is just an application of the law of large numbers: Since output is an average (more precisely, a CES aggregate) of imperfectly correlated individual machine services, proportional changes in productivity become less and less volatile as the number of machines increases. The second case displays an additional effect: The more machine varieties the firm uses, the better they can respond to shocks by employing the remaining inputs at different intensities. Qualitatively, both effects imply that volatility declines with "technological complexity" (n). However, important differences arise when individual inputs are complements rather than substitutes of each other (such as in the O-ring production function of Kremer (1993)). Then the law of large numbers does not apply, because aggregate productivity is no longer an average of individual productivities. Still, as we demonstrate in Appendix B, volatility will fall with technological complexity if we allow for the second margin, variable capacity utilization.

To derive the variance of productivity formally, let a denote the log of productivity when the number of operators *per machine* (X_i) is held constant.

$$a = y - x_i = \frac{1}{\sigma}\ln n.$$

(Lower-case letters denote logarithms.) On the other hand, if we hold the total number of operators constant at L, we have $\tilde{a}$, the log of productivity, allowing for *variable capacity utilization* (VCU).

$$\tilde{a} = y - l = \phi \ln n_i,$$

where we have introduced the notation $\phi = 1/\sigma - 1$.

Our measures of volatility will be the variance of the changes in these two TFP variables:

$$\text{Vol} = \text{Var}(\text{d}y|n, X_i) = \text{Var}(\text{d}a|n) = \left(\frac{1}{\sigma}\right)^2 \text{Var}(\text{dln}\, n|n),$$

$$\text{Vol}_{\text{VCU}} = \text{Var}(\text{d}y|n, L) = \text{Var}(\text{d}\tilde{a}|n) = \phi^2\, \text{Var}(\text{dln}\, n|n).$$

extreme case when such a switch is immediate and fully efficient. In reality, of course, we would see less than perfect flexibility. However, as the skills needed to work with advanced technology are very diverse (for example, Autor, Levy and Murnane (2003) document that computerization increased the demand for non-routine cognitive tasks), we believe that such adjustment is important in practice.

Let us for the moment assume away growth in the number of machines and study what happens to existing machines over time. (We introduce growth in the next section). The number of machines, in the absence of investment, changes because machines break at random.[10] Given that machine lifetimes are exponentially distributed with parameter γ and lifetimes are independently distributed, the first failure comes after an exponentially distributed time with parameter $n_t\gamma$.

$$T_k \sim \exp(\gamma),$$
$$T_{\min} = \min_{k=1,\ldots,n} \{T_k\} \sim \exp(n_t\gamma).$$

The first failure reduces the number of machines by one, so

$$n_{t+h} = \begin{cases} n_t & \text{if } h < T_{\min}, \\ n_t - 1 & \text{otherwise.} \end{cases}$$

The probability of no machine failure over a period of length h is $1 - e^{-n_t\gamma h}$. The expected change in the number of machines is

$$\mathrm{E}(n_{t+h} - n_t | n_t) = -e^{-n_t\gamma h},$$

whereas the variance of the change is

$$\mathrm{Var}(n_{t+h} - n_t | n_t) = e^{-n_t\gamma h}\left(1 - e^{-n_t\gamma h}\right).$$

If we take $h \to 0$, we get the instantaneous mean and variance of the number of machines,

$$\mathrm{E}(\mathrm{d}n_t | n_t) = -\gamma n_t \, \mathrm{d}t,$$
$$\mathrm{Var}(\mathrm{d}n_t | n_t) = \gamma n_{it} \, \mathrm{d}t.$$

More formally, in the absence of investment, the number of machines follows a continuous-time, discrete-space Markov process known as a "pure death" process with death rate γn_t. Such a process can be well approximated by an Itô process if n_t is large. (See Appendix A.) That is, for large n_t, the changes in the number of machines in a dt period of time will be approximately normal. This is just a version of the central limit theorem for discrete-space Markov processes. In the next section we allow for growth in the number of machines. The economy will then exhibit long-run growth in n_t, implying that the approximation will get better and better over time.

[10] Investment in new machines also changes the number of machines, but only gradually, because only a finite flow of investment will be allocated to new machines at any point in time. That is, investment does not contribute to the *volatility* of the number of machines. Formally, investment follows a bounded variation process. Even if we allowed for "jumps" in the number of new machines (say, investors could borrow a whole machine abroad), we are interested in the productivity shock before such an investment response takes place. Note that we are also assuming away integer problems of investment here. See Appendix A for a formal treatment of the number machines as a discrete-state Markov process.

Consequently, abstracting from growth, we can express the evolution of n_t using the following stochastic differential equation,

$$\mathrm{d}n_t = -\gamma n_t \,\mathrm{d}t + \sqrt{\gamma n_t}\,\mathrm{d}z,$$

where $\mathrm{d}z$ is the increment of a standard Wiener process. Given this approximation for $\mathrm{d}n_t$, we can use Itô's lemma to write $\mathrm{d}\ln n_t$ as follows:

$$\mathrm{d}\ln n_t = -\gamma(1 + 0.5/n_t)\,\mathrm{d}t + \sqrt{\gamma/n_t}\,\mathrm{d}z.$$

What is important here is that the volatility of the log number of machines declines with the existing number of machines. Even though as n_t gets big, the first failure gets more and more likely, the proportional (that is, log) drop in the number of machines it induces is less and less important. As is standard in statements of the law of large numbers, the second effect outweighs the first one. In other words, diversification across several machines makes log productivity less volatile.

Given that n_t measures the number of inputs subject to different shocks, we take it as an index of *technological complexity*. It is clear from (3) and the discussion above that technological complexity both increases average productivity and reduces the volatility of productivity. In the next section, we endogenize the investment in new machines, and consequently, the resulting level of technological complexity.

2.2 Endogenous technological complexity

What determines the level of technological complexity in the long run? In this section we endogenize the decision to invest in machines. Much as in models of endogenous growth (Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992), machine owners will be attracted by greater profit opportunities.

We first look at a one-sector economy to bring out the relationship between volatility and development clearly. In Section 2.3, we introduce multiple sectors and investigate how the relative complexity of sectors evolve endogenously. As we have documented in Koren and Tenreyro (2004), intrinsic volatility differences across sectors together with countries' different patterns of specialization are responsible for an important portion of the difference in output volatility between rich and poor countries. As in other multi-sector models of endogenous technology, we will have directed technical change (Acemoglu 2002, Caselli and Coleman 2000). Profits per machine variety will depend on the size of the sector (number of available operators), its relative wage, the degree of competition (number of existing machines), and trade openness.

Technology will be the same as in (1), which results in the following aggregate production function for the final good (4):

$$Y = n^{\phi} L.$$

The economic environment is characterized as follows. The final good sector is perfectly competitive, that is, firms take output and input prices as given. In contrast, machine providers act as monopolistic competitors, that is, they are price setters for their own machine but take the overall price of the composite machine varieties as given.[11]

There is a continuum of symmetric consumers/investors with unit mass. Each consumer has access to the well-diversified mutual fund of all machines. They can trade the share of this mutual fund freely, instantly, and in any quantity (even shorting is allowed). This will ensure that the mutual fund is priced by the consumer's stochastic discount factor. In other words, we assume no frictions in the domestic financial market. Note that there is no positive supply of a riskless asset in the economy, in other words, every production technology is risky.[12] Each consumer supplies L units of labor inelastically in the labor market. Consumers decide how much to consume and how much to invest in the mutual fund of machines, taking the rate of return and wage rate as given.

Time is continuous and consumers maximize lifetime expected utility over consumption with constant relative risk aversion, θ,

$$\mathcal{U}_t = \mathrm{E}_t \int_t^\infty e^{-\rho(s-t)} \frac{C_s^{1-\theta}}{1-\theta} \,\mathrm{d}s, \tag{5}$$

subject to a standard intertemporal budget constraint,

$$\mathrm{d}a_i = [(\mu_P + D/P)a_i + wL - C_i]\,\mathrm{d}t + \sigma_P a_i \,\mathrm{d}z.$$

The change in the asset holding of consumer i, $\mathrm{d}a_i$, comes from capital gains (μ_P) and dividend yield (D/P) on the asset and from labor income (wL) minus consumption (C_i). As we later show, stock prices follow a diffusion process, so asset holdings of consumer i will also follow a diffusion with instantaneous volatility $\sigma_P a_i$. The mean and variance of the rate of return on machines generally depends on the state of the economy, that is, the number of machines.

To derive the equilibrium of the economy, let us first consider the pricing decisions of firms. The demand for machine variety i is

$$X_i = \frac{\chi_i^{-1/(1-\sigma)}}{\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}} Y,$$

which is decreasing in the variety's own price (χ_i), increasing in competitors' prices ($\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}$) and the final good output (Y). Taking the price index $\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}$ and final demand Y as given, the machine owner faces a constant elasticity demand curve with elasticity $1/(1-\sigma)$.

[11] Note that this is a valid assumption even if there is a finite number of different machine varieties. First, the market share of each machine owner falls at the rate $1/n$, whereas the standard deviation of output is of order $1/\sqrt{n}$. That is, even if n is large enough to make monopolistic competition a realistic assumption, we still have positive aggregate volatility. Second, volatility falls with the number of *independent* machine varieties, which may be smaller than the number of machine owners if some of the machines are subject to common shocks or if there are interactions across machine-operating firms.

[12] Alternatively, the rate of return on a riskless asset (for example, storage) is so low that investors do not demand a positive amount.

She will hence follow a constant markup rule when pricing its services. The optimal monopoly price of each capital service will be

$$\chi_i = w/\sigma,$$

where w is the wage rate. Final good prices are, in turn, determined from the price of the services of an individual machine and the number of machines.

Profit maximization implies that price equals marginal cost in the final good sector. We normalize the final good price to unity:

$$1 = \left(\sum_{i=1}^{n} \chi_i^{\sigma/(\sigma-1)} \right)^{1-1/\sigma} = n^{-\phi} w.$$

This implies that wages increase in productivity:

$$w = n^{\phi}.$$

Labor market equilibrium requires that the number of operators exhausts the (exogenous) labor supply,

$$\sum_{i=1}^{n} X_i = nX_i = L.$$

The markup rule also implies that profits are a constant, $(1/\sigma - 1) = \phi$, share of wages. Total wages are wL, hence the wage costs of a single machine operating firm are wL/n (by symmetry), implying that profits are

$$\pi_i = \phi w L/n. \tag{6}$$

The owner of a machine uses this profit flow to calculate the lifetime cash-flow of the machine. Investors take the number of installed capital varieties, the wage rate, capital prices, and the return on equity as given.

