Optimal Monetary Policy in a Channel System of Interest-Rate Control*

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Abstract

This paper studies optimal interest-rate policies when the central bank operates a channel system of interest-rate control. We conduct our analysis in a dynamic general equilibrium model with infinitely-lived agents who are subject to idiosyncratic trading shocks which generate random liquidity needs. In response to these shocks agents either borrow against collateral or deposit money at the central bank at the specified rates. We show that it is optimal to have a strictly positive interest-rate corridor if the opportunity cost of holding collateral is strictly positive and that the optimal corridor is strictly decreasing in the collateral's real return.

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1 Introduction

In this paper we analyze monetary policy when the central bank operates a channel system of interest-rate control. In a channel system a central bank offers two standing facilities to commercial banks that clear transactions through the central bank. A lending facility where it is ready to supply money overnight at a given lending rate and a deposit facility where banks can deposit excess money overnight at a deposit rate. The interest-rate corridor - defined by the difference between the lending and the deposit rates - is chosen to keep the overnight interest rate in the money market close to the target interest rate. Several central banks now operate channel systems.[1] For example, the European Central Bank (ECB) offers a borrowing facility with a lending rate that is 100 basis points higher than its policy interest rate and a deposit facility with a deposit rate, which is 100 basis points below its policy rate.

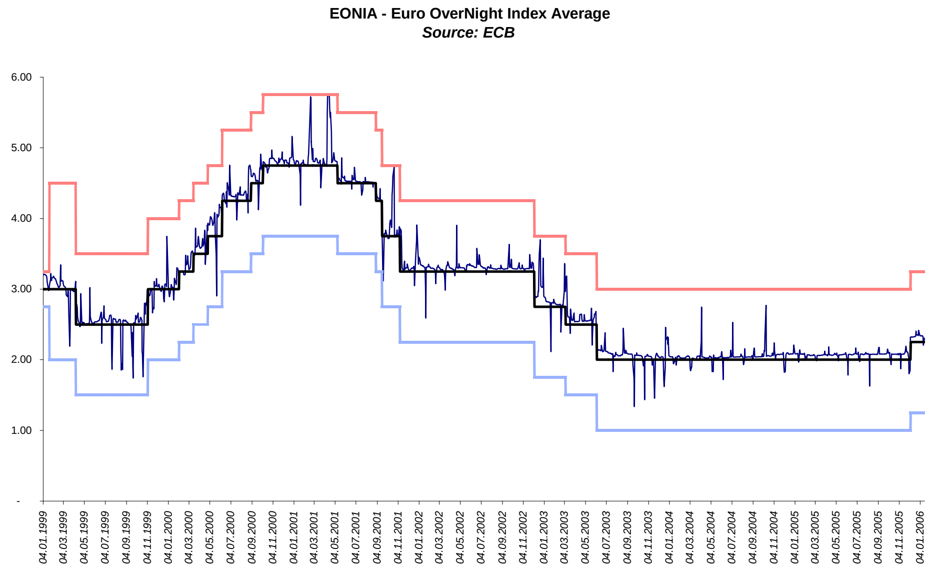

Figure 1 displays the interest-rate corridor operated by the ECB. The solid red curve is the lending rate and the solid blue line the deposit rate. The black line is the overnight interest rate that the ECB targets via its channel system. Central banks

[1] Channel systems of interest rate controls are operated by the Bank of Canada, the European Central Bank, the Reserve Bank of Australia, or the Reserve Bank of New Zealand (see Woodford 2000 for more details).

typically react to changing economic conditions by shifting the interest-rate corridor. Figure 1 illustrates such shifts by the ECB. However, occasionally central banks also change the size of their interest-rate corridor as can be seen from Figure 1 where the ECB increased its corridor dramatically from 50 basis points to 200 basis points around February 1999.

In a channel system there is no limit to the size of deposits on which interest is paid. There is also no limit to the size of a loan that a commercial bank can obtain provided that the loan is collateralized. Collateral are typically low-risk and low-yield assets such as government securities. The rate of return of the collateral determines the opportunity costs for commercial banks of accessing the lending facility of the central bank where a high rate of return implies a small or zero opportunity cost.

We consider three questions in this paper. First, what is the optimal interest-rate corridor in a channel system of interest-rate control? Second, how is the optimal corridor affected by the opportunity cost of holding collateral? Third, how should a central bank modify its corridor when it reacts to changing economic conditions?

To answer these questions we construct a dynamic general equilibrium model with infinitely-lived agents and a central bank.[2] The agents are subject to idiosyncratic trading shocks which generate random liquidity needs. Due to these shocks there is an ex-post inefficiency in that some agents are holding idle balances while others are cash constrained. To reduce or eliminate this inefficiency the central bank operates a standing facility where agents either borrow or deposit money at the specified rates. The central bank cannot force agents to repay their loans and so in accordance with central bank practice we assume that the central bank only provides collateralized loans.

The following results emerge from the model. With respect to the first question we show that it is optimal to have a strictly positive interest-rate corridor if the

[2] We abstract from modelling commercial banks explicitly. Rather, we assume that all agents have direct access to the central bank's lending and deposit facility. The trading shocks are an approximation for the money market where banks receive liquidity shocks at the end of the day. Since there is no trading of reserves feasible after this market, banks who need liquidity have no choice but to use the standing facility offered by the central bank.

opportunity cost of holding collateral is strictly positive. The optimal size of the corridor depends on parameters of the model - such as the nature of the trading shocks, preferences and production technology - and we show how the channel should be adjusted to changes in these parameters. With respect to the second question we show that the optimal corridor is strictly decreasing in the rate of return of the collateral (which means that it is increasing in the opportunity cost of holding collateral). When the rate of return of the collateral attains the point where the opportunity cost of acquiring collateral is zero it is optimal to set deposit and lending rates equal. Finally, with respect to the third question we show that a central bank that must change its policy in response to a change in economic conditions has two options. It can either shift the interest-rate corridor while keeping the size of the band constant as illustrated in Figure 1, or, it can change the size of the interest-rate band. For instance, it can keep the deposit rate constant and increase the borrowing rate.[3]

These results are intuitive. When the opportunity cost of holding collateral is strictly positive, the optimal monetary policy trades-off the cost of holding collateral and the consumption flow from borrowing at the facility. When collateral is costly to hold, agents should optimally not hold too much collateral. This is achieved by increasing the cost of transforming collateral into money, that is by increasing the interest rate corridor. The larger the interest rate corridor, the more costly it will be to transform collateral into money. By modifying the liquidity properties of collateral, monetary policy affects the portfolio decision of agents and as a consequence the real allocation.

