

# Are Patents Discouraging Innovation?

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## Abstract

The strengthening of the U.S. patent regime in the early eighties was followed by a sharp increase in patenting but did not change the R&D expenditure significantly in some industries in the U.S. This “patent paradox” is prominently observed in complex industries, like the semiconductor industry. In this paper I develop a model of invention and product development to examine the effects of a patent regime change on the patenting and R&D decisions of firms in complex industries. Firms in these industries have a greater need to access a large number of ideas to successfully develop an end product. I consider two different environment — one without licensing and one with licensing. While a stronger patent regime leads to higher patenting and R&D activities in both environments, the strategic complementarity between patenting and R&D is relatively weaker in the presence of licensing. A stronger patent regime change that creates incentives for firms to increase patenting activity, therefore, may not lead to a similar increase in R&D activity. (*JEL* L00, L24, O34)

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# 1 Introduction

In 1982 the U.S. Congress established the Court of Appeals for the Federal Circuit (CAFC), a move seen as strengthening the level of patent protection in the United States. The intention on the part of the Government was to create a strong intellectual property regime that will create incentives for firms to conduct R&D. Researchers have debated the pros and cons of this change.<sup>2</sup> Among them, Hall and Ziedonis (2001) have pointed out that in the semiconductor industry patenting increased substantially after 1982 while R&D expenditure maintained the previous trend. They and other researchers have suggested that the strengthening of the patent regime has not changed the incentive to perform R&D significantly, but has provided incentives to create large patent portfolios for bargaining purposes. This observation is particularly relevant for “complex product” industries, such as the semiconductor industry, where the development of the end product is generally achieved by using ideas and products owned by different firms.<sup>3</sup>

Some complex product industries, like electronics and semiconductors, have traditionally relied on licensing and bilateral bargaining (called cross-licensing) to access the knowledge owned by different firms. The number of patented (or potentially patentable) ideas or inventions needed to develop the product produced by these industries is large and the patents are often owned by different firms. Grindley and Teece (1997) note that “with this degree of overlap of technology, compa-

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<sup>2</sup>See Cohen, Nelson and Walsh (2000), Kortum and Lerner (1998), Merges and Nelson (1990).

<sup>3</sup>The term “complex product” is used by Cohen, Nelson and Walsh (2000) to describe “commercializable product or process ... comprised of numerous separately patentable elements.... In complex product industries, firms often do not have propriety control over all the essential complementary components of at least some of the technologies they are developing.”

nies protect themselves against mutual infringement by cross-licensing portfolios of all current and future patents in a field-of-use, without making specific reference to individual patents.” Researchers, who have surveyed the intellectual property (IP) managers of some of the computer, electronics and semiconductor firms, report that after the strengthening of the patenting regime licensing activities in the industry have increased and patents have become more important as a bargaining chip in these agreements.

As mentioned before, the reason for strengthening the U.S. patent regime was to create an incentive system that rewards innovation activity and, thereby, creates incentive for firms to invest more in R&D activities. The “patent paradox”, a term used by Hall and Ziedonis (2001), refers to the empirical observation that the change in the U.S. patent regime was followed by substantial patenting activity by firms, which was not matched by firms’ R&D activity. On an aggregate level the growth of R&D expenditure remained constant, while the growth of patenting increased substantially. The patent paradox raises important questions regarding the role played by the institution of patenting in firms’ decision making process.

No one has so far, to my knowledge, attempted to theoretically analyze the link between patenting, innovation and bilateral licensing in the context of a patent regime change. My paper is an attempt in that direction.

This paper presents a stylized model of basic invention, patenting and product development to study the effects of a stronger patent regime on patenting and R&D in a given licensing environment. I treat the process of acquiring basic inventions (or *ideas*) and the process of developing a new product as two separate activities. R&D expenditure determines the number of new in-house ideas acquired by each firm, but the in-house ideas alone do not guarantee successful product develop-

ment. Access to additional ideas, that are developed by rival firms, increase the probability of successful product development for each firm. Another feature of the model is that each industry is characterized by a complexity parameter. A complex product industry is one where relatively more ideas are needed to successfully develop the final product. In other words for firms in this industry, who have access to only a few ideas, the probability of successful product development is small. I compare the effects of a stronger patent regime on firms' decision to patent and invest in R&D without and with licensing. The model predicts that in complex industries the responsiveness of a firm's R&D decision in response to the strengthening of the patent regime depends on the licensing environment. I consider two different environments — one with no-licensing and the other with licensing. In the presence of licensing the strategic complementarity between R&D and patenting is weaker. Therefore in response to a strengthening of the patent regime, even when the increase in patenting predicted by both the environments are the same, the R&D expenditure will be less affected in the presence of licensing as compared to the case without licensing.