We assume free entry into the machine market. This means that any investor can buy κ units of final good and install a new machine variety. As long as there is positive entry, this pins down the value of a machine at $V = \kappa$.[13] We further assume that the sunk cost required to install a machine is falling with the level of technological progress, n, because of spillovers or learning-by-doing externalities. In particular, it falls at a rate $\phi - 1$ to ensure balanced growth. Expanding variety growth models usually make a similar assumption to ensure long-run growth (Grossman and Helpman 1991, Chapter 3). Alternatively, one could set $\phi = 1$ by restricting the elasticity of substitution across varieties to be 2. Barro and Sala-i-Martin (1995, Chapter 6) put restrictions on the elasticity of substitution across input varieties. Either assumption delivers balanced growth and qualitatively similar results.

[13] If $V < \kappa$, no new machines are built and the growth rate is zero. We will later verify that this is not an equilibrium.

Also, we assume that fixed costs are proportional to the overall size of the economy, L. This assumption ensures that the growth rate is not dependent on country size. (See Jones (1995) on the "scale effect" of endogenous growth models.) Recall that κ measures the unit of a machine *variety* that is subject to variety specific shocks. Arguably, bigger countries use more capital of each variety and therefore require a bigger investment. Our main results are not sensitive to this assumption. The only result that would change without this assumption is that large countries would develop faster.

Formally,

$$\kappa = \kappa_0 L n^{\phi-1}. \tag{7}$$

The dividend yield on a machine is

$$\frac{\pi_i}{V} = \phi \frac{wL}{n\kappa} = \frac{\phi}{\kappa_0}, \tag{8}$$

where we have used $w = n^{\phi}$ and $\kappa = \kappa_0 L n^{\phi-1}$. The dividend yield is higher the higher the profit rate and the lower the fixed cost of installing a machine. The assumption of falling fixed costs ensured that the dividend yield does not vanish as n increases. If the dividend yield tended to zero as n became large, we would obtain a steady-state distribution of n instead of an ever-growing economy.

Note that even if the dividend yield on a machine is constant, the rate of return is random, because there are random capital losses due to machine failures. This results in an average depreciation rate (and hence capital loss) $\gamma \Delta t$ over a period Δt, but this capital loss is random even as we take $\Delta t \to 0$. We next turn to characterizing the stochastic process driving the value of machines.

Let P_{ts} denote the period-s value of all the machines installed at time t. At the time of installation, $P_{tt} = n_t V$. However, after an exponential lifetime the first machine fails and P_{ts} becomes $(n_t - 1)V$. The first failure occurs with probability $1 - e^{-\gamma n h}$ over an interval h. Taking $h \to 0$, we get the following mean and variance for the change in the value of the machines

$$\begin{aligned} \mathrm{E}(\mathrm{d}P_t) &= -\gamma n_t V \,\mathrm{d}t = -\gamma P_t \,\mathrm{d}t, \\ \mathrm{Var}(\mathrm{d}P_t) &= \gamma n_t V^2 \,\mathrm{d}t = \gamma P_t^2/n \,\mathrm{d}t. \end{aligned}$$

We use the same arguments put forward before for the number of machines and approximate the value of machines by a diffusion process:

$$\mathrm{d}P/P = -\gamma \,\mathrm{d}t + \sqrt{\gamma/n}\,\mathrm{d}z. \tag{9}$$

Once we know the rate-of-return process for stocks (which, recall, are the only form of investment), we can use the Euler equation to determine the optimal consumption/saving policy.

The continuous-time equivalent of the fundamental asset pricing equation, $p = \mathrm{E}(Mx)$, is

$$\mathrm{E}\left[M\frac{D}{P}\,\mathrm{d}t + \mathrm{d}(MP)\right] = 0,$$

where M is marginal utility, $M_t = e^{-\rho t}C_t^{-\theta}$, D is the dividend on an asset, and P is its price. This holds whenever investors can freely trade the asset (as is assumed here).

Because of complete domestic financial markets, both marginal utility and the stock price follow a geometric diffusion process driven by the same shock $\mathrm{d}z$. The asset pricing equation then simplifies to

$$\mu_M + D/P + \mu_P + \sigma_M\sigma_P = 0,$$

where μ_M is the proportional drift and σ_M^2 is the proportional variance of marginal utility (for example, Cox and Huang (1989)). The expected change in the discounted asset price (inclusive of dividends) can be zero only if the sum of all drift terms is zero.

Applying Itô's lemma for marginal utility, $M_t = e^{-\rho t}C_t^{-\theta}$,

$$\mathrm{d}M/M = \left[-\rho - \theta\mu_c + \frac{\theta(\theta+1)}{2}\sigma_c^2\right]\mathrm{d}t - \theta\sigma_c\,\mathrm{d}z,$$

where μ_c and σ_c^2 refer to the proportional drift and diffusion of C, respectively. Marginal utility declines with impatience and with the mean consumption growth rate and increases with consumption volatility (that is, the convexity of marginal utility gives rise to precautionary saving motives). Substituting this formula in the asset pricing equation results in the following continuous-time Euler equation

$$\mu_c(n) = \frac{1}{\theta}[r(n) - \rho] - \sigma_c(n)\sigma_P(n) + \frac{\theta+1}{2}\sigma_c^2(n), \tag{10}$$

where $r(n)$ refers to the sum of dividend yield and mean price decrease of stocks.

The mean growth rate of consumption depends positively on the mean rate of return, with a coefficient equal to the elasticity of intertemporal substitution, $1/\theta$. At the same time, because the consumer's portfolio is risky, its covariance with consumption will make saving less attractive and will hence result in lower consumption growth. Given that we have complete markets, in other words, there is only one source of uncertainty in the economy, the instantaneous correlation between stock prices and consumption is 1, so the covariance is $\sigma_c\sigma_P$. Finally, since future consumption is risky, prudent consumers have precautionary savings depending on the volatility of consumption and the degree of prudence, $(\theta+1)/2$.

Recall that the mean return on the portfolio of machines is

$$r(n) = \frac{\phi}{\kappa_0} - \gamma,$$

and the volatility of stock price is

$$\sigma_P^2 = \gamma/n.$$

In general, consumption also depends on the state of the economy, n. Let $C = v(n)$ be the policy function that describes the optimal amount of consumption given the number of machines in the economy. Then by Itô's lemma,

$$\mathrm{d}C = \left[v'(n)n\mu_n + \frac{1}{2}v''(n)n^2\sigma_n^2\right]\mathrm{d}t + v'(n)n\sigma_n\,\mathrm{d}z,$$

and the Euler equation becomes

$$\mu_n\frac{v'n}{v} + \frac{1}{2}\sigma_n^2\frac{v''n^2}{v} = \frac{1}{\theta}(r-\rho) - \sigma_n^2\frac{v'n}{v} + \frac{\theta+1}{2}\sigma_n^2\left(\frac{v'n}{v}\right)^2, \tag{11}$$

where we have omitted the argument n from $v(n)$ for brevity.

The growth rate of machines, μ_n, depends on investment, which, in turn, depends on the consumption policy. By equilibrium in the final good market,

$$Y = n^\phi L = v(n) + (\mu_n+\gamma)n\kappa = v(n) + (\mu_n+\gamma)n^\phi\kappa_0 L. \tag{12}$$

Total output has to equal the value of consumption plus investment. Investment is the sum of net investment (μ_n) and the replacement of broken machines (γ).

Equations (11) and (12) together with $\sigma_n^2 = \gamma/n$ define a second-order ordinary differential equation for v, which has two linearly independent solutions. We therefore need two boundary conditions to pin down the optimal policy function, $v(n)$. One is that no consumption takes place without capital, $v(0) = 0$. The other one comes from the fact that as n becomes arbitrarily large, σ_n^2 becomes zero and the economy resembles a non-stochastic Ramsey model. Consumption growth in the non-stochastic Ramsey model is given by $\tilde{\mu}_c = r - \rho$, so we should have

$$\lim_{n\to\infty}\mu_n\frac{v'n}{v} = \tilde{\mu}_c = r-\rho.$$

To obtain an analytical solution, we put restrictions on the CRRA and the elasticity of substitution across machine varieties and assume that $\theta\phi = 1$.[14] Whether this is plausible depends on how broadly we interpret machine varieties. If these are different intermediate inputs necessary to produce a particular good, the inputs may be strong complements, in which case the elasticity is less than one. This would lead to a negative ϕ which we have ruled out (but see Appendix B for an example of such a production function). However, if we think of machine varieties as representing alternative production techniques that can highly substitute each other, then higher elasticities are more plausible. For example the elasticity of substitution across goods produced in different countries (within a narrow

[14]For other values of relative risk aversion, numerical techniques can be applied to solve (11). If $\theta < 1/\phi$, then the saving rate is increasing in n. Intuitively, low-θ consumers are less prudent, so the precautionary motive is relatively small. This means that risk aversion dominates and saving declines with volatility. In this case, poor countries develop slowly because their excess volatility discourages investment. The reverse is true for $\theta > 1/\phi$. Similarly to Angeletos (2004), we have the cutoff at an IES less than one (RRA greater than one) because capital does not exhaust all income as long as $\phi < 1$.

product category) is estimated to be around 4–7 (Hummels 2001). Estimates based on the time series of U.S. imports are usually lower, in the range of 1–2 (Gallaway, McDaniel and Rivera 2003). For an intermediate range of 3–4, the value of ϕ is 1/2–1/3, resulting in a θ of 2–3. This is plausible both as a measure of relative risk aversion and as an inverse elasticity of intertemporal substitution (Vissing-Jørgensen 2002).

Proposition 1. If $\theta\phi = 1$, the optimal policy function, $v(n)$ takes the form $v_0 n^{\phi}$, where v_0 is given by

$$v_0 = (1 - \phi)L + \rho\kappa_0 L. \tag{13}$$

Proof. Direct substitution reveals that whenever v_0 satisfies (13), $v_0 n^{\phi}$ satisfies (11). For this policy function, $v'' n^2/n = \phi(\phi - 1)$, $v'n/v = \phi$, and μ_n is independent of n. Since $v_0 n^{\phi}$ also satisfies the boundary conditions, it is a unique solution. □

Defining the value of all the machines as $K = n\kappa$, equation (13) can be rewritten in terms of aggregate variables as

$$Y - C = \phi Y - \rho K = (\phi - \rho\kappa_0)\, Y,$$

since the capital output ratio in this economy is $n\kappa_0 L n^{\phi-1}/(n^{\phi}L) = \kappa_0$. Investors save (and invest) a constant fraction of current output. The saving rate is increasing in the profit rate (ϕ) and decreasing in the degree of impatience (ρ) and sunk cost of investment (κ_0).