An interesting aspect of our model is that money growth and hence inflation is endogenous unlike in most theoretical analysis of monetary policy that character-

[3]Interestingly the US Federal Reserve System recently modified the operating procedures of its discount window facility. Prior to 2003, the discount window rate was set below the target federal fund rate, but banks faced penalties when accessing the discount window. As a result, the window was rarely used. In 2003, in an effort to encourage the use of its discount window, the Federal Reserve decided to set the discount window rate 100 basis point above the target federal fund rate and eased access conditions to the discount window. The resulting framework shares some properties with a channel system of interest-rate control, where the deposit rate is zero (which is equivalent of not allowing to deposit) and the lending rate 100 basis point above the target rate.

ize optimal policy in terms of a path for the money supply. In practice, however, monetary policy involves rules for setting nominal interest rates and most central banks specify operating targets for overnight interest rates. This paper therefore is an attempt to break the apparent dichotomy (Goodhard, 1989) between theoretical analysis and central bank practices.

1.1 Background[4]

To understand some of the features of our environment it is useful to have some information on monetary policy procedures at central banks that operate a standing facility. This section does not aim at being general and we will therefore concentrate on the case of the ECB's operating procedures. The ECB has two main instruments for the conduct of its monetary policy.

First, it conducts weekly main refinancing operations that are collateralized loans with a one week maturity. Main refinancing operations are implemented using a liquidity auction where banks bid for liquidity. Bids consist of an amount of liquidity and an interest rate. The total amount to be allocated is announced before the auction. Following the auction, the ECB allocates liquidity according to the rates, in a descending order. The minimum bid rate is the main policy rate used by the ECB to implement monetary policy.

Second, it offers a standing facility with a lending rate that is 100 basis points higher than its minimum bid rate and a deposit rate, which is 100 basis points below its minimum bid rate. Loans are provided overnight at the standing facility and have to be fully collateralized with eligible assets. Eligible banks can access the standing facilities at any time of the day. They may also access the facilities after making a request at the latest 30 minutes after the actual closing time of TARGET, the large value payment system of the Eurosystem through which central bank operations are settled.[5] An overdraft position on a bank's TARGET account is automatically

[4] See ECB (2004) for more details of the ECB's implementation of monetary policy.

[5] On the last Eurosystem business day of a minimum reserve maintenance period, the deposit facility can be accessed for 60 minutes after the actual closing of TARGET.

transformed into an overnight loan via a recourse to the lending facility against eligible assets.

While the main refinancing operations aim at managing the liquidity conditions in the market, the standing facilities is intended to satisfy bank's temporary needs for liquidity. We will concentrate the analysis on the second aspect, and we abstract from modelling liquidity injections in an interbank market. Rather we take the view that transactions on the interbank market have already taken place and we concentrate on bank's temporary needs for liquidity that are satisfied via the standing facilities.

1.2 Literature

There are very few theoretical studies of channel systems of interest-rate control. In a series of papers Woodford (2000, 2001, 2003) discusses and analyses the channel system of interest-rate control.[6] His very careful analysis of the channel system is complementary to ours. We depart from his approach by considering a general equilibrium model where the demand for base money (settlement balances) is endogenous. Moreover, we conduct a welfare analysis and derive the optimal interest-rate corridor and we show how the interest-rate corridor affects the growth rate of settlement balances which is endogenous is such a system.[7]

Some other aspects of our model appear in other papers. These papers however consider issues that are not related with the analysis of a channel system of interest-rate control. For example, Lagos and Rocheteau (2004) and Kiyotaki and Moore

[6] The starting point of his discussion is the possibility that in the future the demand for base money could shrink considerably or even be eliminated due to innovations in the payment system. This possibility has led many authors to speculate that central banks could loose their ability to control aggregate spending via their monetary policy. Friedman (1999) argues along this line in his paper "The Future of Monetary Policy: The Central Bank as an Army with only a Signal Corps." In his 2000 paper Woodford argues that even when base base is "largely or even completely eliminated, monetary policy should continue to be effective. Macroeconomic stabilization depends only upon the ability of central banks to control a short-term nominal interest rate, and this would continue to be possible, in particular through the use of a channel' system for the implementation of policy, like those currently used in Canada, Australia and New Zealand."

[7] It is endogenous in our model because we assume that there are no open market operations. In practice the central bank can operate a channel system and engage in open market operations and by doing so change the stock of settlement balances.

(2003) study how illiquid capital interacts with the use of fiat money. The role of collateral for monetary policy is for example investigated in Buiter and Sibert (2005) and Braggion, Christiano and Roldos (2005).

The paper is structured as follows. Section 2 outlines the environment. The equilibrium is characterized in Section 3 and the optimal monetary policy is derived in Section 4. Section 6 concludes.

2 Environment

There is a [0,1] continuum of infinitively-lived agents. Time is discrete and in each period two perfectly competitive markets open sequentially. The first market is a settlement stage where all agents produce and consume a general good and settle their claims from the previous period with the central bank. General goods are produced solely from inputs of labor according to a constant return to scale production technology where one unit of the consumption good is produced with one unit of labor generating one unit of disutility. Thus, producing h units of the general good implies disutility $-h$, while consuming h units gives utility h.[8]

In the second market agents produce or consume a perishable good. At the beginning of this market, agents receive idiosyncratic preference and technology shocks which determine whether they consume or produce in market 2. With probability $1-n$ an agent can consume and cannot produce. We refer to these agents as buyers. With probability n, an agent can produce and cannot consume. These are sellers. Agents get utility $u(q)$ from q consumption in the second market, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. Producers incur a utility cost $c(q) = q$ from producing q units of output. All trades are anonymous and agents' trading histories are private information. Since sellers require immediate compensation for their

[8] The environment is similar to the one introduced by Berentsen, Camera and Waller (2004). The linear preferences in market 1, first introduced by Lagos and Wright (2005) to get a degenerate distribution of money holdings at the beginning of a period, allows us to interpret transactions that are taking place in the first market as settlement transactions, as in Koeppl, Monnet and Temzelides (2005).

production effort money is essential for trade.[9] The discount factor is β where for technical reasons we assume that $\beta > n$.

