Stock market optimism and participation cost: a mean-variance estimation

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Abstract

This paper estimates the costs of participating to the stock market, together with the cross sectional dispersion of stock market optimism. Our analysis is based on a mean-variance framework, when there is a riskless asset (cash), which makes the allocation of the investment in risky assets (stocks and bonds) independent on preferences. Within this framework, we derive "structural" decision rules for the composition of the risky asset portfolio to be efficient. These rules depend on the amount invested in the risky portfolio and on investors' optimism, which are the determinants of the stock market return expected by a household, when participation involves a fixed cost. Using these rules and the heterogeneity in risky assets holdings and in the degree of optimism, we identify both the fixed costs of stock investment and the variance of optimism. Using the Italian Survey of Household Income and Wealth we find that the risky asset portfolios of Italian households are coherent with a fixed cost of participating to the stock market around 150 euros per year (0.9 % of non-durable expenditure). Having a university degree, owning one's home and living in the North of the country contribute to lower the cost. The standard deviation of investors' optimism is estimated to be high, at around 30 percent.

JEL: D12, D14, G11

Keywords: heterogeneous household portfolios, mean-variance frontier, participation cost, expectation error

1 Introduction[1]

This paper estimates the costs of participating to the stock market, that rationalize the choice of non investing in stocks by a large number of households. Furthermore, we identify the cross-sectional dispersion of the expectation error of investors on stock market returns which can be thought of as a broad measure of their stock market optimism. The analysis is based on a mean-variance framework, motivated by assuming that the rates of return of risky assets are multivariate normally distributed, for arbitrary preferences. Within this framework, when there is a riskless asset, the individual attitude towards risk affects just the split between cash and an efficient portfolio of bonds and stocks. The composition of the portfolio of risky assets is dictated exclusively by mean-variance efficiency considerations, given the first and second moments of returns.

When investing in stocks involves a fixed cost, the stock market return expected by a household depends on the amount it invests and on its optimism. Variation in the (observable) sizes of the risky portfolios maps into parallel shifts of the individual efficient frontiers. Conversely, variation in the (unobservable) degrees of optimism rotates the individual efficient frontiers. In turn, these two sources of variation map into the observed heterogeneity in the composition of the efficient risky portfolios chosen by households, that can exhibit various degrees of diversification.

Within this framework, where risky asset shares are determined exclusively by mean-variance consideration, given the participation cost, we derive "structural" decision rules for the composition of the risky asset portfolio to be efficient, as a function of the amount invested in the risky portfolio and of the investor's expectation error. The rules are conditional on investing in risky assets and are obtained by maximizing the Sharpe ratio of the risky portfolio. The choice is between specialization in bonds, specialization in stocks and full diversification. The probability of investing in stocks, the costly asset, is increasing in the amount invested in risky assets and in the the degree of stock market optmism. Conditional on investing in stocks diversification is most likely, unless the investor's wealth is below a certain threashold, but she is very buoyant about stock returns. Using these rules

[1]We are grateful to Fabio Panetta for providing us with the series of the real returns on Italian equities and long-term government bonds. Cristiana Rampazzi provided skilful assistance with the data. All views expressed belong to the authors and do not necessarily reflect those of the Bank of Italy.

and exploiting the heterogeneity in risky assets holdings, which is observable, and in the degree of optimism, which is unobservable, we identify both the fixed costs of stock investment and the variance of the stock market optimism. Our methodology relies on the information on the sub-sample of households who hold at least one risky asset and is based on the likelihood of the various efficient combinations of risky assets. We present an application based on the Italian Survey of Household Income and Wealth, which suggests that the costs of participating to the stock market in Italy are around 150 euros per year. The standard deviation of the expectation error on the equity premium is estimated at around 30 percent. Overall, the empirical evidence on the nature and on the entity of the costs associated to financial transactions is rather limited, especially at the household level, and a reason is that some of these costs are likely to be figurative and related to information gathering and processing. For the US, Vissing Jørgensen (2002) estimates the median of the distribution of the per-period costs of participating to the stock market to be around $350; Paiella (2002) bounds from below the fixed costs of stock market participation at around $100 per year, corresponding to 1 percent of non-durable consumption; based on the US National Income and Product Account, the ratio of personal expenditure on brokerage charges, investment counseling, bank and trust service charges to expenditure on non-durable goods amounts to 3 percent in 2000 (2 percent in 1996). Our fixed costs of participatig to the stock market correspond to 0.9 percent of mean non-durable household expendidure.

Related papers on the costs of stock holding are Luttmer (1999), Paiella (2002), Vissing-Jørgensen (2002) and Attanasio and Paiella (2003), all focusing on the costs of adjusting consumption. Luttmer (1999) focuses on the losses for leaving unexploited some trading opportunities and proposes a lower bound on the level of fixed transaction costs reconciling per-capita expenditure and asset returns. The frictions that Luttmer identifies are the costs of trading that would justify not taking advantage of temporary changes in returns not matched by changes in the riskiness of assets. Paiella (2002) and Attanasio and Paiella (2003) aim at reconciling the choice of holding an incomplete portfolio of assets with the intertemporal consumption model, by invoking non-proportional costs of financial market participation. By distinguishing between shareholders and non-shareholders, the two papers estimate a lower bound on the costs of entry from the necessary conditions for the optimality of observed behavior of non-participants. Using the implications of the consumption model for shareholders, Attanasio and Paiella are also able to identify households preference parameters and build a powerful test of the theory of the intertemporal allocation of consumption. Vissing-Jørgensen (2002) looks at the stock market participation costs structure from a panel of portfolio choice data. By estimating a censored regression model, based on the solution to the optimization problem for stockholders, she provides evidence on the distribution of the per-period participation costs in the cross-section, under various assumptions for the policy function of the investment in stocks. Other papers on the costs of stockholding are Haliassos and Bertaut (1995) and Bertaut (1998) who carry out theoretical simulations of how large entry and/or per-period participation costs would make households stay out of the stock market, based on the basic expected-utility model.

The main novelty of our paper is that we identify the actual costs of entry to the stock market relying only on portfolio efficiency considerations within a preference-free framework, which makes our results robust to alternative utility specification. Furthermore, within our framework, there is room for individual beliefs, expectations and guesses regarding stock market returns and trends. These unobservable individual features, that we call stock market optimism, determine people's subjective evaluation of the potential returns from investing in stocks and can be expected to represent an important subset of motivations for stock ownership. An interesting paper on the impact of individual cognitive bias and private information on stock ownership is Kézdi and Willis (2001) who construct a measure of optimism and find that it is strongly related to stock purchases.

The rest of the paper is organized as follows. Section 2 outlines the theoretical model of portfolio choice, discusses the set of investment opportunities and posits households' portfolio decision rules. Section 3 presents the empirical model. Section 4 describes the data and discusses the results. Section 5 concludes.

2 Individual problem

In the basic expected-utility model, an agent chooses asset holdings by maximizing her intertemporal, additively separable utility defined over non-durable expenditure subject to a budget constraint that summarizes her investment opportunities. Then, the timing of the problem is as follows: agents choose their consumption and then allocate their savings by maximizing the following-period value function. If we assume that non-financial income (*e.g.* labor income) and asset returns are distributed identically and independently over time, financial wealth turns out to be the only state of the problem. Furthermore, under the assumption that non-financial income and asset returns are normally distributed, from the point of view of every period, beginning-of-next period financial wealth is normally distributed. Then, maximizing the expected value of next period value function is a mean-variance problem, where consumers only care for the mean and standard deviation of their portfolio return, regardless of the functional form of their preferences.[2]

2.1 Investment opportunities set

In every period, after their consumption decisions, households can invest all their financial wealth in three assets: cash, bonds and stocks. Cash earns a riskless rate R. Bonds and stocks are risky and their rates of return are independently (over time) and identically distributed as normal:

$$\begin{bmatrix} R_b \\ R_s \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_b \\ \mu_s \end{bmatrix}, \begin{bmatrix} \sigma_b^2 & \rho\sigma_b\sigma_s \\ \rho\sigma_b\sigma_s & \sigma_s^2 \end{bmatrix} \right).$$

To participate to the stock market, households must pay a fixed fee, F, which reflects the total costs of transaction, commission, information, etc. Such fee is unobservable and its size is the object of inference of this paper.

The existence of a fixed cost makes the rate of return on stocks depend on the amount invested. Let $\hat{w}^h$ be household h's total financial wealth, w^h the portion of financial wealth allocated to the portfolio of stocks and bonds – from here on called the "risky portfolio"– and $(\hat{w}^h - w^h)$ the portion allocated to cash. Let α denote the fraction of w^h invested in stocks; the remaining fraction $(1-\alpha)$ is invested in bonds. After paying the participation fee, conditional on the size of the risky investment w^h and on the share of stocks α, rates of return are

$$\begin{aligned} Cash &: R^h = R; \\ Bonds &: R_b^h = R_b; \\ Stocks &: R_s^h = R_s - \left[F/\alpha w^h \right]. \end{aligned}$$

Borrowing and leveraging one's financial wealth is possible, as there is no restriction that $\hat{w}^h \geq w^h$. Borrowing is at the same riskless rate R as lending.

[2]See Merton (1971, 1973) for more details on these issues.