From (12), we can express the growth rate of the number of machines as

$$\mu_n = \phi/\kappa_0 - \gamma - \rho,$$

resulting in an output growth rate of

$$\phi\mu_n = \phi\left(\phi/\kappa_0 - \gamma - \rho\right).$$

This completes the characterization of the dynamic equilibrium of this economy. Countries with high profit rates and low investment costs will develop faster, implying both a faster growth of output and a faster fall of volatility. In the next section, we extend the model in two directions in order to account for the differences in specialization patterns between rich and poor countries.

2.3 A two-sector model of technological diversification

In this section, we allow for a richer characterization of the economy, by extending the model to a two-sector economy. The sectors differ in the extent of skill intensity. We introduce a multi-country setup, allowing for cross-country differences in endowments and compare the results for closed and open economies. Allowing for international trade, as we later show, can explain the observation that poor countries specialize in less sophisticated sectors. In fact, comparative advantage magnifies the differences in volatility between poor and rich countries through its effect on the patterns of specialization.

Let us assume that there are two sectors, one producing a capital good Y_k, the other producing a consumption good Y_c:

$$Y_k = \left[\sum_{i=1}^{n_k} X_i^\sigma\right]^{1/\sigma},$$

$$Y_c = \left[\sum_{i=n_k+1}^{n_k+n_c} X_i^\sigma\right]^{1/\sigma}.$$

Both sectors use the same CES technology, but they have access to a different set of machines. In particular, the number of machines in the two sectors will (endogenously) be different. The owner of each machine will decide which sector to operate in. The total number of machines $n_k + n_c$ is denoted by n.

We assume that machines in the capital good sector are operated by skilled labor, whereas those in the consumption good sector are operated by unskilled labor.[15] Note that machines are a metaphor for technology in our model so this amounts to assuming that some technologies are skilled labor intensive, whereas others are unskilled labor intensive. Autor, Katz and Krueger (1998) show that computerization has increased the demand for skilled labor. However, previous technological advances such as the industrial revolution and the introduction of the production line relied more on unskilled rather than skilled workers (James and Skinner 1985, Goldin and Katz 1998). In this paper, we think of the skilled-labor intensive sector as a technologically advanced sector, and the unskilled-labor intensive sector as a less technologically complex one.[16]

More formally,

$$X_{it} = \begin{cases} h_{it}, & \text{if } i \leq n_k,\ K_{i0} = \kappa \text{ and } t < T_i; \\ l_{it}, & \text{if } n_k < i \leq n,\ K_{i0} = \kappa \text{ and } t < T_i; \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Here h_{it} denotes the number of skilled operators and l_{it} the number of unskilled operators of variety i, and κ and T_i are defined, as previously, as the fixed cost of building a machine and the random lifetime of machine i.

Let H denote the overall stock of skilled labor in the economy and L the stock of unskilled labor. If the labor markets are in equilibrium, the production functions can be rewritten as

$$Y_k = n_k^\phi H,$$
$$Y_c = n_c^\phi L.$$

[15] Any positive difference in skill intensity is sufficient for our results; we assume this extreme difference in skill intensity for tractability.

[16] In Section 3 we discuss how we identify technological complexity in the data.

Similarly to the one-sector case, the productivity of a firm in sector i will be increasing in the number of machines. This also causes the volatility of productivity to decline with the number of machines:

$$\text{Vol}_k = \text{Var}(\text{dln}\, Y_k | n_k, H) = \phi^2 \gamma / n_k, \tag{15}$$

$$\text{Vol}_c = \text{Var}(\text{dln}\, Y_c | n_c, L) = \phi^2 \gamma / n_c. \tag{16}$$

Both sectors are perfectly competitive. Each producer takes the wage rate and the set of machine varieties available to the sector as given.

What determines the allocation of machines across the two sectors? Again, investors will maximize profits and move toward sectors with better profit opportunities. The price of machine service i will be marked up over the skilled wage in the capital good sector and over the unskilled wage in the unskilled sector:

$$\chi_i = \begin{cases} w_H/\sigma, & \text{if } i \le n_k; \\ w_L/\sigma, & \text{if } n_k < i \le n, \end{cases}$$

where w_H denotes skilled, w_L denotes unskilled wages. This makes profit per machine a constant ϕ fraction of wages per machine:

$$\pi_i = \begin{cases} \phi w_H H/n_k, & \text{if } i \le n_k; \\ \phi w_L L/n_c, & \text{if } n_k < i \le n. \end{cases}$$

In long-run equilibrium, the rate of return on machines in the two sectors have to be equal. Since the values of a machine are the same in each sector, so is the dividend (that is, profit per machine), implying

$$\frac{n_k}{n_c} = \frac{\omega H}{L}, \tag{17}$$

with $\omega = w_H/w_L$ denoting the skill premium.

The number of machines in a sector is proportional to the total wage bill of the sector. Whenever (17) fails to hold, that is, one of the sectors has relatively few machines, that sector has more profits per machine than the other one. Investors will then move machines across sectors (if machines are movable), or invest only in the more profitable sector until the equality is resolved.

We assume that machines are freely and instantly movable across sectors. This assumption ensures that $r_k = r_c$ at any time in any state of the world, implying that (17) holds at any point in time.[17] Since (17) would always hold in the long run, we only need this assumption to simplify the transitional dynamics: If machines can instantly adjust across sectors

[17] Formally, the present discount value of a machine is $p_k \kappa$ in both sectors. If the dividend yield in one sector is higher than in another, an investor could buy one unit of high-dividend stock and short one unit of low-dividend stock. Since the prices of the two stocks always move in parallel, this strategy presents an arbitrage opportunity with positive net dividends in all future periods and no price risk. Absence of arbitrage hence implies the equalization of dividend yields.

then the economy will immediately jump to its balanced-growth path. Because the machines are movable across sectors, the single state variable is the total number of machines, n.

Equation (17) describes a version of *directed technical change*, as in Acemoglu (2002) or Caselli and Coleman (2000): Machine varieties are directed towards the sector with a higher share in employment. On the one hand, this is a size effect: if there are many operators in a sector, it is more profitable to operate machines there and hence more machines will move towards this sector. On the other hand, there is also a relative price effect: If the skill premium is high, the relative price of the capital good is high, so profits are higher in the capital good sector.

Since the final good sectors are competitive, the relative price of the capital good will equal the relative marginal cost:

$$p_k = \omega \left(\frac{n_k}{n_c}\right)^{-\phi}. \tag{18}$$

If there are more machines allocated to the capital sector, it becomes more productive, and its relative price falls.

2.3.1 The closed economy

In a closed economy, the relative price is determined by the relative supply and demand of the two goods. Suppose that the demand for the two goods take the isoelastic form,

$$\frac{Y_k}{Y_c} = Ap_k^{-\varepsilon},$$

where A is a constant and ε is the elasticity of substitution between capital and consumption goods. Then, the skill premium is endogenously determined as

$$\omega = p_k \left(\frac{n_k}{n_c}\right)^{\phi} = A^{1/\varepsilon} \left(\frac{H}{L}\right)^{-\phi/\varepsilon} \left(\frac{n_k}{n_c}\right)^{\phi(1-1/\varepsilon)}, \tag{19}$$

$$\frac{n_k}{n_c} = A^{\frac{1}{\varepsilon 1-\phi+\phi}} \left(\frac{H}{L}\right)^{\frac{\varepsilon-\phi}{\varepsilon(1-\phi)+\phi}}. \tag{20}$$

As we will show, the nominal saving rate in the economy is constant, which means that $p_k Y_k / Y_c$ is constant. This implies that the relative demand for the consumption and the capital good has unitary elasticity, $\varepsilon = 1$. Then equation (20) simplifies to

$$\frac{n_k}{n_c} = A \left(\frac{H}{L}\right)^{1-\phi}. \tag{21}$$

Since we have assumed $0 < \phi < 1$, the relative number of machines is increasing in the relative amount of skilled labor in the economy. However, a 1 percent increase in skill abundance induces a less than 1 percent increase in the relative number of machines in the capital good sector. This is because the abundant factor (more precisely, the good that uses the abundant factor intensively) becomes cheaper and hence less profitable for machine owners. This relative price effect has been pointed out by Acemoglu (2002).

The fixed cost required to build a machine is assumed to arise in capital goods. In terms of consumption goods, the fixed cost is $p_k\kappa$. The mean rate of return on machines is the same in both sectors, so let us take the capital good sector:

$$r = \frac{\pi_i}{V} - \gamma = \phi\frac{w_H H/n_k}{p_k\kappa} - \gamma = \phi\left(\frac{n_k}{n}\right)^{\phi-1}\frac{H/L}{\kappa_0} - \gamma, \tag{22}$$

where we have made use of the facts that capital good prices equal marginal costs ($w_H n_k^{-\phi}$) and that the sunk cost is falling at the rate $\phi-1$, $\kappa = \kappa_0 L n^{\phi-1}$. Relative to (8), the important difference is that the rate of return is falling in the relative complexity of the capital good sector. The number of machines affects profitability in two ways. First, more machines make the capital sector more productive and hence more profitable (productivity is proportional to n_k^ϕ). Second, competition increases in the number of machines, because machines compete for a scarce supply of operators (H). This second effect lowers profits proportionally with n, so it dominates the first for $\phi < 1$.

The relative demand for capital and consumption goods will be determined by the consumption-saving decision. Since the mutual fund holds all machines in both sectors, the Euler equation of the consumer is the same as in (11), but the growth rate of machines and the return on machines may be different.

The mean growth rate of the number of machines, n is

$$\mu_n = \frac{Y_k}{n\kappa} - \gamma = \frac{H/L}{\kappa_0}\left(\frac{n_k}{n}\right)^{\phi} - \gamma. \tag{23}$$

As before, we conjecture that optimal consumption is given by $v_0 n^\phi$, which, by the equilibrium in the consumption good market, implies

$$\frac{n_c}{n} = \left(\frac{v_0}{L}\right)^{1/\phi}, \tag{24}$$

$$\frac{n_k}{n} = 1 - \left(\frac{v_0}{L}\right)^{1/\phi}. \tag{25}$$

The Euler equation simplifies to $\mu_n = r - \rho$. Substituting from (22) and (23), we see that $v_0 n^\phi$ is indeed a solution as long as

$$\frac{H/L}{\kappa_0}\left[1 - \left(\frac{v_0}{L}\right)^{1/\phi}\right]^{\phi} = \phi\frac{H/L}{\kappa_0}\left[1 - \left(\frac{v_0}{L}\right)^{1/\phi}\right]^{\phi-1} - \rho. \tag{26}$$

Note that v_0 does not depend on n, implying that the allocation of machines across sectors as well as the relative prices are independent of development. In other words, the economy exhibits balanced growth. This also means that the saving rate is constant as previously claimed.