Standing facility Since our agents are subject to trading shocks there is an ex-post inefficiency in that sellers are holding idle balances while buyers are cash constrained.[10] To reduce or eliminate this inefficiency the central bank operates a standing facility. It offers nominal loans ℓ at an interest rate i and promises to pay interest $i_d d$ on nominal deposits d with $i \geq i_d$.[11] Since we focus on standing facilities, we restrict financial contracts to overnight contracts. An agent who borrows ℓ units of money from the central bank in market 2, repays $(1+i)\,\ell$ units of money in market 1 of the following period. Also, an agent who deposits d units of money at the central bank in market 2 of period t receives $(1+i_d)\,d$ units of money in market 1 of the following period.

Accordingly, in the absence of open market operations, the money stock evolves endogenously as follows

$$M_{+1} = M - (1-n)i\ell + ni_d d, \tag{1}$$

where M denotes the per capita stock of money at the beginning of period t. In the first market total loans $(1-n)\ell$ are repaid. Since interest rate payments by the agents are $(1-n)i\ell$, the stock of money shrinks by this amount. Interest payments by the central bank on total deposits are $ni_d d$. The central bank simply prints additional money to make these interest payments so the stock of money increases by this amount. The central bank operates the standing facility at zero cost. Consequently, the central bank cannot make profits or losses.

[9] By essential we mean that the use of money expands the set of allocations (Kocherlakota 1998 and Wallace 2001).

[10] Models with this property include Bewley (1980), Levine (1991), Green and Zhou (2005), and Berentsen, Camera and Waller (2004).

[11] This restriction eliminates the possibility for arbitrage where agents borrow and subsequently make a deposit at interest $i_d > i$, thus increasing their money holdings at no cost.

Default In any model of credit, default is a serious issue. Since production is costly, those agents who borrow in market 2 have an incentive to default in market 1 of the following period. To prevent default the central bank requires general goods as collateral for each loan. We assume that general goods that are produced in market 1 can be stored at the central bank with a constant return to scale technology that yields $R \geq 1$ units of general goods in market 1 of the following period. We also impose $\beta R \leq 1$ since when $\beta R > 1$ agents would store infinite amounts of goods which is inconsistent with equilibrium. General goods can *only* be stored at the central bank. Consequently, general goods cannot be used to issue collateralized IOU's among private agents.

First-best allocation The expected lifetime utility of the representative agent for a stationary allocation (q, b) at the beginning of a period is given by

$$(1-\beta)\,\mathcal{W} = (1-n)\,[u(q)-q] + (\beta R - 1)\,b \tag{2}$$

The first term on the right-hand side is the expected utility from consuming and producing the market 2 good. The second term is the utility of producing collateral and receiving the return in the following period.

It is obvious that the first-best allocation (q^*, b^*) satisfies $q = q^*$ where q^* is the value of q that solves $u'(q) = 1$. Moreover, $b^* = 0$ if $\beta R < 1$ and b^* is indeterminate if $\beta R = 1$. Thus, a social planner would never choose a positive amount of collateral when collateral is costly.

3 Symmetric stationary equilibrium

In period t, let $\phi \equiv 1/P$ be the real price of money in market 1. We focus on symmetric and stationary equilibria where all agents follow identical strategies and where real allocations are constant over time. In a stationary equilibrium beginning-of-period real money balances are time invariant

$$\phi M = \phi_{+1} M_{+1}. \tag{3}$$

This implies that $\phi_{+1}/\phi = P_{+1}/P = M/M_{+1} = \gamma$. Moreover, we restrict our attention to stationary equilibria where γ is time invariant.

We let $V(m, b)$ denote the expected value from entering market 2 with m units of money and b collateral. $W(m, b, \ell, d)$ denotes the expected value of entering the first market with m units of money, b collateral, ℓ loans, and d deposits. For notational simplicity we suppress the dependence of the value function on the time index t.

In what follows we look at a representative period t.

3.1 Market 1: settlement

In the first market, the problem of a representative agent is:

$$\begin{aligned} W(m, b, \ell, d) &= \max_{h, m_2, b_2} -h + V(m_2, b_2) \\ s.t. \quad \phi m_2 + b_2 &= h + \phi m + Rb + \phi(1 + i_d)d - \phi(1 + i)\ell. \end{aligned}$$

where h is hours worked in market 1. Using the budget constraint to eliminate h in the objective function, one obtains the first-order conditions[12]

$$V_m = \phi \tag{4}$$

$$V_b \leq 1 \ (= \text{ if } b > 0 \ ) \tag{5}$$

$V_m \equiv \frac{\partial V(m_2, b_2)}{\partial m_2}$ is the marginal value of taking an additional unit of money into the second market in period t. Since the marginal disutility of working is one, $-\phi$ is the utility cost of acquiring one unit of money in the first market of period t. $V_b \equiv \frac{\partial V(m_2, b_2)}{\partial b_2}$ is the marginal value of taking additional collateral into the second market in period

[12] We focus on monetary equilibria where (4) holds with equality. In contrast, there are monetary equilibria where agents do not use the standing facility implying $b = 0$ because $V_b < 1$.

t. Since the marginal disutility of working is 1, -1 is the utility cost of acquiring one unit of collateral in the first market of period t. The implication of (4) and (5) is that all agents enter the following period with the same amount of money and the same quantity of collateral (which can be zero). This is the reason why, as in Koeppl, Monnet and Temzelides (2005), we interpret this market as a settlement stage. By itself, this market does not increase social welfare. Rather, it involves a mere transfer of an asset between participants in order to settle claims from the previous period.

The envelope conditions are

$$W_m = \phi; W_b = R; W_\ell = -\phi\left(1+i\right); W_d = \phi\left(1+i_d\right) \tag{6}$$

where W_j is the partial derivative of $W(m, b, \ell, d)$ with respect to $j = m, b, \ell, d$.