However, we assume that households cannot leverage their position in one risky asset by shorting the other risky asset, *i.e.* that $\alpha \in [0,1]$.[3] For a given participation fee F, household h's rate of return on a stock investment is increasing in the amount αw^h. As αw^h increases from a very small to a very large amount, R_s^h increases from $-\infty$ to R_s. It is not worth to participate to the stock market unless the amount to invest is sizeable. That is, it is not worth to participate to the stock market unless the participation fee F is small relative to the amount αw^h to invest. The return on household h's risky portfolio is determined by the allocation α between bonds and stocks, given the amount w^h invested in the risky portfolio:

$$R_\alpha^h = \begin{cases} R_b & if\ \alpha = 0 \\ \alpha R_s + (1-\alpha) R_b - \left[F/w^h\right] & if\ \alpha \in (0,1) \\ R_s - \left[F/w^h\right] & if\ \alpha = 1 \end{cases} .$$

The subjective distribution of R_s is heterogeneous across households. From the point of view of household h, before paying any participation fees, the distribution of risky returns is

$$\begin{bmatrix} R_b \\ R_s \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_b \\ \mu_s + \varepsilon_s^h \end{bmatrix}, \begin{bmatrix} \sigma_b^2 & \rho\sigma_b\sigma_s \\ \rho\sigma_b\sigma_s & \sigma_s^2 \end{bmatrix} \right).$$

The expectation error ε^h can be interpreted as a measure of the optimism of household h on the return of the stock market. We are assuming that there is no expectation error on bond returns.[4] ε^h is the individual error on the

[3] This assumption is not essential to the model, which can be extended to the case in which α is unconstrained. However, $\alpha^h \in [0,1]$ holds for all the households in the sample that we use to evaluate the cost and restricting the model to this case simplifies the discussion.

[4] Dealing with both expectation errors

$$\begin{bmatrix} R_b \\ R_s \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_b + \varepsilon_b^h \\ \mu_s + \varepsilon_s^h \end{bmatrix}, \begin{bmatrix} \sigma_{bb} & \sigma_{bs} \\ \sigma_{sb} & \sigma_{ss} \end{bmatrix} \right)$$

would require dealing with the bivariate distribution:

$$\begin{bmatrix} \varepsilon_b^h \\ \varepsilon_s^h \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} \sigma_{\varepsilon_b\varepsilon_b} & \sigma_{\varepsilon_b\varepsilon_s} \\ \sigma_{\varepsilon_b\varepsilon_s} & \sigma_{\varepsilon_s\varepsilon_s} \end{bmatrix} \right).$$

Although this would add to the realism of the model, it would complicate identification without adding information to the subject of interest of this paper, which is the relative cost of participating to stock markets vs. bond markets. This depends mostly on excess expected excess return of stocks over bonds and it is normally distributed across the population: $\varepsilon^h \sim N(0, \sigma_\varepsilon^2)$. From the point of view of household h the realization of the expectation error ε^h is a deterministic value, hence for h the return on w^h dollars invested in a risky portfolio with a fraction α of stocks has expectation:

$$\mu_\alpha^h = \begin{cases} \mu_b & if\ \alpha = 0 \\ \alpha\mu_s + (1-\alpha)\mu_b - \left[F/w^h\right] + \alpha\varepsilon^h & if\ \alpha \in (0,1) \\ \mu_s - \left[F/w^h\right] + \varepsilon^h & if\ \alpha = 1 \end{cases}, \tag{1}$$

and standard deviation:

$$\sigma_\alpha^h = \begin{cases} \sigma_b & if\ \alpha = 0 \\ \sqrt{\alpha_s^2\sigma_s^2 + (1-\alpha)^2\sigma_b^2 + 2\alpha(1-\alpha)\rho\sigma_b\sigma_s} & if\ \alpha \in (0,1) \\ \sigma_s & if\ \alpha = 1 \end{cases}. \tag{2}$$

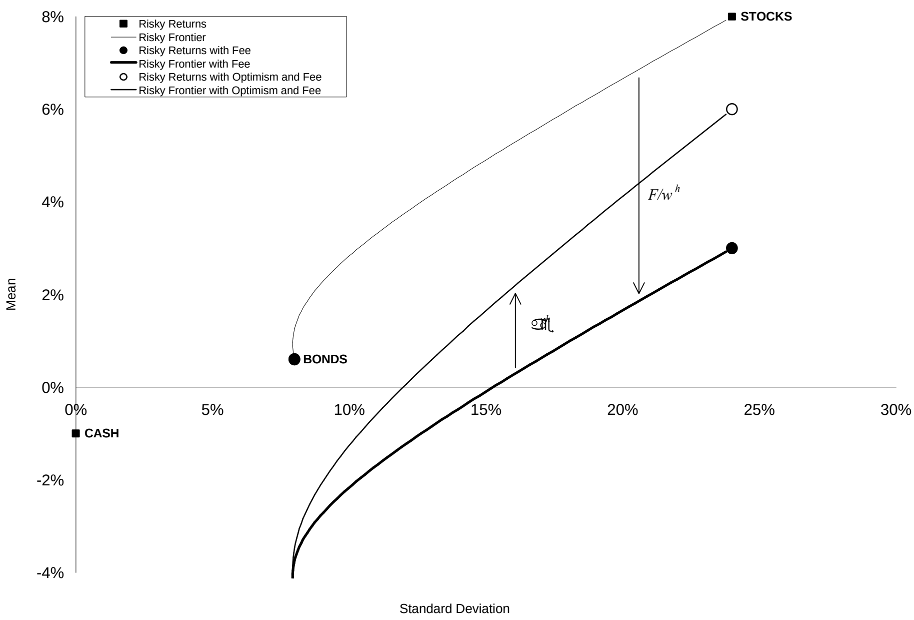
2.2 Optimal portfolio allocation

Regardless of her degree of risk aversion, a risk averse mean-variance investor maximizes the Sharpe ratio of her risky portfolio. Her indifference curves are increasing and convex in the mean / standard deviation plane. It follows that the return of efficient portfolios –of cash, bonds and stocks– must lay on the steepest of the lines connecting the return on cash holdings and the return on the risky portfolio. This is the capital allocation line between cash and the risky portfolio with the highest Sharpe ratio. Preferences determine the optimal split of household h's total financial wealth between cash and risky portfolio. Regardless of preferences, the optimal risky portfolio must maximize the Sharpe ratio.

A positive participation cost F shifts the risky frontier downward and breaks its continuity. By "risky frontier" we mean the mean-variance frontier of risky portfolio returns. Equation 1 implies that from the point of view of household h, given its expectation error ε^h, the expected return μ_α^h of a risky portfolio of bonds and stocks –in which h invests w^h dollars– depends on F. However, from equation 2, F does not affect the standard deviation of the risky portfolio return, σ_α^h. Figure 1 plots hypothetical returns in the mean /

returns, rather than level of returns. Conversely, in order to go from the relative cost to the level of each cost, introducing the expectation error on bond returns would be a fruitful complication.

Figure 1: Mean-variance frontier with participation cost and optimism



standard deviation plane and illustrates that, unless $F = 0$, the risky frontier is not continuous as α varies from zero to one. In the plot, the correlation between bond and stock returns is set to 0.2 and F/w^h is 5 percent. The thin line connecting the square (stocks) and the circle (bonds) symbolizes the mean-standard deviation frontier of asset returns, gross of participation costs, when there is no expectation error, ε^h. For a given level of investment w^h, the thick concave line and the dark circle symbolize the risky frontier net of participation costs. When $F > 0$,there is a point of discontinuity in the risky frontier at $\alpha = 1$, because μ_α^h is shifted downward by F/w^h for any $\alpha \in (0, 1]$. For a given participation costs F, the downward shift and discontinuity are larger the smaller the amount w^h invested. For $\alpha = 0$ we are back on the "thin frontier". A non-zero expectation error ε^h rotates the risky frontier around the axis $\mu = \mu_b$ and F/w^h. The plot shows the case of a optimistic household, with positive expectation error ε^h on the expected rate of return of the stock market. ε^h is set at 3 percent. This household's risky frontier is symbolized by the empty circle and the medium-thick concave line.

Hence, within this framework, in addition to the first two moments of stock and bond returns –difference in Sharpe ratios and correlation of returns– household h's optimal risky portfolio composition depends on the magnitude of the participation fee F/w^h along with the degree of optimism on stock market returns. There are four possible types of optimal behavior with regard to participation: participating to both bond and stock markets, participating to either one of the two, or not participating to any of the two. The intuition is the following. When F/w^h is sufficiently small, household h participates to both risky markets, so long as the difference in Sharpe ratios between the two investments is not too large and the correlation of their returns is sufficiently lower than one. As the costs F/w^h increases, the Sharpe ratio of all risky portfolios is reduced. Beyond some level, it becomes efficient to switch from a fully diversified risky portfolio to one entirely invested in a single risky asset. This is a consequence of the discontinuity introduced by the fee in the risky frontier. Whether the single-asset efficient risky portfolio is in bonds or stocks depends on the relative sizes of the expected return on bonds versus the expected return on stocks net of the cost. For a given level of F, the percentage fee varies with the amount w^h invested in the risky portfolio.

A non-zero expectation error ε^h tilts consumer h's perception of the difference in Sharpe ratios between bond and stock investments. Introducing optimism/pessimism about stock market returns differentiates household h from the average household. For example, for some values of the moments of risky returns, it is possible that although with a positive fee the average household should invest its whole risky portfolio in stocks, pessimism about the stock market tilts this efficient rule towards bonds. This bias towards bonds would also affect the average household without fees in the same manner, if it is pessimistic about the stock market. If its pessimism is large enough, it might find specialization in bonds the efficient strategy. The opposite happens when consumers are optimistic about stock market returns: a positive ε^h tilts investments towards the stock market.

2.3 Decision rules

Household h chooses one of four mutually exclusive investment strategies, with regard to its risky portfolio: not investing in any risky asset, investing only in bonds, investing only in stocks or investing in both. Letting B and S be indicator functions for bond and stock holding respectively, the four mutually exclusive options are $(B = 0; S = 0)$, $(B = 1; S = 0)$, $(B = 0; S =$

1) or $(B=1;S=1)$.