2.3.2 A small open economy

Suppose instead that the country is a small open economy freely trading the output of the two final good sectors at an exogenously given world relative price. We assume that the individual machine varieties cannot be traded. In other words, investors can buy foreign capital goods and install them in their own country as machines, but the physical machines installed abroad cannot contribute to production.[18] This assumption ensures that countries cannot circumvent the fixed costs of machine operation by importing machine services from abroad and hence cannot fully diversify instantly. The number of machines in the country will hence be a state variable that can only be adjusted gradually. At any given point in time, the number of available machines and hence overall technological complexity is given. In the long run, investment in new machines will determine technological complexity, economic development, and volatility.

Trade is balanced at any point in time, ruling out international borrowing and lending. This also means that investment is finite (growth in the number of machines is gradual) in every instant, because the country has only finite flow output to offer in exchange for foreign capital goods. In contrast, if we allow for borrowing, investors can immediately borrow to replace a broken machine, smoothing out some of the shock to productivity. We assume away such consumption smoothing behavior because the current accounts of countries (especially those of less-developed ones) do not seem to act as buffers against productivity shocks.[19]

Let $\tilde{p}_k$ denote the world price of capital. We then have from (18)

$$\tilde{p}_k = \omega \left(\frac{n_k}{n_c} \right)^{-\phi},$$

which results in *conditional factor price equalization*,

$$\omega = \tilde{p}_k \left(\frac{n_k}{n_c} \right)^{\phi}. \tag{27}$$

Conditional on the levels of productivity in the two sectors, the world relative price of the two goods completely determines the relative wage. All else equal, a higher relative price of the capital good (high $\tilde{p}_k$) leads to a higher relative wage of the factor which is used intensively in that sector (high ω). This is the FPE part. At the same time, the more productive the capital good sector is relative to the consumption good sector, the higher the relative wage of skilled labor.

[18] If we interpret machine varieties as different techniques of production, this amounts to very costly imitation and no technology spillovers across countries. Comin and Hobijn (2004) document a relatively slow adoption of leading technologies developed elsewhere. A positive but finite cost of technology adoption could be modeled such that machine varieties already in use abroad have a lower installation cost $\tilde{\kappa} < \kappa$. A $\tilde{\kappa} > 0$ would be sufficient to deliver qualitatively similar results.

[19] Kalemli-Ozcan et al. (2003) show that the beta coefficient of consumption response to output shocks of countries is close to one.

Note that, as standard in small open-economy models with free trade, the production structure is independent of demand considerations. Relative demand for the two sectors (in our case, the consumption/investment decision) will matter only for the patterns of trade. Since $\tilde{p}_k$ is exogenously given, we can substitute (27) into (17):

$$\frac{n_k}{n_c} = \tilde{p}_k^{1/(1-\phi)} \left(\frac{H}{L}\right)^{1/(1-\phi)}. \tag{28}$$

Notice that, similarly to the closed economy case, (21), the relative number of machines in the capital sector increases in skill abundance. However, the dependence on skill abundance is stronger, $1/(1-\phi)$, because we no longer have an offsetting relative price effect. This is just the Rybczynski theorem applied to directed technical change.

The impact is also greater than in the case of pure factor price equalization. The reason for this is that machines flow towards the sector that already has a comparative advantage, making it relatively more productive. This becomes an additional source of comparative advantage. In other words, the initial comparative advantage gets magnified by directed technical change. Our model says that even small human capital differences can account for large differences in specialization patterns and, hence, in the relative volatilities of sectors.

2.4 Extension to multiple sectors

Suppose now that there are S sectors, each using the same CES technology but requiring different levels of skill for machine operation. In particular, sector s requires that each operator possesses at least h_s amount of human capital, and we order sectors such that $h_S > h_{S-1} > ... > h_1$. The output of machine i in sector s is

$$X_{is} = \begin{cases} h_s l_{it}, & \text{if } K_{i0} = \kappa, \text{ and } t < T_i; \\ 0, & \text{otherwise,} \end{cases}$$

where l_{it} is the number of workers on machine i who are "qualified" to operate the machine in the sense that they have a level of human capital higher than h_s.

There are altogether L workers in the economy, and their human capital endowment is distributed according to a cumulative distribution function $F(h)$. The number of workers capable of operating machines in sector s is hence $[1 - F(h_s)]L$. The two-sector case of Section 2.3 is a special case of this framework, with a fraction of people having high human capital (skilled workers) and the rest having low human capital (unskilled workers).

Given the number of machines in each sector, $(n_1, n_2, ..., n_S)$, labor market equilibrium requires that each worker be employed on machines that require the highest skill level that this worker can supply.[20]

[20] To prove this, suppose there exists a worker with human capital level $h_j \geq h_{s+1}$ (that is, capable of working in sector $s+1$) working in sector s. This worker is not willing to switch to sector $s+1$ because $w_{s+1} < w_s$. But all workers in sector $s+1$ are capable of operating sector s machinery, and they would earn higher wages in that sector. Hence this cannot be an equilibrium.

This implies that a fraction $1 - F(h_S)$ of workers is employed in sector S, and a fraction $F(h_{s+1}) - F(h_s)$ in sector s. The output of sector s is hence

$$Y_s = n_s^{\phi} h_s \alpha_s L,$$

where α_s is defined as the share of workers in sector s, $F(h_{s+1}) - F(h_s)$ (defined for all s with $h_0 = 0$, $h_S = \infty$). Profits per machine are a constant, $(1 - \sigma)$, fraction of revenues per machine,

$$\pi_s = (1 - \sigma)\tilde{p}_s n_s^{\phi-1} h_s \alpha_s L,$$

where $\tilde{p}_s$ is the price of product s determined in world markets. Directed technical change will equate per-machine profits across sectors, $\pi_s = \pi_z$, so the relative number of machines in any two sectors is given by

$$\frac{n_s}{n_z} = \left\{ \frac{\tilde{p}_s h_s \alpha_s}{\tilde{p}_z h_z \alpha_z} \right\}^{1/(1-\phi)}. \tag{29}$$

A sector will use relatively more machines if it is producing an expensive good, it is skill intensive, or has a bigger pool of workers with matching skills. Such sectors are also more productive and less volatile. In other words, given the overall number of machines, $n = n_1 + n_2 + ... + n_S$, technological complexity and productivity are increasing, while volatility is decreasing in the sector's skill intensity and its share in total employment.

The variance of sector s in country i is $\phi^2\gamma/n_{is}$, so we can write the log variance as

$$\ln \mathrm{Var}_{is} = 2\ln\phi + \ln\gamma - \ln n_{is} = \nu_i - [\ln\tilde{p}_s + \ln h_s + \ln(L_{is}/L_i)]/(1-\phi), \tag{30}$$

where ν_i is a country fixed effect.

This is a key equation for our empirical exercise. While we can measure a sector's skill intensity and its share in employment, we do not observe $\tilde{p}_s$, the price of the sector's output in world markets. Instead, we interpret it broadly as an unobserved sector-specific variable that affects the level of complexity, capturing not only variations in the value of output but also, for example, technological differences across sectors. Note that this variable is common across countries within a given sector, so we can control for it using either sector fixed effects or observing technological complexity in any given country.

3 Productivity, Volatility, and Technological Complexity: The Empirical Evidence

The model developed in the previous sections leads to a set of predictions concerning the relationships among productivity, volatility, and technological diversification. We discuss these predictions in light of the empirical evidence.

Prediction 1. GDP volatility declines with development.

This is one of the stylized facts in the literature and the main motivation of this paper. There are large cross-country differences in volatility. The standard deviation of annual GDP growth during the period 1970 through 2000 ranges from 1.4 percent to 21.8 percent (a factor of 15) across 167 countries. The most volatile decile of countries had a standard deviation of GDP growth of 12.9 percent. This is seven times as high as the volatility of the least volatile decile (1.8 percent). This cross-country variation in volatility is highly correlated with the cross-country variation in the level of development, gauged by real GDP per capita. More specifically, as shown in Table 4, the elasticity of GDP variance with respect to GDP per capita is -0.326 (with a robust standard error of 0.066).[21]

In the model, investment in new machines brings about development and a gradual decline in volatility. Countries that have few machines are both less developed and more volatile. In the multi-sector version, our model proposes two channels to explain this negative association. First, a within-sector channel, whereby a given sector exhibits higher technological complexity in more-developed countries. This, in turn, implies that a given sector is both more productive and less volatile in developed countries. Second, a compositional channel, whereby poor countries specialize in relatively less complex sectors. This implies that poor countries concentrate in sectors with (absolute) lower productivity and higher variance. In what follows, we check the empirical consistency of the predictions associated with these two channels.

Prediction 2. For any given sector, poor countries utilize less complex technologies. This implies that for any sector, *a*) poor countries are both less productive and more volatile and *b*) productivity and volatility are negatively correlated.

- For a given sector, poor countries utilize less complex technologies.

Various studies have explored the process of technology diffusion across countries. For example, Caselli and Coleman (2000), document that the adoption of computers depends heavily on the level of development of the country, and, more specifically, on the level of human capital. Caselli and Wilson (2004) show that this result extends to a broader set of high-technology equipment (where the extent of technology embodied in capital equipment is measured as the R&D content).

Our model implies that these cross-country differences in technology are also present *within* sectors. Since directed technical change equates the rates of return on machines in all sectors, poor countries will use proportionately fewer machines in all sectors, holding comparative advantage patterns constant.

The two examples mentioned in the introduction suggest important cross-country technological differences for a given sector: Developed countries tend to use more agricultural machinery, fertilizers, and pesticides in agriculture and have access to more types of power plants in the energy sector. Recent empirical studies provide additional support for this

[21] Table 3 presents the list of countries included in the computation.

observation. For example, Comin and Hobijn (2004) document how specific technological innovations have spread across countries. Many of these innovations are relevant only to certain sectors (for example, mule spindle, blast oxygen furnace, internal combustion engine, aviation). The authors show that most innovations originated in developed countries and spread gradually to less-developed countries. This implies that in any given year, in all relevant sectors, poor countries use less sophisticated production techniques than rich ones.

- For a given sector, poor countries are both less productive and more volatile.

In the context of our model, the previous finding, in turn, implies that a given sector is both less productive and more volatile in poor countries. We test this prediction using sectoral data from the United Nations Industrial Development Organization (UNIDO, 2002). The UNIDO data set covers all manufacturing at the 3-digit level of aggregation from 1963 to 1998 for a sample of 64 countries, providing information on employment and value added on an annual basis. Table 3 indicates the countries for which the data are available and Table 5 reports the index of technological diversification for each sector, with the corresponding (average) size of the sector in manufacturing. We compute the sample average of labor productivity for each country and sector. As a measure of volatility, we use the 5-year variance of labor productivity (value added per worker) growth.

To check the consistency of the prediction, we first regress the (log of) sectoral labor productivity on the level of development, proxied by the (log of) real GDP per capita of the country, controlling for sector-specific dummies. The regression yields a positive and significant coefficient: As shown in the first column of Table 6, the point estimate for the elasticity is 0.70 (with a country-clustered standard error of 0.07). This means that, on average, any given sector is significantly less productive in poor countries.