3.2 Market 2: liquidity shocks

At the beginning of market 2, agents receive idiosyncratic shocks which determine whether they are consumers or producers. With probability $1-n$ an agent becomes a consumer and with probability n a producer. Let q and q_s respectively denote the quantities consumed by a buyer and produced by a seller in market 2. Let ℓ_b (ℓ_s) and d_b (d_s) respectively denote the loan obtained and the amount of money deposited by a buyer (seller) in market 2. An agent who has m money and b collateral at the opening of market 2 has expected lifetime utility

$$\begin{aligned} V(m,b) &= (1-n)[u(q) + \beta W(m - pq - d_b + \ell_b, b, \ell_b, d_b)] \\ &\quad + n[-q_s + \beta W(m + pq_s - d_s + \ell_s, b, \ell_s, d_s)] \end{aligned}$$

where q, q_s, ℓ_s, ℓ_b, d_s and d_b are chosen optimally as follows.

It is obvious that buyers will never deposit funds in the central bank and sellers will never take out loans and therefore $d_b = 0$ and $\ell_s = 0$. It is also straightforward to show that sellers will deposit as much money as they can if $i_d > 0$ and therefore

$d_s = m + pq_s$. If $i_d = 0$, they are indifferent and we simply assume $d_s = m + pq_s$. Accordingly, we get

$$\begin{aligned} V(m,b) &= (1-n)[u(q) + \beta W(m - pq + \ell, b, \ell, 0)] \\ &\quad + n\left[-q_s + \beta W\left(0, b, 0, m + pq_s\right)\right] \end{aligned}$$

where q_s and q solve the following optimization problems.

A seller's problem is $\max_{q_s} \left[-q_s + \beta W\left(0, b, 0, m + pq_s\right)\right]$. Using (6), the first-order condition reduces to

$$\beta p \phi_{+1} \left(1 + i_d\right) = 1. \tag{7}$$

If an agent is a buyer, he solves the following maximization problem:

$$\begin{aligned} \max_{q,\ell} \quad & u(q) + \beta W(m - pq + \ell, b, \ell, 0) \\ \text{s.t.} \quad & pq \leq m + \ell \text{ and } \ell \leq \bar{\ell} \end{aligned}$$

where

$$\bar{\ell} = Rb / \left[\phi_{+1}\left(1 + i\right)\right] \tag{8}$$

is the maximal amount that a buyer can borrow from the central bank since b units of collateral transform into Rb units of real goods at the beginning of the following period. These goods can be sold for Rb/ϕ_{+1} units of money. Finally, the collateral must also cover the interest payment.

Using (6) the buyer's first-order conditions can be written as

$$u'(q) = p\beta\phi_{+1}(1 + \lambda_q) \tag{9}$$

$$\lambda_q = \lambda_\ell + i \tag{10}$$

where $\beta\phi_{+1}\lambda_q$ is the multiplier of the buyer's budget constraint and $\beta\phi_{+1}\lambda_\ell$ the one of the borrowing constraint. Using (7) and combining (9) and (10) yields

$$u'(q) = \frac{1 + i + \lambda_\ell}{1 + i_d} \tag{11}$$

If the borrowing constraint is not binding and the central bank sets $i = i_d$, trades are efficient. If the borrowing constraint is binding, then $u'(q) > 1$ which means trades are inefficient even when $i = i_d$.

Using the envelope theorem and (9), the marginal value of money in market 2 is

$$V_m = (1 - n)u'(q)/p + n\beta\phi_{+1}(1 + i_d) \tag{12}$$

The marginal value of money has a straightforward interpretation. An agent with an additional unit of money becomes a buyer with probability $1 - n$ in which case he acquires $1/p$ units of goods yielding additional utility $u'(q)/p$. With probability n he becomes a seller in which case he deposits overnight his money yielding the nominal return $1 + i_d$. Note that the standing facility increases the marginal value of money because agents can earn interest on idle cash.

Since in equilibrium there is no default the real return of collateral is βR. The real return is smaller than the marginal value V_b if $\lambda_\ell > 0$. To see this, use the envelope theorem to derive the marginal value of collateral in the second market

$$V_b = (1 - n)\beta\phi_{+1}\lambda_\ell\frac{\partial \ell}{\partial b} + \beta R \tag{13}$$

Thus, the difference between the real return and the marginal value is $(1 - n)\, \beta\phi_{+1}\lambda_\ell\frac{\partial \ell}{\partial b}$ which is positive if collateral relaxes the borrowing constraints of the buyers. It is critical for the working of the model that $V_b > \beta R$. The reason is that, since $\beta R - 1$ is negative, agents are only willing to hold collateral if the liquidity value as expressed by the shadow price λ_ℓ is positive.

Use (7) and (8) to write (13) as follows:

$$V_b = (1-n)u'(q)\beta R/\Delta + n\beta R. \tag{14}$$

where $\Delta \equiv (1+i)/(1+i_d)$ and $\Delta/\beta R$ is the price of collateral in terms of goods in market 2. A buyer can use the collateral to borrow $\frac{R}{\phi_{+1}(1+i)}$ units of money which allows him to acquire $\frac{R}{p\phi_{+1}(1+i)} = \frac{\beta R(1+i_d)}{1+i} = \beta R/\Delta$ units of goods.

Monetary policy affects the allocation and welfare by its choice of Δ. An increase in Δ increases the cost of acquiring money and hence market 2 goods with collateral.

3.3 Symmetric stationary equilibrium

To define a symmetric stationary equilibrium use the first-order condition (5) and (14) to get

$$\frac{1-R\beta}{R\beta} \geq (1-n)\left[u'(q)/\Delta - 1\right] \;\; (= \text{ if } b > 0 \text{ }). \tag{15}$$

Then (4), (7), (12), and taking into account that in a stationary equilibrium $M_{+1}/M = \phi/\phi_{+1} = \gamma$, yield

$$\frac{\gamma - \beta\left(1+i_d\right)}{\beta\left(1+i_d\right)} = (1-n)\left[u'(q) - 1\right]. \tag{16}$$

Also from (1) we get

$$\gamma = 1 + i_d - (1-n)(i - i_d)\frac{z_\ell}{z_m}, \tag{17}$$

where $z_m = m/p$ and $z_\ell = \ell/p$. To derive this equation we use $d = m + pq_s$, market clearing $nq_s = (1-n)q$ and we we take into account that in symmetric equilibrium all agents hold identical amounts of money when they enter the second market. Then, from the budget constraint of the buyer we have

$$q = z_m + z_\ell. \tag{18}$$

Finally, since $\beta R < 1$ in any equilibrium where agents hold collateral it must be the case that the borrowing constraint is binding and so from (7) and (8)[13]

$$z_\ell = \beta R b / \Delta. \tag{19}$$

We can use these five equations to define a symmetric stationary equilibrium. They determine the endogenous variables $(\gamma, q, z_\ell, z_m, b)$. Note that all other endogenous variables can be derived from these equilibrium values.