As we have illustrated, efficient rules depend on the magnitude of the individual percentage participation fee, for given first and second moments of bond and stock returns and given expectation error. The percentage fee in turn depends on the amount w^h invested in the risky portfolio, for given level of entry cost F. Heuristically, the larger is household h's wealth, hence the larger his potential risky investment w^h, the more likely that it moves from choice 1, to choice 2, to choice 3 or 4.

The above array of choices can be re-mapped into a sequence of two choices:

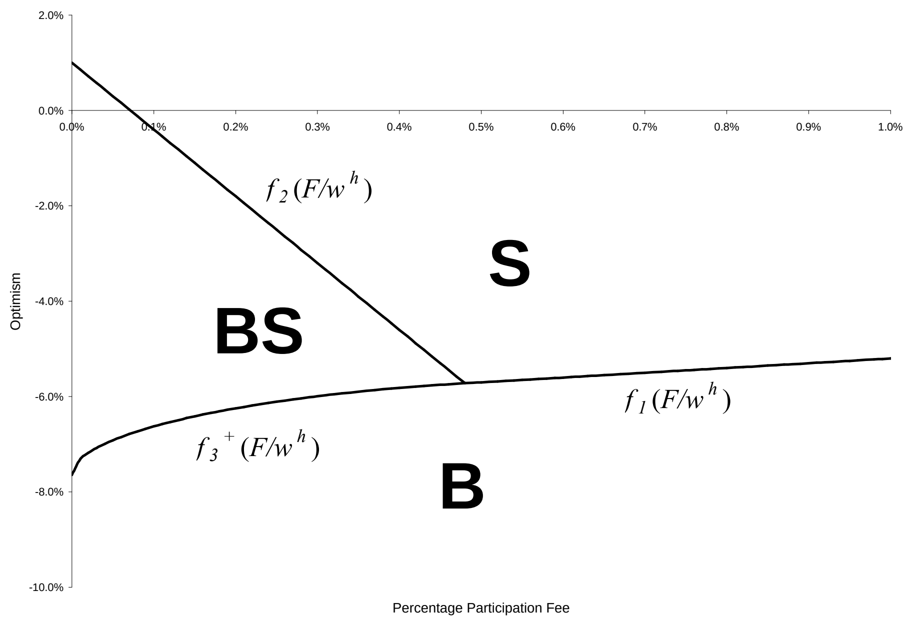
1. choose $(B=0;S=0)$ vs. $\{(B=1;S=0)$, $(B=0;S=1)$ or $(B=1;S=1)\}$;
2. conditional on not choosing $(B=0;S=0)$, choose $(B=1;S=0)$ vs.$(B=0;S=1)$ vs.$(B=1;S=1)$.

Household h's optimal choice can be characterized by backward induction. Let's take as given the first two moments of bond and stock returns, the entry cost F and household h's expectation error. In step 2, for each hypothetical amount w^h to invest, the efficient choice between bonds and stocks would be the one yielding the highest Sharpe ratio.[5] In step 1, household h's problem is to choose the optimal mix of risky and riskless assets for his wealth: $(\hat{w}^h - w^h)$ dollars in cash and w^h dollars in the efficient portfolio of step 2. This choice is made internalizing the fact that the latter portfolio and its return varies with the w^h invested. While choice 1 depends on household h's attitude towards risk, once we condition on its optimal risky investment w^h, its following choices are only driven by efficiency and not preferences. This fact is what will drive our identification strategy.

Suppose that the outcome of choice 1, w^h, is observed. Let the level of participation costs F and the first and second moments of bond and stock returns be a given. Household h's efficient choice among $(B=1;S=0)$, $(B=0;S=1)$ and $(B=1;S=1)$ depends on its optimism on stock market returns and on w^h, which together determine the individual Sharpe ratios of the three alternative portfolios, as explained in Section 2.1 and shown with equations (1) and (2). In order to characterize the decision rules, we draw

[5] Both the Sharpe ratios and the optimal choices vary with the amount w^h to invest.

Figure 2: Decision Regions



in Figure 2[6] an efficiency map: we partition the $\left(F/w^h\right) - \varepsilon^h$ space in three regions corresponding to the three different efficient choices. Such trinomial partition can be obtained as the overlap of three distinct and independent binary partitions of the $\left(F/w^h\right) - \varepsilon^h$ space, each representing the efficient choice when there are only two alternatives, i.e. the choice with the highest Sharpe ratio. In the Figure, $f_1(F/w^h)$ is obtained by comparing (equating) the Sharpe ratios of a portfolio specialized in bonds and of a portfolio specialized in stocks; along $f_2(F/w^h)$ the Sharpe ratio of a portfolio specialized in stocks is the same as the Sharpe ratio of a well-diversified portfolio of bonds and stocks; $f_3(F/w^h)$ considers the choices between diversification and specialization in bonds. Overall, if the percentage participation fee is relatively low, specialization in bonds will occur only if the household is very

[6] Figure 2 is obtained by computing the Sharpe ratio based on the following assumptions: the mean return on stocks, bonds and cash are set to 0.0824, 0.0060 and -0.0038, respectively; the standard deviation are set to 0.2415, 0.0840 and 0. The correlation between the excess returns on stocks and bonds is set to 0.2.

pessimistic. Conditional on investing in stocks diversification is most likely, unless the investor's wealth is below a certain threashold, but she is very buoyant about stock returns. For levels of optimism "in-between", h will fully diversify. Notice that for the average household ($\overline{\varepsilon} = 0$), diversification is the optimal strategy for a percentage participation fee up to 0.07 percent. As F/w^h increases, diversification becomes less likely and the type of specialization will crucially depend on the value taken on by ε^h. The propositions that follow formalize these arguments by deriving explicitely $f_1(F/w^h)$, $f_2(F/w^h)$, and $f_3(F/w^h)$ from the comparisons of the Sharpe ratios based on (1) and (2) associated to the relevant choices.[7] Let $\widetilde{\mu}_s$ and $\widetilde{\mu}_b$denote the mean excess returns of stocks and bonds over the riskless asset.

Proposition 1 $(B = 0; S = 1) \succ (B = 1; S = 0)$ *when*

$$\varepsilon^h > -\widetilde{\mu}_s + \frac{\sigma_s}{\sigma_b}\widetilde{\mu}_b + F\frac{1}{w^h}. \tag{3}$$

Proposition 1 considers the case when the choice is exclusively between specialization in stocks or specialization in bonds, hence when there is no possibility of exploiting diversification. Specialization in stocks is efficient when stock market optimism is above the level that equates the Sharpe ratios. As $1/w^h \to 0$ –because the amount to invest becomes large– the entry costs become irrelevant and the two Sharpe ratios coincide when $\varepsilon^h = \widetilde{\mu}_b\sigma_s/\sigma_b - \widetilde{\mu}_s$.[8] As $1/w^h$ increases because the amount w^h to invest decreases, the expected "after-cost" return of the stock market is reduced and the threshold level of optimism becomes larger. For a very small investment w^h the "after-cost" return of the stock market approaches minus infinity and only an infinite optimism would rationalize preferring the stock market to the bond market. $f_1(F/w^h)$ in Figure 2 can be obtained by replacing the inequality of equation 1 with an equality.

Proposition 2 $(B = 0; S = 1) \succ (B = 1; S = 1)$ *when*

$$\varepsilon^h > -\widetilde{\mu}_s + \frac{1}{\rho}\frac{\sigma_s}{\sigma_b}\widetilde{\mu}_b + \left(1 - \frac{1}{\rho}\frac{\sigma_s}{\sigma_b}\right) F\frac{1}{w^h}. \tag{4}$$

[7] The proofs are avilable upon request.

[8] This is a negative number when the Sharpe ratio of the stock market exceeds that of the bond market.

Proposition 2 considers the case when the choice is between specialization in stocks and full diversification. Again, specialization in stocks is efficient when stock market optimism is above the level that equates the Sharpe ratios. As $1/w^h \to 0$ the two Sharpe ratios coincide when $\varepsilon^h = \widetilde{\mu}_b(\sigma_s/\sigma_b)1/\rho - \widetilde{\mu}_s$.[9] Both the alternatives considered here involve the payment of the cost. The choice between the two is crucially affected by the degree of correlation, ρ, between bond and stock returns. If the returns on the two assets are uncorrelated, specialization in stocks would occur only if optimism were infinite. If the returns are perfectly and positively correlated, specialization in stocks will be generally preferable unless the household is severely pessimistic. If the returns are perfectly, but negatively correlated, household h is the more likely to specialize in stocks the more optimistic and the wealthier. $f_2(F/w^h)$ in Figure 2 is based on equation 3.

Proposition 3 $(B = 1; S = 1) \succ (B = 1; S = 0)$ *when*

$$\begin{aligned} \varepsilon^h &> -\widetilde{\mu}_s + \rho\frac{\sigma_s}{\sigma_b}\widetilde{\mu}_b + \left(1 - \rho\frac{\sigma_s}{\sigma_b}\right) F\frac{1}{w^h} \\ &+ \sqrt{1-\rho^2}\frac{\sigma_s}{\sigma_b}\sqrt{F\frac{1}{w^h}\left(2\widetilde{\mu}_b - F\frac{1}{w^h}\right)}. \end{aligned} \tag{5}$$

Proposition 3 considers the case when the choice is between full diversification and specialization in bonds. The relationship between ε^h and $1/w^h$ is no longer linear, unless asset returns were perfectly correlated ($\rho = \pm 1$). As $1/w^h \to 0$, the entry cost becomes irrelevant and the two Sharpe ratios coincide when $\varepsilon^h = \widetilde{\mu}_b(\sigma_s/\sigma_b)\rho - \widetilde{\mu}_s$.[10] As in Proposition 1, for a very small investment, diversification, which involves the cost payment, occurs only if the degree of optimism is very large. Instead, if w^h is relatively large, specialization in bonds requires a high degree of pessimism. By replacing the inequality in equation 3 with an equality yields $f_3(F/w^h)$ of Figure 2.