Similarly, we regress the (log of) sectoral variance on the level of development, including sector-specific dummies. We obtain a negative and significant coefficient, displayed in the second column of Table 6. The estimated elasticity is -0.30 (with a country-clustered standard error of 0.10), implying that, on average, every sector is significantly more volatile in poor countries.

- For a given sector, productivity and volatility are negatively correlated.

Because poor countries use less complex technologies for any given sector, this implies that the within-sector relationship between volatility and productivity should be negative. To check this implication, we regress the (log) level of volatility on the (log) level of labor productivity, controlling for sectoral dummies. The estimated coefficient, reported in Table 7, is negative and significant: We obtain an elasticity of -0.29 (with a country-clustered standard error of 0.10).

Prediction 3. More complex sectors are both more productive and less volatile. A mean-variance frontier might not exist.

- More complex sectors are both more productive and less volatile.

This is a direct prediction of production with "technological diversification." To test this prediction, we use the measures of labor productivity and volatility computed from the UNIDO data set we referred to before.

Central to this test is the construction of a measure of technological complexity. Following Clague (1991), we measure the technological complexity of a sector by the diversity of inputs it uses. A sector is more complex if it uses more varieties of capital goods. There are two practical shortcomings with this measure of complexity. First, there are no comprehensive data on sector-level input usages for most countries in the sample. Second, even if the data were available, the actual extent of complexity observed would respond endogenously to the level of development of the country and the relative abundance of skilled labor.

To address these issues, we use the approach followed by Clague (1991) and Rajan and Zingales (1998) and calculate the complexity measures for sectors in the U.S. There are two key assumptions for the validity of the test we will perform: First, there are technological reasons why some industries demand a relatively larger number of capital goods than others. Second, these technological differences persist across countries, leading to a positive correlation between the rankings of technological complexity in the United States and any other given country.[22] More formally, as discussed after equation (30), we treat these complexity measures as a proxy for unobserved technological complexity that is not explained by the sector's skill intensity and relative size.

To calculate our measure of technological diversity, we use the 1997 Capital Flow Tables of the Bureau of Economic Analysis. This table distinguishes 180 capital good categories (structures, equipment, and software), each usually corresponding to a 6-digit 1997 NAICS category. We then measure technological diversification as the inverse the Herfindahl index of investment expenditure shares. Table 5 reports the (log) technological diversification index for each of the sectors in our sample.

The simple correlation between (log of) labor productivity and our index of technological diversity is positive and statistically significant (without and with country-specific dummies). However, one might argue that this strong positive correlation might be driven by other determinants. For example, capital intensity is likely to be correlated with the level of technological diversification and also to influence productivity. Incidentally, our model also predicts that the skill intensity of the sector also influences the productivity of the sector. The first column in Table 8 shows the within-country regression results, after controlling for the additional potential determinants of labor productivity. We control for the share of materials in the sector, its skill and capital intensity (measured by the share of skilled or semi-skilled workers in production and the value of equipment per worker, respectively), and the relative

[22] A meaure of technological complexity based on the U.S. is a noisy measure of the complexity of a sector in other countries. As Raddatz (2003) argues, to the extent that the noise corresponds to classical measurement error, the coefficients we are interested in will be biased towards zero, against the hypothesis of our study.

size of the sector. The regression shows that technological diversification is significantly and positively correlated with the level of labor productivity. A one-standard-deviation increase in the measure of technological diversification is associated with a 3 percent increase in the level of productivity. Also in line with our predictions, skill intensity raises productivity.

The same considerations stated before lead us to include a similar set of controls in the regression of (log) variance on the extent of technological diversification. The results are summarized in the second column of Table 8. Technological diversification is significantly and negatively associated with sectoral volatility. A one-standard-deviation increase in technological diversification is associated with a 4 percent decrease in the volatility of the sector. Volatility also decreases with skill intensity and, as we later document in more detail, the size of the sector.

- There is no evidence of a mean-variance frontier.

As discussed before, portfolio-view models predict a positive correlation between mean productivity and variance. However, in the data, the simple correlation between volatility and productivity is negative (-0.10 and significantly different from zero). Controlling for sectoral size, and country- and sector-specific effects yields no positive relationship between the two variables. Using a different approach, Koren and Tenreyro (2004) also reject the notion that countries move along a mean-variance frontier in the data.

Our model is consistent and, in fact, predicts the absence of a mean-variance frontier: As countries develop, they use more sophisticated technologies, which leads both to higher productivity and lower variance.

Prediction 4. Poor countries have a comparative advantage in less complex and hence riskier sectors. Consequently, poor countries specialize in less technologically complex sectors. This also implies that poor countries specialize in more volatile sectors.

- Poor countries have a comparative advantage in less complex and hence riskier sectors. Consequently, poor countries specialize in less technologically complex sectors.

As seen in Sections 2.3 and 2.4, skill intensive sectors will endogenously become more complex. This implies that skill abundant countries have a comparative advantage in complex sectors. Note that even a small difference in skill abundance can result in a large comparative advantage because of directed technical change.

That poor countries have a comparative advantage in less complex sectors was first documented by Clague (1991). He finds that poor countries are relatively less efficient in industries with a lower index of technological complexity (where complexity is measured similarly to the method employed in the present paper).

This pattern of comparative advantage, according to the model, implies that poor countries should specialize in less complex sectors. We checked this implication, by examining the sectoral composition of the economy. Using the UNIDO data set, we regressed the (log) average sectoral shares on *a*) the index of technological diversification of the sector; *b*) the level of development, proxied by the (log) level of GDP of the country; and *c*) the interaction between sectoral variance and the level of development. According to the model, the interaction term should be positive: As countries develop, they should move to more complex sectors. The results are displayed in Table 9. The interaction term is positive and significantly different from zero, consistent with the model.

- Poor countries specialize in more volatile sectors.

To check whether the pattern of comparative advantage might also imply that poor countries specialize in relatively more volatile sectors, we regress the (log) average sectoral shares on *i*) the variance of the sector; *b*) the (log) of GDP of the country; and *c*) the interaction between sectoral variance and the level of development. As the model predicts, the regression yields a negative and significant coefficient for the interaction term, implying that more developed countries tend to specialize in lower-variance sectors. The results are displayed in Table 10, which shows the regressions without and with country-fixed effects.

Prediction 5. Larger sectors, in which the country has a comparative advantage are less volatile.

Profits for an individual machine owner are larger in large sectors (with more machine operators), *ceteris paribus*. Hence more machines will be attracted toward large sectors, making them less volatile. (See equation (30).) The size of the sector, in turn, is determined by its comparative advantage, implying that sectors with a comparative advantage are less volatile than sectors with comparative disadvantage.

Table 11 shows that sectors with a larger share of employment are less volatile even when controlling for country and industry fixed effects. This remains true of we control for other sectoral characteristics such as capital and skill intensity, and technological complexity (Table 8).

Canning, Amaral, Lee, Meyer and Stanley (1998) explored the relationship between GDP volatility and the size of the economy, finding that variance falls with size with an elasticity of about 1/6. We find very similar elasticities for the size of a *sector*. Note that if all risks are idiosyncratic to individual workers or machines, the fall in volatility should be faster, with an elasticity of 1. Canning et al. argue that interactions across economic actors magnify the aggregate importance of idiosyncratic risk. An alternative explanation for why idiosyncratic shocks are important in the aggregate is provided by Gabaix (2004). He shows that if the size distribution of firms has an infinite variance (such as, for example, a Pareto distribution), the decay of idiosyncratic risk with respect to size is slower. In our model, we can account for the slow decay of volatility with size if we assume that each machine has a random productivity drawn from a Pareto distribution. Then, we will have few very productive machines employing many operators. Idiosyncratic shocks to these machines then have a large effect on aggregate productivity.

In our model, the size of a sector is endogenously determined by comparative advantage: sectors that use the abundant factor intensively are relatively larger. Instead of identifying all the necessary factors of production for each sector, we measure the "revealed" comparative advantage (Balassa 1965) of sectors as

$$\mathrm{RCA}_{is} = \frac{X_{is}/X_i}{X_{ws}/X_w},$$

where X_{is}/X_i is the share of sector s in total manufacturing exports of country i and X_{ws}/X_w is the same sectoral share for the world. We use exports data from the Trade and Production Database of the World Bank, which merges product-level trade flow data from UN Comtrade with sector level production data from UNIDO. World export is measured as total exports of the 64 countries for which trade data exist in all 28 manufacturing sectors.

Table 12 reports the results of regressing log variance of sectoral productivity on the log of its revealed comparative advantage. Comparative advantage is associated with significantly lower volatility even when controlling for country and sector fixed effects. A one-standard-deviation increase in revealed comparative advantage leads to 8 to 14 percent lower variance.

3.1 Robustness

In this section, we conduct a number of robustness checks for our empirical results. First, some institutions may facilitate the response to external sectoral shocks. Since rich countries have better institutions, this may contribute to lower output volatility. We therefore look at the role of financial development and labor market flexibility in reducing volatility.

Financial development makes raising capital cheaper and faster. Hence if firms are hit by liquidity shocks, they can borrow the necessary funds without significantly disrupting production. This can make the productivity of firms smoother, especially over shorter horizons. Aghion, Angeletos, Banerjee and Manova (2004) show how the liquidity needs of long-term investments make output volatile in financially underdeveloped countries. Empirically, Braun and Larrain (forthcoming) and Larrain (2004) have shown that financial development makes output less volatile, especially in highly finance dependent sectors.

Our model can easily incorporate the pattern that volatility declines with financial development. The development of new inputs requires financing, because initial development/installation costs have to be covered up front. The value of new machines will hence be higher in financially developed countries where the cost of capital is lower, making these countries less volatile. Across sectors, differences in financing needs ("external finance dependence") lead to similar predictions.

Column 2 of Table 13 reports the regression of sectoral variance on the level of GDP and the degree of financial development, gauged by private credit over GDP. We control for sector-specific fixed effects. Financial development leads to significantly lower volatility, but the effect of general economic development also remains significantly negative.

Our measure of volatility is the variance of labor productivity (value added per worker) growth. This may be higher in countries with rigid labor markets, because firms are less able to react to demand shocks. For example, if the demand for the product of a particular firm falls, optimally it would downsize its workforce. However, firing costs and regulations make this costly, so the firm retains its workers in the hope that the shock is transitory. In the data we would observe this shock as if it were a negative productivity shock; less output is produced with the same number of workers.