Definition 1 *A symmetric stationary equilibrium is a list $(\gamma, q, z_\ell, z_m, b)$ satisfying (15)-(19) with $z_\ell \geq 0$ and $z_m \geq 0$.*

Let

$$\tilde{\Delta} = \frac{1 - n\beta}{1/R - n\beta}. \tag{20}$$

Then we have the following

Proposition 1 *For any (i, i_d) with $i \geq i_d \geq 0$ there exists a unique symmetric stationary equilibrium such that*

$$\begin{aligned} z_\ell > 0 \text{ and } z_m = 0 \quad &\text{if and only if} \quad \Delta = 1 \\ z_\ell > 0 \text{ and } z_m > 0 \quad &\text{if and only if} \quad 1 < \Delta < \tilde{\Delta} \\ z_\ell = 0 \text{ and } z_m > 0 \quad &\text{if and only if} \quad \Delta \geq \tilde{\Delta}. \end{aligned}$$

Several points are worth mentioning. First, the critical element to verify in the proof is under which condition agents acquire collateral. They are willing to borrow at the standing facility if the borrowing rate is not too high, i.e. $\Delta < \tilde{\Delta}$. Second, the critical value $\tilde{\Delta}$ is increasing in R. Moreover, b is increasing in R too. Agents increase their collateral holdings and hence finance a larger share of their consumption

[13] If the borrowing constraint is non-binding ($\lambda_\ell = 0$), equation (13) reduces to $V_b = \beta R$ implying from (5) that $b = 0$ since we have $\beta R < 1$. Consequently, in any equilibrium where agents hold collateral it must be the case that the constraint is binding ($\lambda_\ell > 0$) and so $\ell = \bar{\ell} = Rb / \left[\phi_{+1}(1+i)\right]$ implying $\frac{\partial \ell}{\partial b} = R / \left[\phi_{+1}(1+i)\right]$.

by borrowing if R is increasing. Third, if $\Delta = 1$ agents are not willing to hold money across periods. They just use collateral to borrow money to finance their consumption. This however does not mean that money is not used since it still plays the role of a medium of exchange in market 2. It only means that agents do not want to hold it across periods.

Given a real allocation $(q(\Delta), b(\Delta))$ any pair (i, i_d) satisfying $\Delta = \frac{1+i}{1+i_d}$ is consistent with this allocation. Thus, there are many ways to implement a given policy Δ. The allocations only differ in the rate of inflation. This can be seen from (17) which can be written as follows

$$\frac{\gamma}{1+i_d} = 1 - (1-n)(\Delta - 1)\frac{z_\ell}{z_m}$$

Since the right-hand side is a constant for a given Δ the inflation rate $\gamma - 1$ is increasing in i_d.

In the introduction we have seen that the Reserve Bank of New Zealand (see Figure 1) reacts to changing economic condition by shifting the interest rate corridor $\delta = i - i_d$. An upwards shift of δ increases Δ and so reduces aggregate output q and borrowing z_ℓ. Another way to tighten monetary policy is by increasing the size of the band δ since increasing δ also reduces both q and b.

For the rest of the paper we focus on the real allocation (q, b) since only consumption q and collateral b affect the expected lifetime utility (2). In the proof of Proposition 1 we show that when $1 \leq \Delta < \tilde{\Delta}$, b and q solve

$$\frac{1}{R\beta} = (1-n)u'(q)/\Delta + n \tag{21}$$

$$q = \beta R b F(\Delta) \tag{22}$$

where

$$F(\Delta) = \frac{1}{\Delta}\left[1 + \frac{(1-n)(\Delta-1)}{1+\beta n(\Delta-1) - \Delta/R}\right].$$

These two equations have an intuitive interpretation. (21) defines the liquidity premium on the collateral. While collateral costs -1 to produce, it returns $\beta R \leq 1$. Hence, if $\beta R < 1$, agents need an incentive to hold collateral. This is provided by making collateral liquid. The liquidity premium from holding collateral is $(1 - n)\beta R(u'(q)/\Delta - 1)$. As the return on collateral increases, its liquidity premium will increase. Hence in equilibrium agents will be given less incentives to hold it. This means that the marginal benefit from an additional unit of collateral $u'(q)/\Delta$ will drop as R increases. When interest rates are fixed, q must increase. Alternatively, an increase in Δ affects negatively the liquidity premium, since it is now more difficult to transform collateral into goods. To establish the liquidity premium to its equilibrium level, the marginal benefit of an additional unit of good must rise and therefore q decreases. In this way monetary policy affects the quantities that are traded in equilibrium.

Given the liquidity premium, (22) gives us the amount of collateral an agent holds, which in turn determines the composition of an agent's portfolio. Indeed, we know that $q = z_\ell + z_m = \beta RbF(\Delta)$ and $z_\ell = \beta Rb/\Delta$ from (19). Therefore, $z_m = \beta RbF(\Delta) - \beta Rb/\Delta = \frac{\beta Rb}{\Delta}\frac{(1-n)(\Delta-1)}{1+\beta n(\Delta-1)-\Delta/R}$. Given an amount of collateral b and its return R, a tightening of monetary policy - an increase in Δ - will decrease the liquidity of collateral, so that agents will have more incentives to increase their money holdings z_m.

4 Optimal policy

We now derive the optimal policy. The central bank's objective is to maximize the expected lifetime utility of the representative agent. It does so by choosing consumption q and collateral holding b to maximize (2) subject to constraint that its choice is consistent with the allocation given by (15)-(18). The policy is implemented by choosing Δ.

Assume first that it is optimal to set $\Delta \geq \tilde{\Delta}$. In this case no agent is borrowing at the standing facility which implies that $b = 0$. Moreover, from (16) and (17) q satisfies

$$\tilde{q} = u'^{-1}\left(\frac{1/\beta - n}{1-n}\right).$$

Thus, any $\Delta \geq \tilde{\Delta}$ implements the same real allocation $(b, q) = (0, \tilde{q})$.