The result can be understood by observing that in this paper a stronger patent regime encourage patenting by lowering the cost of patenting. Increased patenting activity, in turn, provides incentives for higher R&D investment because with increased patenting firms now know that imitation activity will decrease which will allow the owners of innovations a better chance to develop the final product. This creates the strategic complementarity between firms' decision to patent and their decision to invest in R&D. The complementarity result hold in both kind of licensing environments considered in this paper. In the absence of licensing, firms can improve their chance of developing the final product either via innovation or

via imitation. In the presence of licensing, firms have an additional source of obtaining innovations — via licensing. The presence of the licensing option reduces the complementarity between patenting and R&D investment decisions made by each firm. Although a stronger patent regime provides incentive to engage in higher levels of patenting, that increase in patenting elicits a smaller increase in R&D investment in the presence of licensing. Empirically, therefore, it is possible to observe a stronger patent regime leading to a substantial increase in patenting activity without observing a similar increase in R&D activity.

## **1.1 Related Literature**

There has been substantial theoretical work on cumulative innovation. This paper also deals with cumulative innovation, but the notion of cumulative innovation used here is slightly different than that found in the literature. In the literature cumulative innovations generally refers to the notion of time-lagged complementarity. The basic idea is that today's innovations are not only valuable for the immediate benefits they provide, but also valuable inputs for future innovations. For cumulative innovations complementarity exists between today's innovations and tomorrow's innovations. In that sense, cumulative innovations refer to time-lagged, unidirectional complementarity. In complex product industries like the semiconductor industry innovations are not only cumulative in the above sense but they also show temporal and bi-directional complementary, i.e., the mutual exchange of innovations at the same period of time enhances the chance of success for all parties concerned.

Bessen and Maskin (2000) address the issue of complementary innovations and their paper relates most closely to my work. They investigate a firm's incen-

tive to invest in R&D by comparing two scenarios, one without a patent system and another with a patent system. Complementarity is modeled by assuming that the expected number of innovations increases when both firms invest in R&D. In the no-patent case two firms make the decision whether to invest or imitate. In the presence of a patent system there is no imitation. Instead firms can invest in new innovation only if the patent-holders agree to license their innovations. Bessen and Maskin identify conditions under which patent-holders will not license their innovations and will, therefore, lead to an outcome that reduces social welfare.

This paper develops a model where imitation of innovations, as in Bessen and Maskin (2000), has a positive social value. In Bessen and Maskin, imitation increases the expected number of future innovations. In my paper imitation increases the chance of successfully developing a final product which is valuable to the consumers. The patent system plays an important role by altering the cost of patenting and, thus, altering firm's choice of in-house R&D and outside R&D (obtained either by imitation or licensing).

The three-stage structure in my model is similar to that in Katz and Shapiro (1985). Katz and Shapiro, however, focus on process innovations and do not consider cumulative innovations. They find that firms will license small innovations and effect of licensing on research incentives is ambiguous. Although, I model cumulative innovation and take the licensing environment as given, I obtain the result that the effect of a strong patent regime on R&D in the presence licensing is less pronounced than that in the absence of licensing.

This paper is organized in the following way. In the next section I describe the baseline model. In section 3, I discuss the no-licensing equilibrium. In section 4 the planner's problem is described while in section 5 the licensing equilibrium is

discussed. Section 6 concludes.

## 2 Basic Model

I focus on an industry with two firms in an economy. This is one period model with multiple stages. The consumers care only about the new product and are represented by a demand function,  $P = D(Q)$ , where  $Q$  is the total quantity of the new product demanded at price  $P$ .  $D(\cdot)$  is a downward sloping, well-behaved demand function, with  $D^{-1}(0) < \infty$ .

In this model, therefore, firms make a profit only if they have a new product at the end of the period. *Ideas* (also called innovations in this paper) are developed by firms at an earlier stage and are the building blocks of the new product. The probability of successful product development by each firm depends on the total number of new ideas a firm has access to.