To see whether this measurement problem contaminates our results, we control for the costs of modifying and terminating employment contracts across countries, as compiled by Botero, Djankov, La Porta, Schleifer and Lopez-de-Silanes (2004). As Column 3 of Table 13 shows, labor market rigidities significantly increase the volatility of labor productivity within any given sector. However, this does not alter our prediction that volatility declines with development; in fact, the point estimate of the coefficient of GDP is greater in absolute value. Intuitively, some highly developed countries have rather rigid labor markets (notably European countries) but are still highly stable in terms of labor productivity.

In Column 4, we control for both financial development and labor market rigidities. The effect of overall economic development is still highly significant with a coefficient very similar to our benchmark estimates.

An alternative explanation for the decline of volatility with development is that high-income countries specialize in differentiated products, which are subject to idiosyncratic demand and supply shocks. This could result in lower volatility because idiosyncratic shocks wash out when aggregated over many products. Also, if sectors producing multiple differentiated products use a wider variety of inputs, then "output diversification" is correlated with "input diversification," which potentially biases our results on technological diversification.

To test for the presence of output diversification, the use of firm-level data would be desirable. Lacking such data, however, we can use data on the number of establishments reported by UNIDO. If products are differentiated by producer firms and these products are subject to idiosyncratic demand or supply shocks, the volatility of a sector should decline with the number of firms.

Our model also predicts that larger sectors should be less volatile. The distinction between the two theories relies in the margin through which this takes place. Output diversification takes place across firms, hence volatility declines with the number of firms (extensive margin) but not with the average size of firms (intensive margins). In our model, larger firms attract proportionately more machines, and hence both margins are equally important.

To test the relative importance of the extensive and intensive margins, we decompose the size of sector s in country i, L_{is}, into the number of firms and the average size of firms,

$$\ln L_{is} = \ln N_{is} + \ln(L_{is}/N_{is}).$$

We then regress the log variance of a sector on the log number of firms and the log of their average size, controlling for both country- and industry-specific fixed effects. The output diversification would suggest that the number of firms decreases volatility while firm size does not. Table 14 reports our results. Both the extensive and the intensive margins of sector size contribute to lower volatility, and, in line with our theory, there is no significant difference in the importance of the two. Moreover, when we only focus on "complex" sectors, where product differentiation may be more prevalent, there are still no significant differences. This suggests that "output diversification" does not significantly contribute to the decline in volatility.

4 Conclusion

This paper proposes a model in which the production process makes use of different input varieties, which are subject to imperfectly correlated shocks. As in other expanding variety growth models, technological progress takes place as an expansion in the number of input varieties, increasing productivity. The new insight we develop is that the expansion in varieties also leads to lower volatility of production via two channels. First, as each individual variety matters less and less in production, the contribution of idiosyncratic fluctuations to overall volatility declines. Second, each additional input provides a new adjustment margin in response to external shocks, making productivity less volatile.

In the model, the number of varieties evolves endogenously in response to profit incentives. The consequent change in volatility associated with changes in the number of varieties feeds back into the investment and savings decisions of producers.

The model yields empirical predictions concerning the relationships among productivity, volatility, technological complexity, and comparative advantage. We discuss these predictions in light of the existing empirical evidence and provide novel findings supporting the results.

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A Approximating the Discrete State Space Markov Process with a Diffusion

In this section we formalize the diffusion approximation for the number of machines, relying on limit theorems for birth and death processes in Stone (1963) and Iglehart (1965).

Let n follow a continuous-time birth and death process with birth rate A and death rate bn. That is, we assume that a new machine is finished after an exponential time, with arrival parameter A measuring the investment into new machines. Making the arrival of machines random takes care of potential integer problems: since the arrival of a finished machine only depends on the current intensity of investment, we do not need to track how much investors have invested since the building of the last machine. In what follows, we assume A is constant.

The instantaneous mean and variance of this process are

$$\begin{aligned} \mathrm{E}(\mathrm{d}n) &= (A - bn)\,\mathrm{d}t, \\ \mathrm{Var}(\mathrm{d}n) &= (A + bn)\,\mathrm{d}t. \end{aligned}$$

Let $\xi_A = (n - A/b)/\sqrt{A}$ denote a transformed Markov process. Obviously,

$$\begin{aligned} \mathrm{E}(\mathrm{d}\xi_A) &= -b\,\mathrm{d}t, \\ \mathrm{Var}(\mathrm{d}\xi_A) &= \left(2 + \frac{b}{\sqrt{A}}\xi_A\right)\mathrm{d}t. \end{aligned}$$

What happens as A tends to infinity? The process ξ_A weakly converges in the Markov sense to a diffusion process

$$\mathrm{d}\xi_\infty = -b\,\mathrm{d}t + \sqrt{2}\,\mathrm{d}z.$$

Proof. This requires (Stone 1963) that, in any compact interval of $\mathcal{R}$, (1) the state space of ξ_A becomes dense and (2) the instantaneous mean and variance of ξ_A converge uniformly to $-b$ and 2.

(1) Let $[I_1, I_2] \subset \mathcal{R}$ be a compact interval and $x \in [I_1, I_2]$ an arbitrary point in the interval. For each A there exists an integer k_A for which

$$(k_A - A/b)/\sqrt{A} \leq x < (k_A + 1 - A/b)/\sqrt{A}.$$

The distance of x to the closest element of the state space of ξ_A is hence smaller than $1/\sqrt{A}$. That is, for any $\varepsilon > 0$, there exists an element of the state space in the ε-neighborhood of x as long as $A > 1/\varepsilon^2$. This proves that the state space becomes dense in $[I_1, I_2]$ as $A \to \infty$.

(2) Clearly, the instantaneous mean and variance of ξ_A converge to $-b$ and 2, we just need to establish uniformity. Let $\xi_a \in [I_1, I_2]$. The for any ε there exists an $A_0 = (bI_2/\varepsilon)^2$ such that $|b\xi_a/\sqrt{a}| < \varepsilon$ as long as $a > A_0$.

□

Note that if ζ_A is a diffusion process described by

$$\mathrm{d}\zeta_A = (A - bn)\,\mathrm{d}t + \sqrt{A + bn}\,\mathrm{d}z,$$

its transformation $(\zeta_A - A/b)/\sqrt{A}$ also weakly converges to ξ_∞.

That is, for large enough A, the birth and death process will be well approximated by a diffusion process with corresponding instantaneous mean and variance.

B An Example with Fixed-Coefficients Technology

In the benchmark model we assume that $\sigma \in (0, 1)$, that is, the elasticity of substitution across machine varieties is bigger than 1 (the machines are gross substitutes). This is a standard assumption in the expanding variety literature and is needed to ensure that the varieties not yet invented (or installed) are not essential in production.

However, complementarities across different inputs (or tasks) may be an important feature of the development process. As Kremer (1993) points out, many production processes feature an "O-ring" technology: even if a single input fails, it may jeopardize the whole outcome. (Also see Young (1993) and Grossman and Maggi (2000)) on the importance of complementarities for productivity patterns.) We hence consider an example in which all the machine varieties are essential in production. We show that even in the extreme form of complementarity (O-ring), technological diversification may still take place via variable capacity utilization.

In particular, the production function takes the Leontief form:

$$Y = \min_{i=1,\ldots,n} \{x_i\}.$$

The services of individual machine varieties are produced as before, with skilled operators, $x_i = h_i$. We assume, however, that the failure of the machine does not render it completely useless (otherwise log output would become minus infinity), but, rather, makes it more expensive to operate. In particular, while good machines require 1 unit of skill labor, broken machines require $\delta > 1$ units.

Let us first focus on the case without variable capacity utilization, that is, when the number of operators per machines is constant at h. This implies that when the first machine fails, output drops from h to h/δ. (Further failures have no impact on output.) So the change in log output is

$$\Delta \ln Y = -\ln \delta.$$

Since the first failures arrives after an exponentially distributed working time with an arrival rate of γn, the instantaneous variance is

$$\mathrm{Var}(\mathrm{d}\ln Y)/\,\mathrm{d}t = (\ln \delta)^2 \gamma n.$$

This is in fact increasing in n; the more complex the technology, the more likely a machine failure is. We do not have an offsetting effect from the law of large numbers because the working machines do not substitute for the broken one.

Consider now the case with variable capacity utilization. In this case, we let firms reshuffle operators across machine varieties, only holding the total number of operators fixed at $H = nh$. If a machine "fails," the firm allocates more operators to that machine to partially offset this negative productivity shock. With free reallocation of operators, it is optimal to equalize the services of each machine variety at, say, x. This requires δx operators on the broken machine and $(n-1)x$ operators on the rest. The total number of operators is unchanged, so

$$\delta x + (n-1)x = nh.$$

The change in log output is hence

$$\Delta \ln Y = \ln n - \ln(n - 1 + \delta),$$

which is negative, that is, output still drops but it drops by less than without VCU. The firm can successfully mitigate some of the impact of the shock with VCU.

The instantaneous variance in this case is

$$\mathrm{Var}(\mathrm{d}\ln Y)/\,\mathrm{d}t = [\ln n - \ln(n - 1 + \delta)]^2 \gamma n.$$

The first part decreases with n. The more machine varieties there are, the more possibilities there exist to reshuffle operators without affecting output too much. The second part is increasing in n because more complex technologies fail more often. In general the effect of technological complexity on volatility is hence ambiguous. Nonetheless, as complexity increases without bound ($n \to \infty$), the first effect dominates and volatility goes to zero,

$$\lim_{n\to\infty} \mathrm{Var}(\mathrm{d}\ln Y)/\,\mathrm{d}t = 0.$$

To see this, use the intermediate value theorem to rewrite $[\ln n - \ln(n - 1 + \delta)]$ as $(\delta - 1)/[n + \xi_n(\delta - 1)]$, where $\xi_n \in [0, 1]$. Since ξ_n is bounded, $[\ln n - \ln(n - 1 + \delta)]^2 = O(n^{-2})$ and $\gamma n = O(n)$.

In summary, the ability to vary capacity utilization can make more complex technologies less volatile even in the case of fully complementary inputs.

C Data Appendix

Variable Definitions

GDP per capita GDP per capita of the country in 1997, measured in 1995 international dollars. [WDI, PWT]

Population Population of the country in 1997. [WDI]

Yield volatility Variance of the log of annual wheat yield. [FAOSTAT]

Rainfall volatility Variance of cumulated log changes in precipitation. Precipitation data are recorded monthly at several meteorological stations within a country. Because many stations do not report data in all months, we take the average of year-on-year changes for all months and all stations within the country. We cumulate these changes to obtain the country's deviation from long-run precipitation trends. [Global Historical Climatology Network]

Temperature volatility Variance of cumulated changes in temperature, calculated in the same way as rainfall volatility. [Global Historical Climatology Network]

Change in oil price Two-year change in the U.S. CPI-deflated price of West Texas Intermediate oil.