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[9] For a positive ρ, if the Sharpe ratio of the stock market exceeds that of the bond market, $(\widetilde{\mu}_b\sigma_s/\sigma_b)1/\rho - \widetilde{\mu}_s > 0$ for ρ sufficiently small. For a negative ρ, the inequality never holds. The coefficient on the percentage fee is negative for any value taken on by ρ (unless $\sigma_b > \sigma_s$).

[10] $\widetilde{\mu}_b(\sigma_s/\sigma_b)\rho - \widetilde{\mu}_s < 0$ for any value taken on by ρ (unless the Sharpe ratio of bonds is much greater than that of stocks).

3 Probabilistic structure and identification

The structural decision rules in Propositions 1 through 3 allow us to identify the parameters of interest F and σ_ε, by imposing a probabilistic structure on the investment choice problem. Let's focus on risky asset holders. Let D denote an index function that takes on value 1 if (besides investing in cash) the household invests just in bonds ($B = 1; S = 0$), 2 if it invests in bonds and stocks ($B = 1; S = 1$), and 3 if it invests just in stocks ($B = 0; S = 1$). To simplify the notation, let's adopt the following definitions:

$$
\begin{aligned}
\alpha_1 &= -\widetilde{\mu}_s + \frac{\sigma_s}{\sigma_b}\widetilde{\mu}_b, \\
\alpha_2 &= -\widetilde{\mu}_s + \frac{1}{\rho}\frac{\sigma_s}{\sigma_b}\widetilde{\mu}_b, \\
\beta_2 &= \left(1 - \frac{1}{\rho}\frac{\sigma_s}{\sigma_b}\right), \\
\alpha_3 &= -\widetilde{\mu}_s + \rho\frac{\sigma_s}{\sigma_b}\widetilde{\mu}_b, \\
\beta_3 &= \left(1 - \rho\frac{\sigma_s}{\sigma_b}\right), \\
g\left(\frac{1}{w^h}\right) &= \sqrt{1-\rho^2}\frac{\sigma_s}{\sigma_b}\sqrt{F\frac{1}{w^h}\left(2\widetilde{\mu}_b - F\frac{1}{w^h}\right)}.
\end{aligned}
\tag{6}
$$

Then, formally, based on the criterion functions of Propositions 1 through 3, the index function can be characterized as follows:

$$
\begin{aligned}
D^h &= 1 : if \quad : \varepsilon^h < \alpha_1 + \frac{F}{w^h} \cap \varepsilon^h < \alpha_3 + \beta_3\frac{F}{w^h} + g\left(\frac{1}{w^h}\right); \\
D^h &= 2 : if \quad : \varepsilon^h < \alpha_2 + \beta_2\frac{F}{w^h} \cap \varepsilon^h > \alpha_3 + \beta_3\frac{F}{w^h} + g\left(\frac{1}{w^h}\right); \\
D^h &= 3 : if \quad : \varepsilon^h > \alpha_1 + \frac{F}{w^h} \cap \varepsilon^h > \alpha_2 + \beta_2\frac{F}{w^h}.
\end{aligned}
\tag{7}
$$

Given w_h the conditional probability that for household h we observe $D^h = 1$ ($B = 1; S = 0$) is given by:

$$
\begin{aligned}
\Pr\left(D^h = 1 | w^h\right) = \; & \Phi\left(\frac{\alpha_1}{\sigma_\varepsilon} + \frac{F}{\sigma_\varepsilon}\frac{1}{w^h}\right) I\left(\frac{1}{w^h} \geq \frac{1}{w^h}^*\right) + \\
& \Phi\left(\frac{\alpha_3}{\sigma_\varepsilon} + \frac{\beta_3 F}{\sigma_\varepsilon}\frac{1}{w^h} + \frac{1}{\sigma_\varepsilon}g\left(\frac{1}{w^h}\right)\right) I\left(\frac{1}{w^h} < \frac{1}{w^h}^*\right);
\end{aligned}
\tag{8}
$$

where Φ is the cumulative standard normal distribution function. $I(\ )$ is an index function that takes on value 1 depending on the value of w^h relative to a threshold $w^{h\,*}$. At $w^{h\,*}$ the household is indifferent between holding either just bonds or just stocks and fully diversifying. The indifference condition yield:

$$w^{h*} = \left(\frac{1-\rho}{F}\mu_b\right)^{-1}.$$

The first term in equation (8) allows for the choice between specialization in bonds and specialization in stocks (and identifies the area above $f_1(F/w^h)$ in Figure 2); the second term allows for the choice between specialization in bonds and full diversification (and identifies the area above $f_3(F/w^h)$ in the Figure). If w^h is sufficiently low, the household will prefer bonds to stocks unless it is very optimistic about the stock market. If w^h is above the threshold $w^{h\,*}$, it will prefer bonds to diversifying only if it is relatively pessimistic.

The conditional probability of $D^h = 2$ $(B = 1; S = 1)$ is given by:

$$\Pr\left(D^h = 2|w^h\right) = \left(\Phi\left(\frac{\alpha_2}{\sigma_\varepsilon} + \frac{\beta_2 F}{\sigma_\varepsilon}\frac{1}{w^h}\right) + -\Phi\left(\frac{\alpha_3}{\sigma_\varepsilon} + \frac{\beta_3 F}{\sigma_\varepsilon}\frac{1}{w^h} + \frac{1}{\sigma_\varepsilon}g\left(\frac{1}{w^h}\right)\right)\right) I\left(\frac{1}{w^h} < \frac{1}{w^h}^*\right), \tag{9}$$

(which identifies the area between $f_2(F/w^h)$ and $f_1(F/w^h)$ in Figure 2). The first term in equation (9) allows for the choice between full diversification and specialization in stocks and implies that the household will diversify unless it is super-optimistic. The second term allows for the choice between full diversification and specialization in bonds and gives the probability of investing just in bonds. Hence, if its optimism is sufficiently *low* not to induce to invest just in stocks, the household will diversify unless it is so pessimistic about the stock market to invest just in bonds. Notice that the probability of $D^h = 2$ is defined just for w^h above the threshold $w^{h\,*}$ For lower wealth, full diversification is never efficient. If its wealth is low, the household will invest in stocks only if it is very optimistic about the stock market, because the investment in stocks involves a fixed costs. However, if it so optimistic about the stock market to be willing to pay the cost despite the relatively small investment, it will want to put all its money in the stock market, disregarding bonds. If its wealth is low and it is not very optimistic about the stock market, the household will invest just in bonds (conditioning on risky asset holding), which do not involve fixed costs.

Finally, the conditional probability that $D^h = 3$ ($B = 0; S = 1$) is given by:

$$\begin{aligned} \Pr\left(D^h = 3|w^h\right) &= \left(1 - \Phi\left(\frac{\alpha_1}{\sigma_\varepsilon} + \frac{F}{\sigma_\varepsilon}\frac{1}{w^h}\right)\right) I\left(\frac{1}{w^h} \geq \frac{1}{w^h}^*\right) + \\ & \left(1 - \Phi\left(\frac{\alpha_2}{\sigma_\varepsilon} + \frac{\beta_2 F}{\sigma_\varepsilon}\frac{1}{w^h}\right)\right) I\left(\frac{1}{w^h} < \frac{1}{w^h}^*\right). \end{aligned} \quad (10)$$

The first term in equation (10) allows for the choice between specialization in stocks and specialization in bonds (and identifies the area above $f_1(F/w^h)$ in Figure 2) and implies that if w^h is sufficiently low, diversification is not an efficient alternative. For low w^h the household will specialize in stocks if it is very optimistic about stock market returns. The second term (which identifies the area above $f_2(F/w^h)$ in the Figure) implies that if w^h is above the threshold $w^{h\,*}$, the household will either specialize in stocks or diversify depending on how optimistic it is.

Given (8), (9) and (10), the probability on our observed sample is:

$$\begin{aligned} &\prod_{h=1}^{H_1}\left[\Phi\left(\frac{\alpha_1 + F/w^h}{\sigma_\varepsilon}\right) I^+ + \Phi\left(\frac{\alpha_3 + \beta_3 F/w^h}{\sigma_\varepsilon} + \frac{1}{\sigma_\varepsilon} g\left(\frac{1}{w^h}\right)\right) I^-\right] \quad (11) \\ &\prod_{h=H_1+1}^{H_2}\left[\Phi\left(\frac{\alpha_2 + \beta_2 F/w^h}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\alpha_3 + \beta_3 F/w^h}{\sigma_\varepsilon} + \frac{1}{\sigma_\varepsilon} g\left(\frac{1}{w^h}\right)\right)\right] I^- \\ &\prod_{h=H_2+1}^{H}\left[\left(1 - \Phi\left(\frac{\alpha_1 + F/w^h}{\sigma_\varepsilon}\right)\right) I^+ + \left(1 - \Phi\left(\frac{\alpha_2 + \beta_2 F/w^h}{\sigma_\varepsilon}\right)\right) I^-\right], \end{aligned}$$

where we assume that the first H_1 households in our sample have a preference for specializing in bonds, the following $(H_2 - H_1)$ prefer to diversify fully, while the latter $(H - H_2 - H_1)$ hold just stocks. I^+ denotes $I\left(\frac{1}{w^h} \geq \frac{1}{w^h}^*\right)$, while I^- denotes $I\left(\frac{1}{w^h} < \frac{1}{w^h}^*\right)$.

Given the first and second moments of asset returns and the definitions in (6), we can maximize the log- of the likelihood function in (11) with respect to F and σ_ε and obtain asymptotically efficient estimates both of the cost of stock market participation and of the variance of stock market optimism.