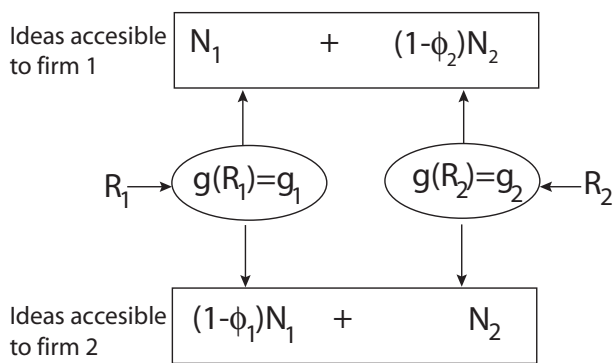
The number of ideas that a firm gets is a random variable. Each firm can acquire ideas in two different ways. It can either invest in R&D and generate in-house ideas or can imitate ideas that have been generated by other firms.

To generate in-house ideas each firm  $i$  chooses R&D expenditure,  $R_i$ , which affects the distribution of the number of new ideas,  $N_i$ , developed by firm  $i$ . I assume that  $N_i \sim \text{Poisson}(g_i)$ , where  $g_i \equiv g(R_i)$  and  $g(\cdot)$  is a monotonically increasing function of  $R_i$ . I also assume that  $N_1$  and  $N_2$  are independent random variables and no two ideas are alike.

Firms can also acquire ideas via imitation. The act of imitation is an attempt to access the other firm's knowledge pool without making a payment. The extent of successful imitation by a firm is determined by the patent strategy adopted by

its rival firm. Higher patenting negatively affects the number of ideas that can be imitated by rival firms. Firms patent to protect their ideas. By protecting ideas, firms make it harder for other firms to imitate. Therefore in this paper patenting is a firm-level decision that reduces imitation. Specifically, patenting is modeled in the following way : firm 1 chooses a patenting strategy,  $\phi_1$ , such that  $0 \leq \phi_1 \leq 1$ .  $\phi_1$  is a measure of the strength of firm 1's IP policy. It can be a function of the total number of patents firm 1 receives and its emphasis on hiring lawyers and IP managers to successfully defend its intellectual property. Similarly firm 2 chooses  $\phi_2$ .

If firm 1 chooses  $\phi_1$  then the firm 2 can only successfully imitate  $(1 - \phi_1)N_1$  of firm 1's ideas. Similarly firm 2's patenting strategy,  $\phi_2$ , determines the number of ideas that can be imitated by firm 1. Therefore, by choosing the patenting strategy, firms decide (deterministically) the fraction of ideas that will be imitated by the other firm.



Since  $\phi$  can take any value between 0 and 1, this model does not have the restriction that the number of ideas imitated by a firm must be an integer. The



assumption of perfect divisibility of ideas simplifies the analysis considerably, although it may not be realistic. One way to interpret this is that ideas are complex entities themselves. An idea might have several different components that work together to generate a particular form of usable knowledge. Usable knowledge of a different form (or quality) may also be obtained by combining some, but not all, of the above mentioned components. Therefore an idea, when transmitted via imitation, may not represent the same unit of knowledge that is available to the original innovator. This model assumes that the IP policy of the firm determines how much of the knowledge content will be transmitted to the imitator.

Firms's R&D and patenting strategies are also affected by the external patent policy environment. The patent regime is parameterized by a non-negative parameter,  $S$ . This parameter is taken as given in this model. The patent regime directly affects the cost of patenting. A higher value of  $S$  suggests a stronger patent regime, in the sense that it lowers the cost of patenting and, thereby, reduces the cost of enforcing IP rights. A lower value of  $S$  will have the opposite effect.

The cost of firm 1's patenting strategy is denoted by a monotonically increasing, convex function,  $C(\cdot)$ , of  $\phi_1$ .  $S > 0$  is a parameter of the cost function, with  $\frac{\partial C}{\partial S} < 0$ . Also  $C(0) = 0$  and  $C'(0) > 0$ . Firm 2 has an identical cost function for patenting.

Firms are ex-ante symmetric. Each firm chooses R&D and patenting strategies, engages in licensing (only when that option is available) and introduces a new finished product with some probability. There are three stages to the process.

**Stage 1 (R&D and IP Protection Stage):** Firms choose R&D strategies,  $R_1$

and  $R_2$ , and patenting strategies,  $\phi_1$  and  $\phi_2$ , by maximizing expected profit.

For firm 1, the choice of  $R_1$  affects the number of in-house new ideas,  $N_1$ , obtained. The choice of  $\phi_1$  determines firm 1's extent of patent protection. The cost of choosing  $\phi_1$  is  $C(\phi_1)$ . Firm 2 faces a similar problem.