Diverse powerplants dummy Takes the value of one if the concentration of powerplants by type (conventional, hydroelectric, nuclear, renewable) in the country is below the median. [International Energy Annual]

Technological Diversification The log of the inverse of the Herfindahl index of concentration of equipment purchases across different varieties of capital goods. A sector has a high Technological Diversification index if it purchased many different capital goods. [1997 Capital Flow Tables]

Average Share in Manufacturing The sector's share in manufacturing employment, averaged across the sample period, 1963–1998. [UNIDO]

Labor Productivity Value added per worker in 1995 dollars, averaged across the sample period, 1963–1998. [UNIDO]

Variance of Productivity The variance of 5-year growth of value added per worker in 1995 dollars across the sample period, 1963–1998. [UNIDO]

Skill Intensity The fraction of production workers in the 3-digit ISIC sector that are employed in skilled or semi-skilled occupations. [Occupational Employment Statistics]

Share of Materials The share of intermediate inputs in total sales. [NBER-CES Manufacturing Industry Database]

Equipment per Worker, Structure per Worker [NBER-CES Manufacturing Industry Database]

Revealed Comparative Advantage The share of sector s in country i's manufacturing export relative to the world average. [Trade and Production Database]

Data References

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(5) FAOSTAT Agricultural Data 2004. Food and Agriculture Organization of the United Nations, 2004.

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(8) International Energy Annual 2002. U.S. Department of Energy, 2004.

(9) Occupational Employment Statistics 1998. Bureau of Labor Statistics, 2004.

(10) Trade and Production Database. World Bank, 2001.

(11) UNIDO Industrial Statistics Database 2002, 3-digit ISIC, Revision 2. United Nations Industrial Development Organization, 2002.

(12) World Development Indicators, World Bank, 2002.

Table 1. Yield Volatility and Development

	Variance of wheat yield		
GDP per capita	– 0.4004***	– 0.3731***	– 0.3862***
	(0.1189)	(0.1291)	(0.1325)
Population		0.0634	
		(0.1041)	
Temperature volatility			– 0.2413
			(0.1480)
Rainfall volatility			0.0319
			(0.1159)
Observations	20	20	20
Adjusted R–squared	0.3524	0.329	0.377

Notes: Standard errors are in parentheses. *, **, *** denote significance at 10, 5, 1%. All variables are in logs. Dependent variable is the variance of log wheat yield per acre. (Source: FAOSTAT, Food and Agriculture Organization of the UN.) Temperature volatility is the variance of annual temperature changes. Rainfall volatility is the variance of percentage annual rainfall changes. (Source: Global Historical Climatology Network.)

Table 2. Power Plant Diversity and Energy Production

	Electricity production	Share of oil in energy
Change in oil price	– 0.0235*	0.0003
	(0.0121)	(0.0038)
Change in oil price × Diverse power plant types	0.0400**	– 0.0103**
	(0.0169)	(0.0053)
Time trend	– 0.0021***	0.0007***
	(0.0005)	(0.0002)
Observations	4169	4164
Country Fixed Effects	217	217
Adjusted R–squared	0.089	0.017

Notes: Standard errors are in parentheses. *, **, *** denote significance at 10, 5, 1%. Dependent variables are (1) the log change in electricity production in kWh, (2) the %point change in the share of oil consumption in total energy consumption (in British thermal units). Oil price change is the 2– year change in U.S. CPI– deflated price of West Texas Intermediate oil. The concentration of power plants is measured as the Herfindahl index of shares of power generation techniques (conventional, nuclear, hydroelectric, renewable) in total electricity production of the country. Countries with "diverse power plant types" are those with below-median concentration. (Source: International Energy Annual 2002, Energy Information Administration.)

Table 3. List of Countries. Development and Size

Country	Real GDP per capita	Population (thousands)	Sectoral Data	Country	Real GDP per capita	Population (thousands)	Sectoral Data	Country	Real GDP per capita	Population (thousands)	Sectoral Data
Albania	2789	3339	No	Gambia, The	1489	1186	No	Niger	729	9770	No
Algeria	4798	29000	No	Georgia	2130	5431	No	Nigeria	830	118000	No
Angola	1853	12000	No	Germany	22636	82100	Yes	Norway	27782	4404	Yes
Antigua and Barbuda	9624	66	No	Ghana	1767	18000	No	Pakistan	1802	128000	Yes
Argentina	12342	35700	Yes	Greece	14382	10500	No	Panama	5478	2719	Yes
Armenia	2124	3786	Yes	Grenada	5823	96	No	Papua New Guinea	2379	4761	No
Australia	22967	18500	Yes	Guatemala	3549	10500	Yes	Paraguay	4625	5085	No
Austria	23471	8072	Yes	Guinea	1862	6922	No	Peru	4667	24400	Yes
Azerbaijan	2103	7838	No	Guinea–Bissau	926	1126	No	Philippines	3872	71300	Yes
Bahamas	15190	289	No	Guyana	3969	749	No	Poland	7703	38700	Yes
Bahrain	14561	620	No	Haiti	1420	7492	No	Portugal	15103	9945	Yes
Bangladesh	1388	124000	No	Honduras	2471	5939	Yes	Romania	6512	22600	Yes
Barbados	13590	265	No	Hong Kong	23734	6502	Yes	Russian Federation	7184	147000	No
Belarus	6073	10100	No	Hungary	10244	10200	Yes	Rwanda	798	7895	No
Belgium	24151	10200	No	Iceland	25557	272	No	Samoa	4213	167	No
Belize	4810	217	No	India	2034	962000	Yes	Saudi Arabia	11033	19200	No
Benin	897	5794	No	Indonesia	3217	200000	Yes	Senegal	1341	8777	No
Bhutan	1239	737	No	Iran	5416	60900	Yes	Sierra Leone	511	4726	No
Bolivia	2309	7767	Yes	Ireland	21287	3670	Yes	Singapore	20560	3794	Yes
Botswana	6552	1533	No	Israel	18382	5836	No	Slovak Republic	9902	5383	No
Brazil	7065	164000	No	Italy	21499	57500	Yes	Slovenia	14586	1986	No
Brunei	17086	313	No	Jamaica	3563	2554	No	Solomon Islands	2279	404	No
Bulgaria	4860	8312	Yes	Japan	25405	126000	Yes	South Africa	8978	40700	Yes
Burkina Faso	874	10500	No	Jordan	3899	4459	Yes	Spain	16623	39300	Yes
Burundi	575	6418	No	Kazakhstan	4713	15300	No	Sri Lanka	3041	18600	Yes
Cambodia	1345	11200	No	Kenya	1038	28000	Yes	St. Kitts and Nevis	10937	41	No
Cameroon	1535	13900	Yes	Korea, Rep.	15295	46000	Yes	St. Lucia	5358	150	No
Canada	24557	30000	Yes	Kuwait	15748	1809	Yes	St. Vincent	4779	112	No
Cape Verde	4091	404	No	Kyrgyzstan	2403	4681	No	Sudan	1465	29300	No
Central African Rep.	1077	3529	No	Lao PDR	1384	4919	No	Suriname	3669	412	No
Chad	839	7086	No	Latvia	5793	2469	Yes	Swaziland	4038	960	No
Chile	8740	14600	Yes	Lebanon	4276	4146	No	Sweden	21231	8849	Yes
China	3152	1230000	Yes	Lesotho	1865	1945	No	Switzerland	25973	7088	No
Colombia	5891	40000	Yes	Lithuania	6530	3706	No	Syria	3265	15000	No
Comoros	1618	518	No	Luxembourg	36355	422	No	Tajikistan	941	6017	No

Table 3 continued.

Country	Real GDP per capita	Population (thousands)	Sectoral Data	Country	Real GDP per capita	Population (thousands)	Sectoral Data	Country	Real GDP per capita	Population (thousands)	Sectoral Data
Congo, Dem. Rep.	791	46800	No	Macao, China	18970	418	No	Tanzania	484	31300	No
Congo, Rep.	997	2767	No	Macedonia	4433	1997	No	Thailand	6591	59400	Yes
Costa Rica	6993	3577	Yes	Madagascar	784	14100	No	Togo	1526	4135	No
Cote d'Ivoire	1617	14700	No	Malawi	575	9665	Yes	Trinidad and Tobago	7417	1278	Yes
Croatia	7140	4447	No	Malaysia	8562	21700	Yes	Tunisia	5481	9215	No
Cyprus	17560	744	Yes	Maldives	6316	256	No	Turkey	6626	62500	Yes
Czech Republic	13265	10300	No	Mali	723	10100	No	Turkmenistan	2620	4779	No
Denmark	24897	5284	Yes	Malta	14355	383	No	Uganda	1078	20400	No
Dominican Republic	4903	7968	No	Mauritania	1601	2415	No	Ukraine	3408	50700	No
Ecuador	3273	11900	Yes	Mauritius	8425	1148	No	United Arab Emirates	20561	2580	No
Egypt, Arab Rep.	3141	60400	Yes	Mexico	7839	93900	Yes	United Kingdom	21006	59000	Yes
El Salvador	4135	5911	No	Moldova	2261	4312	No	United States	30123	272000	Yes
Equatorial Guinea	3478	421	No	Mongolia	1619	2331	No	Uruguay	8850	3265	Yes
Eritrea	821	3773	No	Morocco	3335	27300	Yes	Uzbekistan	2137	23700	No
Estonia	8087	1427	No	Mozambique	705	16600	No	Vanuatu	2778	179	No
Ethiopia	629	59800	Yes	Namibia	5778	1648	No	Venezuela	6211	22800	Yes
Fiji	4716	783	No	Nepal	1221	21400	No	Vietnam	1711	75500	No
Finland	20979	5140	Yes	Netherlands	22464	15600	Yes	Yemen	773	16100	No
France	21647	58200	Yes	New Caledonia	22241	202	No	Zambia	784	9443	No
French Polynesia	20541	224	No	New Zealand	18256	3761	Yes	Zimbabwe	2810	11900	No
Gabon	6535	1137	No	Nicaragua	2138	4680	No				

The list includes all countries used in the analysis. The sectoral– data column indicates the countries for which sectoral data are available

Table 4. Volatility and Development

	Variance of GDP growth (log)	Variance of GDP growth (log)
GDP per capita	– 0.2952***	– 0.3259***
(1995 international dollars, log)	(0.0699)	(0.0655)
Population (log)		– 0.1353***
		(0.0376)
Constant	– 3.6316***	– 1.2573
	(0.5968)	(0.8663)
Observations	167	167
Adjusted R–squared	0.10	0.15

Notes: Robust standard errors are in parentheses. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 5. List of Sectors. Technological Diversification Index and Average Share in Manufacturing

ISIC code	Description	Index of Technological Diversification	Average Share in Manufacturing
311	Food products	2.898	0.1581
313	Beverages	2.975	0.0282
314	Tobacco	2.666	0.0148
321	Textiles	1.733	0.1210
322	Wearing apparel, except footwear	2.303	0.0818
323	Leather products	3.278	0.0089
331	Wood products, except furniture	2.368	0.0345
332	Furniture, except metal	2.909	0.0223
341	Paper and products	2.433	0.0242
342	Printing and publishing	2.340	0.0371
351	Industrial chemicals	2.835	0.0235
352	Other chemicals	2.808	0.0342
353	Petroleum refineries	2.726	0.0099
354	Miscellaneous petroleum and coal products	2.726	0.0023
355	Rubber products	2.217	0.0167
356	Plastic products	1.847	0.0251
361	Pottery, china, earthenware	3.006	0.0065
362	Glass and products	3.006	0.0093
369	Other non– metallic mineral products	3.006	0.0438
371	Iron and steel	2.618	0.0297
372	Non– ferrous metals	3.111	0.0117
381	Fabricated metal products	2.849	0.0589
382	Machinery, except electrical	2.817	0.0662
383	Machinery, electric	2.487	0.0637
384	Transport equipment	2.722	0.0548
385	Professional and scientific equipment	2.999	0.0116

Notes: Sectors correspond to the 3-digit manufacturing sectors from Revision 2 of the International Standard Industrial Classification of all Economic Activities (ISIC). Technological diversification measures the diversity of capital goods a sector purchases in the U.S. Average share is the sector's share in manufacturing employment averaged across countries. (See Data Appendix.)