Now consider the largest q that the central bank can implement. From (15) the largest q is attained when $\Delta = 1$. It satisfies

$$\hat{q} = u'^{-1}\left[\frac{1/(\beta R) - n}{1-n}\right].$$

Thus, the policy $\Delta = 1$ attains the allocation $(b, q) = (\hat{q}/(\beta R), \hat{q})$ since no agent is holding money across period when $\Delta = 1$. Accordingly, the central bank's is constrained to choose quantities q such that $\hat{q} \geq q \geq \tilde{q}$.

Finally, if the optimal policy satisfies $\tilde{\Delta} > \Delta \geq 1$ the central bank is constrained to choose an allocation that satisfies (21) and (22). Accordingly, the central bank's maximization problem is

$$\begin{aligned} \max_{q,b} \quad & (1-n)\left[u(q) - q\right] + (\beta R - 1)\, b \\ \text{s.t.} \quad & q = \beta b R F\left(\frac{R\beta(1-n)u'(q)}{1 - nR\beta}\right) \\ & \text{and } \hat{q} \geq q \geq \tilde{q}. \end{aligned} \tag{23}$$

where to derive (23) we use (21) to replace Δ in (22).

Proposition 2 *There exists a critical value $\overline{R}$ such that if $R < \overline{R}$, then the optimal policy is $\Delta \geq \tilde{\Delta}$. Otherwise the optimal policy is $\Delta \in \left(1, \tilde{\Delta}\right)$.*

The striking result of Proposition 2 is that it is never optimal to set a zero interest rate band $\delta = i - i_d$ since the optimal interest rate band satisfies $\Delta > 1$. The reason is that for society the use of collateral is costly since $\beta R - 1$ is negative. However, without collateral consumption $q = \tilde{q}$ is small since agents economize on their cash holdings. The central bank thus faces a trade-off. It can encourage the use of costly collateral to increase consumption. The optimal policy simply equates the marginal benefit of additional consumption to the marginal cost of holding collateral. It is interesting to note that the use of fiat money is not costly for society, as money can be produced without cost. Nevertheless each agent economizes on its use because it must be acquired before it can be spend.

If R is small ($R < \overline{R}$) the cost of holding collateral is so high that the central bank's optimum is to discourage the use of collateral.[14] It does so by implementing an interest rate policy that satisfies $\Delta \geq \tilde{\Delta}$. In contrast if the rate of return is high it sets $\Delta \in \left(1, \tilde{\Delta}\right)$ so that agents finance some of their consumption through borrowing at the standing facility. An increase in R reduces the optimal Δ. In the limit as $R \rightarrow 1/\beta$ the holding of collateral becomes costless and we now consider the optimal policy in this limiting case.

Costless collateral Holding collateral is costless when $R = 1/\beta$ since the cost of acquiring one unit is equal to the discounted return βR. To avoid indeterminacies of the equilibrium allocation we consider the limiting allocation when the rate of return of the collateral satisfies $R \rightarrow 1/\beta$.[15] In this limiting case the critical value is $\tilde{\Delta} = \frac{1-\beta n}{\beta - \beta n} > 1$ and Proposition (1) continues to hold. We define the allocation that is attained under the optimal policy as the limiting allocation that is attained when $i \rightarrow i_d$. We find the following results.

Proposition 3 *With costless collateral, the optimal policy $i \rightarrow i_d$ implements the first-best allocation q^*. The price level approaches infinity.*

[14] This is similar as in Lagos and Rocheteau (2004) albeit in a very different context. They construct a model where capital competes with fiat money as a medium of exchange. They show that when the socially efficient stock of capital is low (which is the case when the rate of return is low) a monetary equilibrium exists that dominates the nonmonetary one in terms of welfare. So in this case it would be optimal to discourage the use of capital as a medium of exchange.

[15] We consider the limiting allocation since at $R = 1/\beta$ agents are indifferent of how much collateral they acquire even if they plan not to use it to obtain goods. If $\lambda_\ell > 0$ agents are strictly better off by increasing their collateral holdings up to the amount where $\lambda_\ell = 0$. However, they are indifferent between any amount of collateral that yields $\lambda_\ell = 0$. In the limiting allocation attained when $R \rightarrow 1/\beta$ agents acquire the smallest amount consistent with $\lambda_\ell = 0$.

The proof of the first part is an immediate consequence of equation (15) which implies that $\lim_{\beta R \to 1} u'(q) = \Delta$. Since the first-best allocation requires that $u'(q) = 1$ the result is established.

To understand why the price level approaches infinity under the optimal policy note that if $i = i_d > 0$, then money is strictly dominated in return by collateral. The reason is that the collateral can costlessly be transformed into money and so any consumption level that can be achieved with money can be achieved with collateral at no additional cost. However, the collateral has the intrinsic return $\beta R = 1$ while the return on money is $\frac{\beta}{\gamma} < 1$.[16] Consequently, the demand for money approaches zero. To encourage agents to hold the stock of money its price must approach zero. This immediately implies that $p \to +\infty$ and therefore $z_m = M_{+1}/p \to 0$. Only at the Friedman rule $i = i_d = 0$ the returns are equal and so agents are indifferent between holding money, collateral or both.

5 Conclusion

t.b.a.

[16] This follows from (16) together with $u'(q) = \Delta$.