Problems arise if the mean of the expectation error on stock returns, ε, is not truly zero. Let z denote a set of observable covariates such that

$\varepsilon = \gamma z + \upsilon$, with $\upsilon \rightarrow N(0, \sigma_\upsilon^2)$. Then, the estimate of F based on the maximization of (11) will be biased if the variables in z are correlated with $1/w^h$. Furthermore, $\sigma_\varepsilon^2 = \gamma^2\sigma_z^2 + \sigma_\upsilon^2 + 2\gamma\sigma_{\upsilon,z}$. The problem can be addressed by redefining the choices in (7) in terms of υ which implies:

$$\begin{aligned}
D^h &= 1 : if : \upsilon^h < \alpha_1 + \gamma z^h + \frac{F}{w^h} \cap \\
\upsilon^h &< \alpha_3 + \gamma z^h + \beta_3 \frac{F}{w^h} + g\left(\frac{1}{w^h}\right); \\
D^h &= 2 : if : \upsilon^h < \alpha_2 + \gamma z^h + \beta_2 \frac{F}{w^h} \cap \\
\upsilon^h &> \alpha_3 + \gamma z^h + \beta_3 \frac{F}{w^h} + g\left(\frac{1}{w^h}\right); \\
D^h &= 3 : if : \upsilon^h > \alpha_1 + \gamma z^h + \frac{F}{w^h} \cap \\
\upsilon^h &> \alpha_2 + \gamma z^h + \beta_2 \frac{F}{w^h}.
\end{aligned} \tag{12}$$

The likelihood based on the probabilities implied by (12) yields an unbiased estimate of F unless F depends on a set of covariates that are correlated with the variables in z If $F = F(x)$, with x coinciding or not with z, the likelihood becomes:

$$\begin{aligned}
&\prod_{h=1}^{H_1} \left[\Phi\left(\frac{\alpha_1 + \gamma z^h + F\left(x^h\right)/w^h}{\sigma_\upsilon} \right) I^+ + \right. \\
&\qquad \left. + \Phi\left(\frac{\alpha_3 + \gamma z^h + \beta_3 F\left(x^h\right)/w^h}{\sigma_\upsilon} + \frac{1}{\sigma_\upsilon} g\left(\frac{1}{w^h}, x^h\right) \right) I^- \right] \\
&\prod_{h=H_1+1}^{H_2} \left[\Phi\left(\frac{\alpha_2 + \gamma z^h + \beta_2 F\left(x^h\right)/w^h}{\sigma_\upsilon} \right) + \right. \\
&\qquad \left. - \Phi\left(\frac{\alpha_3 + \gamma z^h + \beta_3 F\left(x^h\right)/w^h}{\sigma_\upsilon} + \frac{1}{\sigma_\varepsilon} g\left(\frac{1}{w^h}, x^h\right) \right) \right] I^- \\
&\prod_{h=H_2+1}^{H} \left[\left(1 - \Phi\left(\frac{\alpha_1 + \gamma z^h + F\left(x^h\right)/w^h}{\sigma_\upsilon} \right) \right) I^+ + \right. \\
&\qquad \left. \left(1 - \Phi\left(\frac{\alpha_2 + \gamma z^h + \beta_2 F\left(x^h\right)/w^h}{\sigma_\upsilon} \right) \right) I^- \right].
\end{aligned} \tag{13}$$

Maximizing (13) with respect to x^h and σ_v yields asymptotically consistent and efficient estimates of the costs of participating to the stock market and of the variance of (the unobservable component of) optimism.

4 Empirical Application

4.1 Data

4.1.1 The Shiw

The estimation of the financial participation costs is based on data from the Survey of Household Income and Wealth (SHIW), which is run biannually by the Bank of Italy.[11] Each wave surveys a representative sample of the Italian resident population and covers about 8,000 households. It collects detailed information on the composition of Italian households' wealth, both real and financial, in addition to data on households' income, consumption and demographics. For our analysis, we rely on a sequence of 5 waves, covering the period 1991-2000.

The SHIW was first run in the mid-60s but has been available on tape only since 1984. Over time, it has gone through a number of changes in sample size and design, sampling methodology and questionnaire. However, sampling methodology, sample size and the broad contents of the information collected have been unchanged since 1989. We choose to start our analysis in 1991 because, in the 1989 wave, for each asset, respondents report just the percentage share of financial wealth[12], whereas in all subsequent waves they report the asset bracket in a list of 14 possible brackets. The problem of bracketing can be handled by assuming that households own the mid-point of the interval or by applying more sophisticated imputation procedures. Imputation requires modelling the responses within each bracket and its advantage diminishes when the number of brackets is relatively large, as in the case at hand. We thus proceed using the mid-point.

[11] An exception was the 1998 wave which was run three years after the previous survey.

[12] For cash and bank deposits they are asked to report also the amount. Hence, one can estimate the amount invested in each financial asset combning this information with the fact that portfolio shares add up to one. The validity of this procedure for eliciting asset values relies on the assumptions that households are less reluctant to report portofolio shares rather than amounts and they are less reluctant to report small amounts of assets, such as with currency.

For the purpose of the analysis, we need to define separately the investments in currency and deposits, in bonds and in stocks. Currency is defined as the sum of cash, bank and postal accounts and deposits, certificates of deposits and postal saving certificates.[13] Bonds include government securities, corporate bonds and loans to coops. Stocks include direcly-held shares of listed (at end-of-year market value) and unlisted companies and partnerships (at end-of-year estimated realizable value). An asset that has become increasingly popular in Italy in the 1990s is mutual funds, whose classification is problematic because of lack of information on their composition. To avoid any assumption regarding the composition of mutual funds, we carry out our estimates of the costs by dropping those households who do not hold both stocks and bonds in addition to mutual funds.[14] We ignore foreign asset holdings, which in year 2000 were held by just 1.3 percent of households. All balance sheet items are end-of-year values; they are in thousands of euros, at prices of year 2000.

Tables 1 to 3 report some summary statistics regarding the wealth of the sample used for the analysis (inclusive of all mutual funds holders). Table 1 groups households based on the year of the survey and provides evidence on time trends in asset holdings; Table 2 groups them by area of residence and provides evidence on regional differences; Table 3 distinguishes them according to their portfolios. The picture that emerges from Table 1 is well-known. Household net worth has increased by over 30 percent over the decade and mean financial assets have almost doubled. After peaking in 1995, bond holdings in 2000 were about the same as in 1991. The share of bond owners has somewhat fallen, with the sharp drop in government bond holders partly offset by a rise in corporate bond holders. However, most of the action has concerned equity and mutual funds holdings: the amount of wealth invested in these assets has increased steadily over the 1990s and in 2000 it was 7 times the amount held in 1991 with the share of investors gone from 6 percent of

[13] All households in the sample hold some riskless asset in the form of cash. In the original data set there were 175 household (out of 39,558) who reported zero cash and deposits and they have been excluded.

[14] The same applies to managed accounts, whose composition is also non-reported, but whose diffusion is limited. The mutual fund and managed accounts holders whose classification in terms of risky portfolio composition is problematic represent about 6 percent of the whole sample: 3 percent hold just mutual funds; just over 2 percent report bonds and mutual funds holdings (but no directly-held stocks) and less than 1 percent hold mutual funds and directly-held equity (but no bonds). As a share of risky asset holders, they amount to 10, 8 and 3 percent, respectively.

the population to 20 percent. The increase can be attributed both to a rise in direct ownership of stocks and to the increasing popularity of mutual funds. If we look at financial asset shares, the percentages held in cash and deposits and in bonds have fallen by over 10 percentage points each; the share held in equity has more than doubled; that in mutual funds has increased five-fold. The picture based on (non-asset) income shares is similar.

Table 2 looks at the differences in asset ownership across geographical areas and distinguish among households depending on whether they live in the North, in the Center or in the South (and islands). People living in the North are much wealthier than those living in the South and the discrepancy is particularly strong when it comes to financial assets, with the amount held by the representative family in the North about three times as large as that held by the representative household in the South. With respect to the latter, the average household in the North holds 4 times as many bonds, 6 times as much directly-held equity and 8 times as many mutual funds. The differences in terms of participation rates are as sharp. When looking at portfolio shares, households in the South hold almost 90 percent of their financial wealth in cash, bank deposits and bonds, versus 70 percent in the North. The picture based on income shares is similar apart from the percentages of cash and deposits which do not exhibit any significant regional variation.

Finally, Table 3 distinguishes between riskless asset holders (first column), who hold just currency, and those who also hold some risky assets: bond holders are in the second column, bond and stock holders (with or without mutual funds) are in the third, stock holders are in the fourth and those with mutual funds alone, or with either just stocks or just bonds are in the last. On average, riskless asset holders are the less wealthy and the most heavily indebted. They tend to be older, less educated, more likely to be headed by a woman, less likely to be married and tend to live in the South. Among risky asset holders, those specialized in bond holdings are the poorest, both in terms of asset holding and in terms of income and appear quite heavily indebted. Furthermore, they are older, less educated, more likely to be retired and less likely to be self-employed than the rest of the risky asset holders. Those who are fully diversified are the wealthiest, the most educated, the most likely to be married and the most likely to live in the North. Those specialized in stock holdings are somewhere in between. They are younger than the other risky asset holders, less likely to be retired and more likely to be self-employed. This picture is coherent with the framework outlined in the previous sections. Stock market participation is costly: hence households must have a sufficiently large amount of resources to invest. However, the super-rich are unlikley to specialize in stocks and more likely to allocate at least a small amount of their resource to bonds. Among the moderately rich, specialization will occur among the most optimistic and risk-loving, such as the young and the self employed. Those in the last column are in between the fully diversified and those specialized in stocks.