Table 6. Productivity, Volatility and Development

	Log Productivity	Log Variance
GDP per capita (log)	0.7047***	– 0.3005***
	(0.0690)	(0.1020)
Sector Fixed Effects	Yes	Yes
Observations	1429	1429
Adjusted R–squared	0.73	0.14

Notes: The regressions use sectoral data at the 3– digit level and include sector–specific effects. Mean labor productivity and variance of labor productivity growth rates correspond to the period 1963– 1998. Robust standard errors are adjusted for country clustering. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 7. Productivity and Volatility within Sectors

	Log Variance
Labor productivity (log)	– 0.2871***
	(0.1049)
Sector Fixed Effects	Yes
Observations	1521
Adjusted R–squared	0.13

Notes: The regression uses sectoral data at the 3– digit level and include sector–specific effects. Mean labor productivity and variance of labor productivity growth rates correspond to the period 1963– 1998. Robust standard errors are adjusted for country clustering. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 8. Productivity, Volatility and Technological Diversification

	Log Productivity	Log Variance
Technological Diversification	0.0660***	– 0.0999**
	(0.0234)	(0.0448)
Sectoral Skill Intensity (log)	0.2351***	– 0.3005***
	(0.0441)	(0.0919)
Share of Materials in Production (log)	0.2155***	0.0289
	(0.0634)	0.0939
Equipment per Worker (log)	0.4613***	0.0062
	(0.0184)	0.0211
Labor Share (Log)	– 0.0532***	– 0.1727***
	(0.0151)	(0.0240)
Country Fixed Effects	Yes	Yes
Observations	1535	1535
Adjusted R–squared	0.81	0.56

Notes: The equations use sectoral data at the 3– digit level and include country–specific effects. Mean labor productivity and variance of labor productivity growth rates correspond to the period 1963– 1998. Technological diversification measures the diversity of capital goods a sector purchases in the U.S. Skill intensity is the share of skilled and semi-skilled workers. Material share is the ratio of material costs to total shipments. Labor share is the sector's share in manufacturing employment. (See Data Appendix.) Robust standard errors are adjusted for country clustering. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 9. Sectoral Shares, Technological Diversification, and Development

	Sectoral Share	Sectoral Share
Technological Diversification × Real GDP per capita (Log)	0.0083*	0.0089**
	(0.0044)	(0.0044)
Technological Diversification	– 0.1015**	– 0.1075***
	(0.0404)	(0.0408)
Real GDP per capita (Log)	– 0.0248**	
	(0.0115)	
Country Fixed Effects	No	Yes
Observations	1429	1429
Adjusted R–squared	0.05	0.12

Notes: The equations use sectoral data at the 3– digit level. The dependent variable is the share of the sector in total manufacturing employment, averaged over 1963–1998. The second column includes country– specific effects. Technological diversification measures the diversity of capital goods a sector purchases in the U.S. (See Data Appendix.) Robust standard errors clustered by country are shown in parentheses. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 10. Sectoral Shares, Volatility, and Development

	Sectoral Share	Sectoral Share
Sectoral Variance × Real GDP per capita (Log)	– 0.0006***	– 0.0015***
	(0.0002)	(0.0004)
Sectoral Variance	– 0.0075	– 0.0018
	(0.0061)	(0.0060)
Real GDP per capita (Log)	– 0.0064***	
	(0.0014)	
Country Fixed Effects	No	Yes
Observations	1429	1429
Adjusted R–squared	0.04	0.10

Notes: The equations use sectoral data at the 3– digit level. The dependent variable is the share of the sector in total manufacturing employment, averaged over 1963–1998. The second column includes country– specific effects. Variance of labor productivity growth rates correspond to the period 1963– 1998. Robust standard errors clustered by country are shown in parentheses. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 11. Relative Size and Volatility

	Log Variance	Log Variance	Log Variance
Labor Share	– 0.1554***	– 0.1743***	– 0.1893***
	(0.0256)	(0.0233)	(0.0293)
Country Fixed Effects	No	Yes	Yes
Sector Fixed Effects	No	No	Yes
Observations	1521	1521	1521
Adjusted R–squared	0.03	0.55	0.59

Notes: The equations use sectoral data at the 3– digit level. Labor share is the sector's share in total manufacturing employment, averaged over 1963–1998. Robust standard errors are adjusted for country clustering. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 12. Comparative Advantage and Volatility

	Log Variance	Log Variance	Log Variance
Revealed Comparative Advantage (log)	– 0.0893***	– 0.0721***	– 0.0557***
	(0.0215)	(0.0149)	(0.0148)
Country Fixed Effects	No	Yes	Yes
Sector Fixed Effects	No	No	Yes
Observations	1621	1621	1621
Adjusted R–squared	0.01	0.43	0.47

Notes: The equations use sectoral data at the 3– digit level. Revealed comparative advantage (Balassa 1965) is the country's export share in the given sector relative to the world average export share. Robust standard errors are adjusted for country clustering. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 13. Other Institutions Reducing Volatility

	Log Variance			
Real GDP per capita (log)	–0.264*** (0.082)	–0.163* (0.092)	–0.350*** (0.111)	–0.256** (0.120)
Private credit / GDP		–0.523* (0.289)		–0.473 (0.330)
Labor market rigidities			0.846** (0.420)	0.819* (0.431)
Sector Fixed Effects	Yes	Yes	Yes	Yes
Observations	1607	1607	1319	1319
Adjusted R–squared	0.11	0.12	0.16	0.17

Notes: Robust standard errors (in parentheses) allow for clustering within countries. * Significant at 10%; ** significant at 5%; *** significant at 1%. Complex sectors are those with above median investment good diversification. Labor market rigidities are measured by an index that combines the costs of firing workers and changing employment terms (Botero, Djankov, La Porta, Schleifer and Lopez–de–Silanes, 2004).

Table 14. Output versus Input Diversification

	Log Variance (all sectors)	Log Variance (complex sectors)
Number of firms (log)	–0.1140*** (0.0382)	–0.1357*** (0.0487)
Average size of firms (log)	–0.1550*** (0.0516)	–0.1887*** (0.0603)
Country Fixed Effects	Yes	Yes
Sector Fixed Effects	Yes	Yes
Observations	1586	932
Adjusted R–squared	0.48	0.43

Notes: Robust standard errors (in parentheses) allow for clustering within countries. * Significant at 10%; ** significant at 5%; *** significant at 1%. Complex sectors are those with above-median investment good diversification.

Figure 1. GDP Volatility and Development

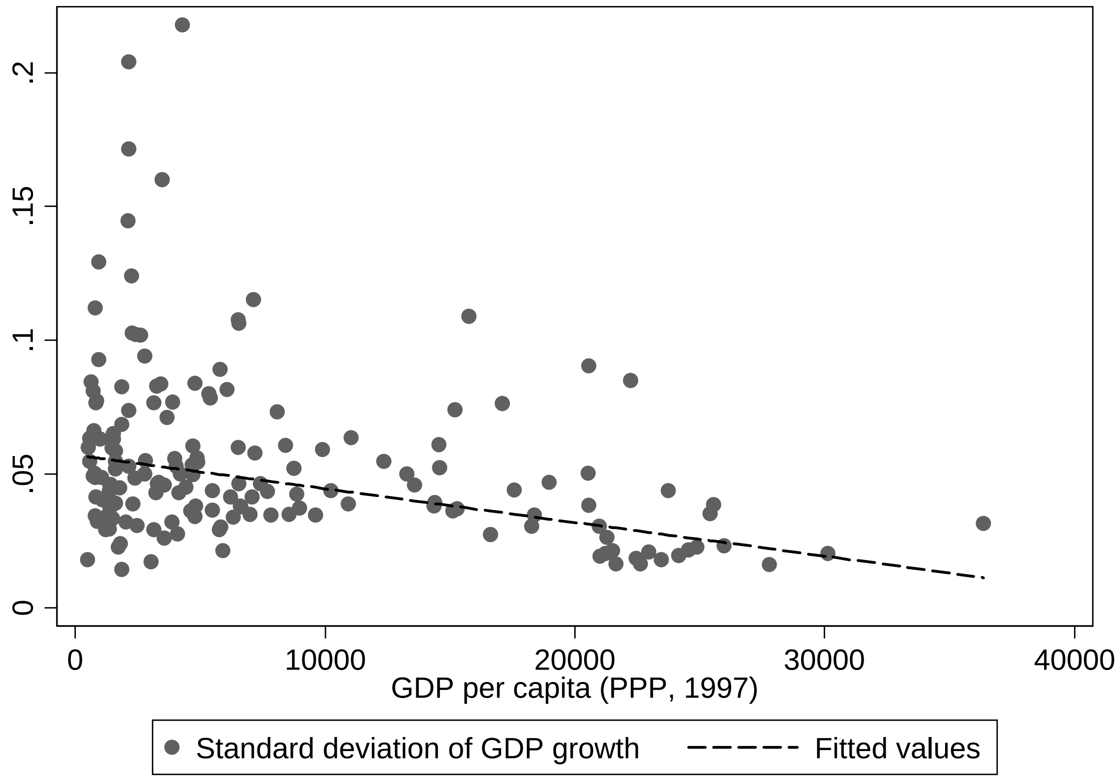

Figure 2. Wheat Yield Volatility and Development