At the end of this stage in-house ideas  $N_1$ ,  $N_2$  and imitated ideas  $(1-\phi_2)N_2$ ,  $(1-\phi_1)N_1$  are realized.

**Stage 2 (Licensing Stage):** The firms go into the licensing stage knowing  $N_1$ , and  $N_2$ . To abstract from the discussion of optimal licensing mechanism choice, I assume a licensing structure where each firm is either a licensor (one who gives the license) or licensee (one to whom a license is given) of ideas. A licensor firm permits a licensee firm to use the ideas developed by the former, but licensing does not preclude the licensor firm from using its own ideas. Licensing, therefore, allows all firms to access the same ideas simultaneously. This is different from the generally accepted notion of buying and selling goods, where the buyer of the good can exclude the seller from using the good. In this paper, however, I am going to use the words buyer and seller to indicate the licensee and the licensor respectively. This is done for simplicity.

Two types of environments are considered in this model. In the first type, licensing is not permitted. This extreme case is used to understand the strategic interactions between different firm-level decisions in an environment where there are institutional or technological impediments to licensing.

The other environment is one in which licensing is allowed. The licensing process is given. Each firm is a buyer with probability  $\frac{1}{2}$  and a seller with probability  $\frac{1}{2}$ . The buyer firm offers to buy all the other firm's un-imitated ideas by proposing a payment that makes the seller indifferent between selling and not-

selling.  $P_1^L$  is the payment offered by firm 1 when firm 1 is the buyer and  $P_2^L$  is the payment offered by firm 2 when firm 2 is the buyer.  $N_1^L$  denotes the number of ideas obtained by firm 1 (when firm 1 is the buyer) after successful licensing. The licensing process works in the same way for firm 2.

Firms start developing the new product at the end of the second stage.

**Stage 3 (Product Development Stage):** At the beginning of this stage firms either successfully complete the development of a new product or they fail. The probability of successful product development by firm 1 is given by  $f_1 = f(N_1, (1 - \phi_2)N_2)$ , where  $N_1$  is the number of new ideas invented by firm 1 and  $(1 - \phi_2)N_2$  is the number of firm 2's ideas imitated by firm 1. Firm 2 faces a similar problem. Once the new product is developed, profits are realized. The cost of production is zero. If only one firm is able to develop the product then that firm gets the monopoly revenue of  $\pi_M > 0$ .<sup>4</sup> If both firms develop the product simultaneously, they engage in Bertrand competition, which implies that each of them charges a price of zero for the product and earns zero profit. I assume that when the firms are indifferent between producing and not producing they choose to produce the amount dictated by consumer demand. If none of the firms are successful in developing the product, then the revenue for each of them is zero.

## 2.1 Equilibrium

An equilibrium is defined here.

In stage 1, firms choose their R&D and patenting strategies before innovations are actually realized. Firms at this stage, therefore, maximize expected profit by

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<sup>4</sup>The monopoly profit is obtained in the usual way from the consumer demand function  $P = D(Q)$  and the cost of production of the firm. The cost of production is zero in this case.

taking into account that there will be a licensing stage after the innovations are realized.

If there is no licensing, firm 1's expected revenue conditional on the number of ideas of each firm is given by

$$\pi_{1nl} = \pi_M f(N_1, (1 - \phi_2)N_2)[1 - f(N_2, (1 - \phi_1)N_1)] \quad (1)$$

The expected revenue structure for firm 2 is similar.

If there is licensing and firm 1 is the buyer (which happens with probability  $\frac{1}{2}$ ) then firm 1 offers to buy  $N_1^L$  ideas by making a payment  $P_1^L$ .  $P_1^L$  is a function of  $(N_1, N_2, N_1^L)$ , and is chosen in accordance with the given licensing environment. An offer  $(N_1^L, P_1^L)$  is always accepted by the seller because the structure of the offer. However, the buyer firm will make an offer only if  $\pi_{1B} \geq 0$ . For firm 1 that means the unconditional probability of success without licensing must be less than  $\frac{1}{2}$ . I assume  $\lambda$  to be small enough so that this condition is always satisfied. After licensing, firm 1 now has access to ideas  $(N_1 + (1 - \phi_2)N_2 + N_1^L)$ , while firm 2 has access to the same ideas as before, which is  $(N_2 + (1 - \phi_1)N_1)$ . The probability that firm 1 succeeds in developing the final product and firm 2 fails is  $f(N_1, (1 - \phi_2)N_2, N_1^L)[1 - f(N_2, (1 - \phi_1)N_1)]$ . Firm 1's expected revenue as a buyer, conditional on the number of ideas of each firm, is