4.1.2 The Italian financial markets[15]

Annual returns on stocks and bonds are taken from Panetta and Violi (1999). In their paper, Panetta and Violi reconstruct the series of the real returns on Italian equities and long-term government bonds from 1860 to today. The returns include both capital gains and losses and dividends or coupons paid.[16] Overall, the Italian equity market provided long-term returns comparable to those of the other major countries. However, a large fraction of the risk premium for the period starting from 1860 can be accounted for by the performance following the high inflation episodes of the wars. As a consequence owing to the much larger volatility, the risk-return trade-off compares unfavorably with other markets. Because of this, for our analysis we have chosen to focus on the past fifty years of return, based on the presumption that households form their expectations of stock returns and risk premia, using just the information that go as far back as the 1950s. This is

[15]This section draws heavily from Panetta and Violi (1999).

[16]For their total return index for shares, Panetta and Violi used various sources. From 1860 to 1895 their index is based on the general index of the Genoa Stock Exchange. From 1896 to 1907 it was calculated with reference to all the shares listed on the Milan Stock Exchange. From 1907 to 1911 it was obtained from Aleotti (1990). From 1912 to 1977 it was based on the Bank of Italy's indeces of prices and dividends for a sample of 40 companies, which accounted for over three quarters of the total market capitalization. After 1977 the index was calculated using the Bank of Italy Research Department's data base on the share market, which covers all listed shares. As to long-term government securities, Panetta and Violi's index is based on the return on the consolidated debt of the Treasury for the period 1862 to the second world war (the 5% consolitated Rendita issues, the 4th, 5th and 6th national loans and the Littorio loan). For the period 1945 onwards, they constructed an index of total returns on the Treasury bonds (BTPs) listed on the stock exchange. The calculation of returns takes into account the witholding tax on government securities coupons and on dividends, allowing for the frequent changes occurred during the period. See Panetta and Violi (1999) for further details on the sources and methods used for the caluclation of the indeces. See also Siciliano (2001), who uses basically the same data, for a discussion on the issues of taxation.

the same as assuming that after the war there was a structural break such that the hypothesis of i.i.d. returns holds over the pre-1950 period and over the post-1950 period, but not over the two sample periods taken jointly. We will check the robustness of our results by considering also longer and shorter windows.

Table 4 reports summary statistics for the returns used in the analysis: the first column refers to after-tax real returns on bank deposits (the riskless asset); the second and third columns refer to the after-tax real total returns on long-term government bonds and on the stock market; the last column reports the correlation between the excess returns of stocks and bonds over the riskless asset. Stock and bond returns are also plotted in Figure 3. Over the period running from 1914 to 1998, the excess returns on equity and long-term government bonds over bank deposits amounted to almost 11 and over 2 percent respectively, and the equity premium over bonds was around 8 percent, with a correlation between bond and stock (excess) returns of 0.24. Restricting the period to 1951-1998, the excess returns drop to just below 9 and 3 percent, respectively, with a correlation of 0.18. In the first half of the XX century, asset prices went through two periods of high inflation, during the first and the second world war, which caused major drops of real returns, and the Great Depression when share prices dropped by almost 40 percent in real terms. In the 1950s share prices rose sharply by a factor of 8 in conjunction with the rapid income growth of the post-war era. During the same years, the price of government securities rose by approximately 50 percent. The sixties were characterized by a much slower growth of the Italian economy and the seventies registered a sharp rise of inflation. Furthermore, the share market was depressed by a series of other factors such as the nationalization of the electricity companies and the introduction of the withholding tax on dividends. In the early sixties share prices declined sharply and in 1964 they were about half their record high reached in 1961. After a short period of fluctuations, they began to fall again and rock-bottomed at the end of 1977 after the first oil crisis. The seventies are characterized by a sharp fall of the price also of government bonds as a result of the capital losses caused by the rise in nominal interest rates. Nonetheless, their return remained above that of shares throughout the period. The 1980s represent a positive era for the Italian economy and for the share market. In 1986, stock prices rose by over 200 percent in nominal terms as result of the introduction of investment funds, of the several initial public offerings and of the general enthusiasm that followed. Stock returns began to fall again towards the end of the decade as a consequence of world-wide recession, whereas government-bond prices were not much affected. The second half of the nineties is characterized by a new phase of rising share prices. Several factors contributed to the bull market: these include the privatizations of the utilities, the public offerings of several small companies, which benefited of the fiscal incentives wanted by Tremonti, and the new legislation (*testo unico della finanza, d.lgs. n. 58/1998*) which brought about far-reaching changes in the structure of the stock market.

4.2 Results

Tables 5 through 7 report the results from the maximization of the likelihood in equation (13), which is reported here for convenience:

$$\prod_{h=1}^{H_1}\left[\Phi\left(\frac{\alpha_1+\gamma z^h+F\left(x^h\right)/w^h}{\sigma_v}\right)I^{+}+\right.$$
$$\left.+\,\Phi\left(\frac{\alpha_3+\gamma z^h+\beta_3 F\left(x^h\right)/w^h}{\sigma_v}+\frac{1}{\sigma_v}g\left(\frac{1}{w^h},x^h\right)\right)I^{-}\right]$$
$$\prod_{h=H_1+1}^{H_2}\left[\Phi\left(\frac{\alpha_2+\gamma z^h+\beta_2 F\left(x^h\right)/w^h}{\sigma_v}\right)+\right.$$
$$\left.-\,\Phi\left(\frac{\alpha_3+\gamma z^h+\beta_3 F\left(x^h\right)/w^h}{\sigma_v}+\frac{1}{\sigma_\varepsilon}g\left(\frac{1}{w^h},x^h\right)\right)\right]I^{-}$$
$$\prod_{h=H_2+1}^{H}\left[\left(1-\Phi\left(\frac{\alpha_1+\gamma z^h+F\left(x^h\right)/w^h}{\sigma_v}\right)\right)I^{+}+\right.$$
$$\left.\left(1-\Phi\left(\frac{\alpha_2+\gamma z^h+\beta_2 F\left(x^h\right)/w^h}{\sigma_v}\right)\right)I^{-}\right],$$

where α_1, α_2, α_3, β_2, β_3 and $g(.)$ depend on the distribution of asset returns according to the definitions in (6).

The coefficients reported in the upper part of each Table refer to the cost of participating to the stock market and correspond to the coefficients of the polynomial $F(x)$ based on the assumption that the participation cost depends on the set of covariates x. We set $F(x) = F + \phi x$, with $\phi = 0$ implying that all households in the sample face the same cost F. $F(x)$ enters the likelihood as the “coefficient” of $1/w$. The coefficients reported in the bottom part of the tables refer to the expectation error regarding the equity premium, which was parameterized as $\varepsilon = \gamma z + v$ They correspond to the standard deviation σ_v and to the coefficients γ on the covariates z, shifting the mean of the error. Clearly when $\gamma = 0$, $\sigma_\varepsilon = \sigma_v$. Given the lack of theoretical priors regarding x and z we set them equal and let them include a second-order polynomial in age, education dummies, a gender dummy, occupation dummies and dummies for homeownership, area of residence and year of survey. Computational constraints limit our ability to saturate the regressions with more household-specific as well as “supply” variables.

Tables 5, 6 and 7 are based on a sample that excludes those households who do invest in mutual funds but not also in both bonds and stocks. The moments of asset returns are computed using the data for the period 1951-1998, which implies excess returns on stocks and bonds over bank deposit (the riskless asset) of 8.73 and 2.70 percent with standard deviations of 28 and 6 percent, respectively, and a correlation between the returns of 0.1755.

The evidence reported in the first column of Table 5 is based on the assumption that neither the cost of participating, nor the expectation error depend on household characteristics, i.e. $F(x) = F$ and $\gamma = 0$. In this instance the fixed cost consistent with observed portfolio choices, i.e. the coefficient on $1/w^h$ reported in the first row of the Table, is estimated to be around 160 euros (prices of year 2000) per year. This is the fixed costs in which households incur for investing and managing their investment in stocks. It includes monetary charges and opportunity costs of investor's time for choosing, carrying out and managing the investment. The standard deviation of the expectation error, σ_v, is estimated to be quite high at over 30 percent. Both F and σ are estimated with great precision. As a matter of fact, our estimate of the standard deviation of the expectation error is likely to be overstated because it has to compensate for the lack of flexibility of the theoretical model. To gain in tractability, we have assumed that there is no entry cost in the bond market. This makes the stock market investment undesirable, for anyone with a low amount to invest, unless optimism is high. As indeed there are several people with small portfolio who actually hold a pure stock investment, the estimate of the standard deviation of optimism must be large to fit the data. Had the bond market entry cost not been restricted to zero, this bias would not be a problem.

When we allow F to vary across households with a set of observable socio-demographic charcteristics and include a set of covariates in the mean of ε, the across household average of the estimated cost turns out to be around 150 euros (with a standard deviation of almost 50 euros). The analysis reported in the third column of Table 5, suggests that the cost is convex in age and peaks around 60 years of age, which might reflect some opportunity cost of time: for those in their fifties, the costs are about 50 euros higher than the costs for those in their thirties. As expected, education lowers the cost of participation (see Haliassos and Bertaut (1995) for a discussion), but what really makes a difference is having a university degree: for such investors the costs are on average around 30 euros lower than the costs faced by those with elementary school or less (the benchmark, omitted education dummy). The coefficient on the gender dummy for male-headed households is positive, but insignificant; that on the single-person household dummy is also positive and suggests that for singles the fixed cost of stock market participation are higher by just over 25 euros. Interestingly, the costs are lower for homeowners (-25 euros) which might reflect deeper "investment culture/knowledge" and greater familiarity with financial instruments, although a dummy for having a mortgage (not included in this specification) is never significant. The coefficient on the dummy for self emplyed head is positive and significant, which might once again reflect the opportunity cost of time. The dummies for the area of residence suggest that with respect to those living in the North (omitted benchmark), for those living in the South investing in stocks is over 70 euros costlier. Finally, and quite surprisingly, the year dummies are not consistent with any regular time trend in the costs and suggest that the fixed costs of investing in 2000 (omitted benchmark) are slightly higher than in 1995 and even than in 1991.