6 APPENDIX

In this Appendix we show that if the central bank's objective is to maximize the expected discounted utility of the representative agent, the central banks objective is to maximize (2). To derive (2) we must first calculate hours worked in market 1. The money holdings at the opening of the first market are $\tilde{m} = 0$ having bought and $\tilde{m} = m + pq_s$ having sold. Hence, hours worked are

$$\begin{aligned} h_b &= \phi[m_{+1} + (1+i)\ell] - (R-1)b \\ h_s &= \phi[m_{+1} - (1+i_d)(m+pq_s)] - (R-1)b \end{aligned}$$

Since $h = nh_s + (1-n)h_b$, we get

$$\begin{aligned} h &= -(R-1)b + \phi m_{+1} + (1-n)\phi(1+i)\ell - n\phi(1+i_d)(m+pq_s) \\ &= -(R-1)b + \phi m_{+1} + \phi m - \phi m + (1-n)\phi(1+i)\ell - n\phi(1+i_d)(m+pq_s) \\ &= -(R-1)b + \varphi + (1-n)\phi\ell - n\phi(m+pq_s) + \phi m \\ &= -(R-1)b + (1-n)\phi\ell - n\phi(m+pq_s) + \phi m \end{aligned}$$

where the last equality follows from (1) and the fact that $d = m + pq_s$, so that

$$\varphi = \phi m_{+1} - \phi m + (1-n)\phi i\ell - n\phi i_d(m+pq_s) = 0.$$

Hence we get

$$h = -(R-1)b + (1-n)\phi\ell + (1-n)\phi m - n\phi pq_s = -(R-1)b.$$

where the last equality follows from the fact that $pq = m + \ell$ and market clearing requires $q_s = \frac{1-n}{n}q$. Then, welfare is given by

$$\begin{aligned} \mathcal{W} &= -b + (1-n)\left[u\left(q\right) - q\right] + \sum_{j=1}^{\infty} \beta^j \left\{(1-n)\left[u\left(q\right) - q\right] + (R-1)\, b\right\} \\ &= \frac{(1-n)[u(q) - q] + (\beta R - 1)b}{1-\beta} \end{aligned}$$

To calculate welfare, it is also useful to consider the economy that starts at date $t = 0$, at the beginning of the centralized market when agents having no financial obligations toward the central bank. From then on, the economy is in steady state. At $t = 0$, agents do not hold any collateral and have to produce the steady state level b. Hence, at $t = 0$, $h(0) = b$, while for all $t \geq 1$, $h(t) = -(R-1)b$.

The expected discounted payoff from date 0 onward of an agent who starts with m money holding in period 0 at the beginning of the centralized market is

$$W(m, 0, 0, 0) = -h(0) + V(m_{+1}, b_2)$$

In a steady state equilibrium the expected discounted payoff of an agent at the end of market 1 with money holding m is

$$V(m, b) = (1-n)[u(q) + \beta W(m - pq + \ell, b, \ell, 0)] + n[-q_s + \beta W(m + pq_s - d, b, 0, d)]$$

where

$$\begin{aligned} W(m - pq + \ell, b, \ell, 0) &= -h_b + V(m_{+1}, b_{+1}) \\ W(m + pq_s - d, b, 0, d) &= -h_s + V(m_{+1}, b_{+1}) \end{aligned}$$

Using the definitions for $W(m - pq + \ell, b, \ell, 0)$ and $W(m + pq_s - d, b, 0, d)]$ we get

$$\begin{aligned}(1-\beta)V(m,b) &= (1-n)[u(q) - h_b] - n[q_s + h_s] \\ &= (1-n)[u(q) - q] + (R-1)b\end{aligned}$$

Hence, using the fact that $h(0) = b$, the expected discounted payoff of a representative agent with m money holding in period 0 at the beginning of the centralized market is

$$W(m,0,0,0)(1-\beta) = (1-n)[u(q) - q] + (\beta R - 1)b$$

which is equal to equation (2).

Proof of Proposition 1. We first prove the *if* part. Assume first $z_\ell = 0$ and $z_m > 0$. Then from (16) and (17) we get

$$\frac{1-\beta}{\beta} = (1-n)\left[u'(q) - 1\right]. \tag{24}$$

and from (15) we have

$$\frac{1-R\beta}{R\beta} \geq (1-n)\left[u'(q)/\Delta - 1\right]. \tag{25}$$

Use (25) to replace $u'(q)$ in (24) and rearrange to get $\Delta \geq \tilde{\Delta}$.

Assume now that $z_\ell > 0$ and $z_m > 0$. Then from (17) $z_\ell > 0$ implies that $1+i_d > \gamma$. Use (16) to replace γ and rearrange to get $\Delta < \tilde{\Delta}$. Next divide (17) by $1 + i_d$ and solve for Δ to get

$$\Delta = 1 + \frac{z_m}{z_\ell}\frac{1 + i_d - \gamma}{(1-n)(1+i_d)} > 1$$

since $1 + i_d > \gamma$. Hence we have $1 < \Delta < \tilde{\Delta}$ if $z_\ell > 0$ and $z_m > 0$.

Finally, assume that $z_\ell > 0$ and $z_m = 0$. Then, the previous equation immediately implies that $\Delta = 1$.

We now prove the *only if* part. From (16) and (17) we get

$$1 - n\beta - \beta\,(1-n)\,u'(q) = (1-n)\,(\Delta - 1)\,\frac{z_\ell}{z_m}. \tag{26}$$

and from (15) we have

$$\Delta\left(\frac{1}{R} - n\beta\right) \geq \beta\,(1-n)\,u'(q) \tag{27}$$

Assume first that $1 < \Delta < \tilde{\Delta}$. Use (26) to rewrite (27) as follows

$$1 - n\beta - \Delta\left(\frac{1}{R} - n\beta\right) \leq (1-n)\,(\Delta - 1)\,\frac{z_\ell}{z_m}.$$

Rearrange to get

$$0 < \tilde{\Delta} - \Delta \leq \frac{(1-n)\,(\Delta - 1)}{(1/R - n\beta)}\,\frac{z_\ell}{z_m}.$$

Hence, $1 < \Delta < \tilde{\Delta}$ implies $\frac{z_\ell}{z_m} > 0$.

Assume next that $\Delta \geq \tilde{\Delta}$. Then from (26) we have

$$1 - n\beta - \beta\,(1-n)\,u'(q) \geq (1-n)\left(\tilde{\Delta} - 1\right)\frac{z_\ell}{z_m}.$$

Then $z_\ell > 0$ immediately implies that

$$0 > \tilde{\Delta} - \Delta \geq \frac{(1-n)\left(\tilde{\Delta} - 1\right)}{(1/R - n\beta)}\,\frac{z_\ell}{z_m}.$$

a contradiction. Hence $\Delta \geq \tilde{\Delta}$ implies $z_\ell = 0$.

Existence and uniqueness when $\tilde{\Delta} \leq \Delta$: In this case $z_\ell = b = 0$ and from (17) $\gamma = 1 + i_d$. Then, from (16) and (17) we get (24). Since right-hand side of (24) is strictly decreasing in q there exists a unique q that solves (24). Finally, from (18) we have $z_m = q$.