The role of age, education, gender, occupation, area of residence, etc. turn out to be more important in determining households' optimism. For the most educated, the male, the homeowners, the self employed, optimism seems to play a relatively smaller role in determining stock ownership, via the effect of such characteristics on the expected equity premium, which is negative making these households relatively less bullish about the stock market. If anything, the importance of optimism has fallen over time. σ_v is estimated at 0.42.

The last columns of the Table report the results of the estimation based on a smaller set of covariates, which is the specification adopted in most of the rest of the analysis due primarily to computational limitations. No important difference can be detected.

Next, we split the sample and distinguish between households living in the North, Center and South of the country in Table 6 and between pre- and after-1996 in Table 7. Notice that by splitting the sample, we are not only allowing for costs to vary across the splits, but we are also assuming that the distribution of optimism varies across the splits and this might explain some of the unexpected differences in the estimates. Without controls (panel (a) of Table 6), the costs of investing in stocks turn out to be around 175 euros for those living in the North of the country, 190 euros for those living in the Center and 260 euros for those living in the South. The costs do not appear to vary much across socio-demographic groups. University education turns out to be a significant cost-reducing factor, but only for those living in the North or in the Center. This might suggest that most of the fixed costs associated to direct stock ownership and captured by our set up are monetary (as opposed to figurative). When we look at time trends in Table 7, costs appear to have fallen from over 300 euros in the first half of the decade to around 160 euros and σ_v has increased.[17] Interestingly, over time, the North-South difference in the costs of direct investment in stocks (not reported) has increased, although the costs have fallen in all regions.

To conclude, it must be mentioned that the results are quite sensitive to the equity premium used in the estimation and clearly the lower the premium, the lower the costs consistent with observed portfolio choices. Specifically, ignoring household heterogeneity, considering the returns over the period 1914-1998, F is estimated at around 80 euros and σ_v turns out around 20 percent. Considering the returns over the period 1961-1998 and over the period 1971-1998, F and σ_v turn to be around 100 euros and 30 percent and 150 euros and 50 percent, respectively.

5 Concluding Remarks

This paper estimates the costs of participating to the stock market that rationalize the choice of non investing in stocks. We also identify the cross-sectional dispersion of the expectation errors of investors on the size of the

[17] The increase in σ implies a restriction on the covariance between the time dummies and the truly unobservable component of ε. Formally, let $\varepsilon = \alpha d + \upsilon$, where d takes value 1 if the survey takes place before 1998 and 0 otherwise. The unconditional variance of ε would be: $Var(\varepsilon) = \alpha^2 Var(d) + Var(\upsilon) + 2\alpha Cov(d, \upsilon)$. Conditioning on d, $Var(\varepsilon|d) = Var(\upsilon)$. It follows that: $Var(\varepsilon|d) > Var(\varepsilon)$ only if: $\alpha^2 Var(d) + 2\alpha Cov(d, \upsilon) < 0$, i.e. if $2Cov(d, \upsilon) < -\alpha Var(d)$.

equity premium, which can be thought of as a broad measure of their stock market optimism. Our analysis is based on a mean-variance framework, when there is a riskless asset (cash), which makes the allocation of the investment in risky assets (stocks and bonds) independent on preferences and based exclusively on mean-variance eficiciency considerations. Within this framework, we derive "structural" decision rules for the composition of the risky asset portfolio to be efficient. These rules depend on the amount invested in the risky portfolio and on investors' optimism or pessimism regarding asset returns, which are the determinants of the stock market return expected by a household, when participation involves a fixed cost. Using these rules and the heterogeneity in risky assets holdings and in the degree of optimism, we are able to identify both the fixed costs of stock investment and the variance of optimism and to estimate the parameters of interest via maximum likelihood.

We then use the Bank of Italy's Survey of Household Income and Wealth to analyze the cost that rationalize Italian households portfolio choices and find that the risky asset portfolios of individual investors are coherent with a fixed cost of participating directly to the stock market around 150 euros per year, which corresponds to 0.9 percent of mean non-durable household expendidure. Having a university degree, owning one's home and living in the North of the country contribute to lower the cost. Investors' optimism as measured by the standard deviation of the expectation error is estimated to be high, at over 30 percent. As mentioned, this estimate is likely to be biased, hence it should be looked into further.

Overall, the empirical evidence on the nature and on the entity of the costs associated to financial transactions is rather limited, especially at the household level, and a reason is that some of these costs are likely to be figurative and related to information gathering and processing. For the US, Vissing Jørgensen (2002) estimates the median of the distribution of the per-period costs of participating to the stock market to be around \$350; Paiella (2002) bounds from below the fixed costs of stock market participation at around \$100 per year, corresponding to 1 percent of non-durable consumption. Based on the US National Income and Product Account, the ratio of personal expenditure on brokerage charges, investment counseling, bank and trust service charges to expenditure on non-durable goods amounts to 3 percent in 2000 (2 percent in 1996). None of these evidence distinguishes between direct and indirect stockholding.

Table 1: Household portfolios over time

	1991	1993	1995	1998	2000
Net worth	129.660	152.303	150.308	163.208	171.614
Financial wealth	16.041	19.395	19.081	25.187	27.882
Real assets	116.160	136.242	134.491	140.885	147.174
Cash and deposits	9.406	9.271	8.898	12.604	13.471
Owners of deposits (share)	*0.811*	*0.838*	*0.835*	*0.860*	*0.805*
Bonds	5.292	6.540	7.699	4.683	5.396
Owners of government bonds and bills (share)	*0.233*	*0.227*	*0.263*	*0.118*	*0.118*
Owners of private bonds (share)	*0.015*	*0.021*	*0.039*	*0.063*	*0.068*
Equity and mutual funds	1.309	2.891	2.489	7.649	8.640
Owners of equity and funds (share)	*0.060*	*0.082*	*0.077*	*0.154*	*0.186*
Directly held equity	0.644	1.120	0.902	1.856	2.679
Owners of directly held equity (share)	*0.035*	*0.041*	*0.039*	*0.079*	*0.097*
Mutual funds	0.421	1.166	1.009	3.195	4.093
Owners of mutual funds (share)	*0.025*	*0.044*	*0.042*	*0.097*	*0.117*
Managed accounts	0.244	0.605	0.577	2.598	1.868
Owners of managed accounts (share)	*0.010*	*0.010*	*0.010*	*0.029*	*0.030*
Financial asset shares:					
Cash and deposits	0.586	0.478	0.466	0.500	0.483
Bonds	0.330	0.337	0.403	0.186	0.194
Directly held equity	0.040	0.058	0.047	0.074	0.096
Mutual funds	0.026	0.060	0.053	0.127	0.147
(Non-financial) income share:					
Total financial assets	0.643	0.799	0.795	1.025	1.094
Cash and deposits	0.377	0.382	0.371	0.513	0.529
Bonds	0.212	0.270	0.321	0.190	0.212
Directly held equity	0.026	0.046	0.038	0.076	0.105
Mutual funds	0.017	0.048	0.042	0.130	0.161

Note: all figures are mean values. They have been deflated to 2000 currency and are in thousands of euros. 175 households (out of 39,558) have been dropped from the sample because they reported zero cash and zero bank and postal deposits. Real assets include real estate, land and business assets. Financial wealth includes also foreign assets which are ignored in the rest of the analysis because they account for a negligible share of households wealth. Deposits include both bank and postal current and saving accounts and saving certificates. Equity includes both listed and private shares. Asset and income shares are computed as ratios of averages. Financial asset shares do not add up to 1, because of foreign assets and managed accounts which have been included in the denominator.

Table 2: Household portfolios across geographical area

	N	C	S
Net worth	178.738	167.759	106.663
Financial wealth	29.068	20.821	10.445
Real assets	153.400	150.240	98.244
Cash and deposits	11.971	12.160	7.903
Owners of deposits (share)	*0.912*	*0.864*	*0.686*
Bonds	8.844	5.406	2.003
Owners of government bonds and bills (share)	*0.274*	*0.184*	*0.081*
Owners of private bonds (share)	*0.060*	*0.040*	*0.012*
Equity and mutual funds	7.527	3.089	0.888
Owners of equity and funds (share)	*0.170*	*0.095*	*0.031*
Directly held equity	2.376	0.769	0.409
Owners of directly held equity (share)	*0.088*	*0.048*	*0.018*
Mutual funds	3.218	1.334	0.413
Owners of mutual funds (share)	*0.100*	*0.054*	*0.016*
Managed accounts	1.933	0.986	0.066
Owners of managed accounts (share)	*0.031*	*0.011*	*0.001*
Financial asset shares:			
Cash and deposits	0.412	0.584	0.757
Bonds	0.304	0.260	0.192
Directly held equity	0.082	0.037	0.039
Mutual funds	0.111	0.064	0.040
(Non-financial) income share:			
Total financial assets	1.060	0.780	0.539
Cash and deposits	0.436	0.456	0.408
Bonds	0.322	0.203	0.103
Directly held equity	0.087	0.029	0.021
Mutual funds	0.117	0.050	0.021

Note: see note to Table 1.

Table 3: Household wealth and demographic characteristics by portfolio holdings

	B=0, S=0	B=1, S=0	B=1, S=1	B=0, S=1	Others with mutual funds
Total assets	111.459	209.409	505.505	308.879	348.48
Liabilities	2.949	2.591	5.22	4.99	4.466
Bonds	0	23.32	44.972	0	13.034
Stocks	0	0	27.06	20.135	3.727
Mutual funds	0	0	28.128	0	36.433
Non-financial income	20.945	30.322	47.462	35.356	39.066
Consumption	16.288	21.956	32.687	25.542	27.743
Age	54.875	54.834	52.967	50.935	50.967
Gender	0.692	0.783	0.84	0.764	0.784
Up to 5th grade	0.485	0.343	0.117	0.195	0.158
8th grade	0.266	0.263	0.182	0.241	0.261
High school diploma	0.204	0.296	0.416	0.43	0.413
University degree	0.045	0.098	0.285	0.134	0.167
Married	0.665	0.751	0.804	0.739	0.773
Household components	2.786	2.853	2.952	2.947	2.864
Income recipients	1.644	1.88	2.034	1.919	1.92
Pensioner	0.427	0.441	0.318	0.251	0.304
Self-employed	0.156	0.153	0.29	0.317	0.255
Publicic sector employee	0.148	0.174	0.171	0.149	0.185
North	0.404	0.651	0.788	0.63	0.758
Center	0.195	0.194	0.141	0.198	0.166
South	0.401	0.155	0.07	0.172	0.177
Nobs	28,319	6,659	1,246	699	2,460

Note: see note to Table 1. Column 3 refers to the holders of bonds and stocks with or without mutual funds; column 5 refers to those with mutual funds, but no stocks, nor bonds, those with mutual funds and stocks, but no bonds and those with mutual funds and bonds, but no stocks. Age, gender, the education dummies, marital status and occupation refer to the household head. North, Center and South refer to the area of residence. South includes the islands.

Table 4: Asset returns

	Bank deposits (R)	Shares (R_s)	Gov. bonds (R_b)	Corr(R_s – R, R_s – R)
1914-1998	-0.0382	0.0709	-0.016	0.2419
	(0.1288)	(0.3182)	(0.1548)	
1951-1998	-0.0051	0.0822	0.0219	0.1755
	(0.0329)	(0.2866)	(0.0776)	
1971-1998	-0.0056	0.0712	0.0290	0.1379
	(0.0384)	(0.3288)	(0.0977)	

Note: real after tax returns. The last column reports the correlation between the excess returns on shares over the riskless asset and the excess returns on bonds over the riskless asset. Annual frequencies. Standard deviations in parentheses.

Table 5: Stock market participation costs and optimism: pooled estimation

		Coef.	S. E.	Coef.	S. E.	Coef.	S. E.
1/w		312.000	0.698	-32.205	93.474	56.559	92.689
″	* age			10.116	3.843	4.998	3.631
″	* age2			-6.543	3.726	-1.939	3.524
″	* 8th grade			-15.304	26.844	-3.426	19.077
″	* diploma			-9.078	24.865	-3.405	17.965
″	* degree			-65.287	28.629	-48.317	21.436
″	* gender			30.695	17.357	9.749	15.614
″	* single			55.216	27.688		
″	* home			-56.075	19.710		
″	* public			5.567	17.019		
″	* self			41.671	19.755		
″	* Center			49.356	18.888	13.110	16.670
″	* South			150.733	22.333	73.203	16.153
″	* yr 1991			-136.872	20.498	-114.929	18.996
″	* yr 1993			26.327	30.175	4.761	20.367
″	* yr 1995			-50.686	20.174	-53.622	17.288
″	* yr 1998			-21.378	19.850	16.074	20.287
σ_v		0.304	0.007	0.421	0.013	0.452	0.013
age				-0.014	0.004	-0.017	0.004
age2				0.014	0.003	0.017	0.003
8th grade				-0.148	0.023	-0.139	0.024
diploma				-0.304	0.022	-0.307	0.023
degree				-0.395	0.026	-0.395	0.027
gender				-0.054	0.019	-0.095	0.019
single				0.044	0.024		
home				-0.040	0.017		
public				0.068	0.019		
self				-0.165	0.018		
North				0.617	0.100	0.674	0.105
Center				0.663	0.100	0.725	0.106
South				0.667	0.101	0.736	0.106
yr 1991				0.504	0.026	0.540	0.027
yr 1993				0.372	0.023	0.409	0.024
yr 1995				0.429	0.023	0.459	0.025
yr 1998				0.076	0.021	0.084	0.022
N. obs		8,494		8,486		8,495	
Wald χ^2		(1) 199,771		(17) 1,671		(13) 2,424	

Note: The sample does not include those holding just mutual funds, mutual funds and just bonds and mutual funds and just stocks. Omitted (benchmark) dummies: less-than-8th-grade (interacted with 1/w); North (interacted with 1/w); yr 2000 (alone and interacted with 1/w). Gender takes on value 1 if the head is a male. The degrees of freedom of the Wald χ^2 are in parentheses. In the estimation w is measured in thousands of lira of year 2000. Hence, the coefficients of the variables interacted with 1/w should be divided by 1.92736 to obtain the costs in euros.

Table 6: Geographical differences in stock market participation costs and optimism

Panel(a)

	North		Center		South	
	Coef.	S. E.	Coef.	S. E.	Coef.	S. E.
1/w	339.283	2.371	363.621	2.907	505.452	4.444
σ_v	0.285	0.008	0.255	0.013	0.261	0.015
N. obs	5,172		1,841		1,479	
Wald χ^2 (1)	20,470		15,648		12,938	

Panel (b)

	North		Center		South	
	Coef.	S. E.	Coef.	S. E.	Coef.	S. E.
1/w	375.510	137.726	795.833	350.333	-46.204	362.334
″ * age	-4.121	5.392	-17.573	14.567	8.356	14.430
″ * age2	7.207	5.176	19.265	15.269	-4.171	13.551
″ * 8th grade	27.577	27.362	-99.534	98.895	33.999	83.377
″ * diploma	10.813	24.006	-217.133	90.868	168.834	109.269
″ * degree	-56.861	23.999	-267.614	104.275	90.256	107.026
″ * gender	-29.373	29.091	19.966	60.476	-14.264	97.418
″ * yr 1991	-202.191	25.197	-0.100	66.258	-48.084	91.838
″ * yr 1993	-24.376	31.907	139.484	96.174	-46.543	85.147
″ * yr 1995	-61.329	26.244	86.268	57.543	-135.088	94.081
″ * yr 1998	-41.270	27.954	163.300	64.032	39.522	74.709
σ_v	0.384	0.014	0.428	0.034	0.531	0.051
age	0.008	0.001	0.003	0.003	0.001	0.004
age2	-0.006	0.002	0.000	0.003	0.005	0.005
8th grade	-0.114	0.025	-0.138	0.048	0.045	0.076
diploma	-0.271	0.024	-0.204	0.046	-0.154	0.065
degree	-0.376	0.029	-0.305	0.054	-0.186	0.070
gender	-0.084	0.020	-0.028	0.039	-0.194	0.066
yr 1991	0.451	0.028	0.615	0.071	0.712	0.093
yr 1993	0.328	0.026	0.418	0.055	0.640	0.085
yr 1995	0.359	0.026	0.435	0.054	0.842	0.104
yr 1998	0.047	0.024	0.153	0.047	0.174	0.065
N. obs	5,173		1,842		1,479	
Wald χ^2 (11)	1,059		206		123	

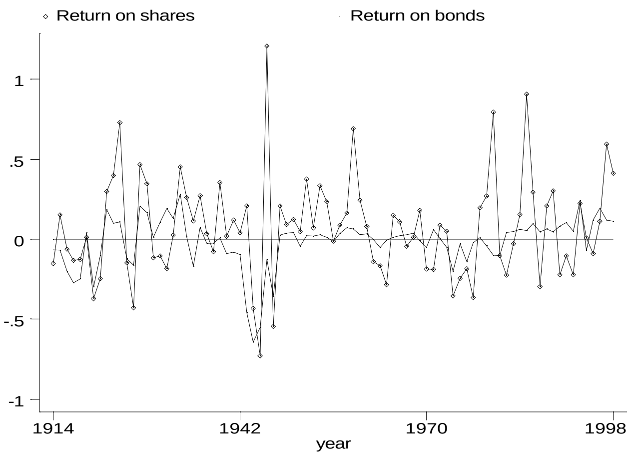
Note: see note to Table 5.

Table 7: Time trends in stock market participation costs and optimism

	1991 - 1995				1998 - 2000			
	Coef.	S. E.	Coef.	S. E.	Coef.	S. E.	Coef.	S. E.
1/w	597.505	4.094	436.514	172.167	321.216	6.602	-161.741	114.997
″ * age			-13.368	6.309			15.314	4.733
″ * age2			14.351	5.864			-11.770	4.798
″ * 8th grade			26.606	23.987			-25.845	30.371
″ * diploma			-2.723	26.032			-1.091	36.858
″ * degree			-39.392	32.767			-48.537	39.973
″ * gender			8.726	28.250			10.334	22.619
″ * Center			89.641	26.024			13.591	24.160
″ * South			59.926	25.416			102.982	27.397
σ_v	0.140	0.003	0.533	0.024	0.394	0.013	0.401	0.015
Age			-0.023	0.006			-0.009	0.005
Age2			0.021	0.006			0.012	0.005
8th grade			-0.143	0.037			-0.151	0.032
Diploma			-0.365	0.037			-0.269	0.031
Degree			-0.496	0.043			-0.321	0.036
Gender			-0.178	0.034			-0.036	0.023
North			1.538	0.180			0.379	0.137
Center			1.608	0.183			0.407	0.138
South			1.692	0.185			0.366	0.139
N. obs	5,882		5,885		2,610		2,610	
Wald χ^2	(1) 21,304		(9) 682		(1) 367		(9) 1,364	

Note: see note to Table 5.

Figure 3: Returns on an index of Italian equities and on an index of long-term government bonds



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