Existence and uniqueness when $1 < \Delta < \tilde{\Delta}$: The system of equations (15)-(18) with $z_\ell = \beta Rb/\Delta$ can be reduced as follows. Equations (18) and $z_\ell = \beta Rb/\Delta$

imply $z_m = q - \beta Rb/\Delta$. Then, multiply both sides of (17) by z_m and replace z_m to get

$$(q - \beta Rb/\Delta)\left[\gamma - (1 + i_d)\right] = -(1 - n)z_\ell(i - i_d)$$

Use (16) to eliminate γ as follows

$$\begin{aligned}
&(q - \beta Rb/\Delta)(1 + i_d)(1 - \frac{\gamma}{1 + i_d}) = (1 - n)z_\ell(i - i_d) \\
&(q - \beta Rb/\Delta)(1 + i_d)\{1 - (1 - n)\beta[u'(q) - 1] - \beta\} = (1 - n)z_\ell(i - i_d) \\
&(q - \beta Rb/\Delta)\{1 - (1 - n)\beta[u'(q) - 1] - \beta\} = (1 - n)\frac{\beta Rb}{(1 + i)}(i - i_d)
\end{aligned}$$

Hence, an equilibrium is defined by the following two equations:

$$\begin{aligned}
&\frac{1}{R\beta} = (1 - n)u'(q)/\Delta + n \\
&(q - \beta Rb/\Delta)\{1 - (1 - n)\beta[u'(q) - 1] - \beta\} = (1 - n)\frac{\beta Rb}{(1 + i)}(i - i_d)
\end{aligned}$$

We can use the first equation to replace for $u'(q)$ in the second to get

$$\frac{1}{R\beta} = (1 - n)u'(q)/\Delta + n \tag{28}$$

$$q = \beta RbF(\Delta) \tag{29}$$

If we substitute q in the first expression, we get

$$\frac{1}{R\beta} = (1 - n)u'\left[\beta RbF(\Delta)\right]/\Delta + n \equiv RHS \tag{30}$$

The left-hand side of (30) is constant while the right-hand side is decreasing in b for a given $1 \leq \Delta < \tilde{\Delta}$. Moreover, we have $\lim_{b\to 0} RHS = +\infty$ and $\lim_{b\to\infty} RHS = n < \frac{1}{R\beta}$. Hence, for any policy Δ with $1 \leq \Delta < \tilde{\Delta}$ a unique $b > 0$ exists. Then, from (22) a unique value for q exists. Accordingly a unique symmetric stationary equilibrium exist.

Finally, we have $\lim_{\Delta\to\tilde{\Delta}} F(\Delta) = +\infty$ and so $b \to 0$. ■

Proof of Proposition 2. Substituting (23) into the objective function the problem becomes

$$\max_{q} \ (1-n)\left[u(q)-q\right]+(\beta R-1)\frac{q}{\beta RF\left(\frac{R\beta(1-n)u'(q)}{1-nR\beta}\right)}$$

$$\text{s.t.} \ \ \hat{q}\geq q\geq \tilde{q}$$

After rearranging the first-order condition is

$$(1-n)\left[u'(q)-1\right]+\frac{1-\beta R}{\beta RF(\Delta)}\left[\frac{F'(\Delta)\,\Delta}{F(\Delta)}\frac{u''(q)\,q}{u'(q)}-1\right]=\hat{\lambda}-\tilde{\lambda}$$

where $\hat{\lambda}$ is the multiplier of the first inequality and $\tilde{\lambda}$ the one of the second inequality. Consider the first-order condition[17]and note that

$$\Delta(q)=\frac{R\beta(1-n)\,u'(q)}{1-nR\beta}$$

Suppose that the optimal q is such that $\Delta=1$, i.e., $q=\hat{q}$. In this case $\tilde{\lambda}=0$ and $\hat{\lambda}>0$ implying that $\Theta(\hat{q},R)>0$. Then we have $F(1)=1$, $F'(1)=\frac{1-nR}{R-1}$ and so

$$\Theta(\hat{q},R)=\frac{1-\beta R}{\beta R}\frac{1-nR}{R-1}\frac{u''(\hat{q})\,\hat{q}}{u'(\hat{q})}<0$$

which is a contradiction. Thus, in any equilibrium $q<\hat{q}$ implying $\Delta>1$.

Now suppose that the optimal q is such that $\Delta=\tilde{\Delta}$, i.e., $q=\tilde{q}$. In this case $\tilde{\lambda}>0$ and $\hat{\lambda}=0$ implying that $\Theta(\tilde{q},R)<0$. One can show that $\lim_{\Delta\to\tilde{\Delta}}F(\Delta)=\infty$, $\lim_{\Delta\to\tilde{\Delta}}F'(\Delta)=\infty$, $\lim_{\Delta\to\tilde{\Delta}}\frac{F'(\Delta)\Delta}{F(\Delta)}=\infty$ and $\lim_{\Delta\to\tilde{\Delta}}\frac{F'(\Delta)\Delta}{F(\Delta)F(\Delta)}=\frac{(1-1/R)}{(1/\Delta)^2(1-n)(\Delta-1)^2}$. Moreover, $(1-n)\left[u'(q)-1\right]=1/\beta-1$. Accordingly, we get

$$\Theta(\tilde{q},R)=1/\beta-1+\frac{1-\beta R}{\beta R}\frac{R\,(1-\beta n)^2}{(R-1)\,(1-n)}\frac{u''(\tilde{q})\,\tilde{q}}{u'(\tilde{q})}$$

[17] The following proofs omit many intermediate steps. A file containing the full proof is available upon request.

Consider first $R \to 1$. Then we have $\lim_{R \to 1} \Theta(\tilde{q}, R) = -\infty$. Now consider $R \to 1/\beta$. Then we have $\lim_{R \to 1/\beta} \Theta(\tilde{q}, R) = 1/\beta - 1 > 0$. Since $\frac{1-\beta R}{\beta} \frac{(1-\beta n)^2}{(R-1)(1-n)}$ is monotonically decreasing in R we have a unique critical value $\overline{R}$ such that $\Theta\left(\tilde{q}, \overline{R}\right) = 0$. Thus if $R < \overline{R}$, $q = \tilde{q}$ and if $R > \overline{R}$, q solves $\Theta(q, R) = 0$. ■

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