$$\pi_{1B} = \pi_M f(N_1, (1 - \phi_2)N_2, N_1^L)[1 - f(N_2, (1 - \phi_1)N_1)] - P_1^L(N_1, N_2, N_1^L) \quad (2)$$

such that

$$N_1^L \geq 0.$$

When firm 1 is the seller (which happens with probability  $\frac{1}{2}$ ), firm 1 is offered  $(N_2^L, P_2^L)$ — a payment of  $P_2^L$  for licensing  $N_2^L$  ideas to firm 2. Here  $N_2^L \geq 0$  and

$P_2^L$  is a function of  $(N_1, N_2, N_2^L)$ . The offer is such that firm 1 always accepts. Therefore after licensing firm 1 has access to the same ideas as before  $(N_1, (1 - \phi_2)N_2)$ , while firm 2 has now has access to ideas  $(N_2, (1 - \phi_1)N_1, N_2^L)$ . Hence, firm 1's expected revenue as a seller conditional on the number of ideas of each firm is

$$\pi_{1S} = \pi_M f(N_1, (1 - \phi_2)N_2)[1 - f(N_2, (1 - \phi_1)N_1, N_2^L)] + P_2^L(N_1, N_2, N_2^L) \quad (3)$$

such that

$$N_2^L \geq 0.$$

The equilibrium for this model is defined as follows :

**DEFINITION :** An *industry equilibrium* is a collection of R&D strategies  $\{R_1^*, R_2^*\}$  and IP strategies  $\{\phi_1^*, \phi_2^*\}$  which are obtained as follows:

Firm 1 chooses  $R_1(R_2, \phi_2)$  and  $\phi_1(R_2, \phi_2)$  by maximizing expected profit. The expectation is over the number of ideas for each firm conditional on R&D and patent protection. Firm 2 solves an identical problem and obtains  $R_2(R_1, \phi_1)$  and  $\phi_2(R_1, \phi_1)$ .  $R_1^*, R_2^*, \phi_1^*$  and  $\phi_2^*$  are the Nash equilibrium values of R&D and patent protection.

I only look at the symmetric Nash equilibrium of the model to keep the analysis simple, i.e.,  $R_1^* = R_2^* = R^*$  and  $\phi_1^* = \phi_2^* = \phi^*$ . In a symmetric Nash equilibrium, the equilibrium solution can be expressed in terms of  $(R^*, \phi^*)$ , which makes the analysis a lot easier.

## 2.2 The Optimal Solution

The optimal choice problem is formulated in the following way : the planner chooses  $R_1, R_2$  and  $\phi_1, \phi_2$  in stage 2 to maximize the expected sum of consumer

surplus (CS) and producer surplus (PS). The planner intervenes at the R&D and patenting stages, but does not intervene in the product market. This assumption is maintained because the focus of this model is to understand the effects of the patent system on innovation and patenting only, and not on the market structure.

In the production stage there is no intervention. If only one firm innovates then the innovating firm gets the monopoly profit,  $\pi_M$ , the rival firm gets 0 and the consumer surplus is small. If both firms innovate, then both firms engage in Bertrand competition, each get a revenue of 0 and the consumer surplus is the maximum. If none of the firms innovate then both producer and consumer surplus are zero.

Therefore, the **optimal solution** is given by the set  $\{R_1^{SP}, R_2^{SP}, \phi_1^{SP}, \phi_2^{SP}\}$  such that in stage 2, the planner chooses  $R_1, R_2$  and  $\phi_1, \phi_2$  that maximize the expected sum of the producers' and the consumers' surplus.

To make both the industry equilibrium and the planner's problem more tractable some specific functional forms are introduced.

### 2.3 Functional Assumptions

The following functional assumptions are made :

$$f_i = f(N_i, N'_i) = f(N_i + N'_i) = 1 - (1 - \lambda)^{(N_i + N'_i)} \quad \forall i, N_i, N'_i, \quad 0 < \lambda < 1.$$

$N'_i$  takes the value of  $(1 - \phi_j)N_j$ ,  $\forall i \neq j$  in the absence of licensing and takes the value  $N_i^L$  in presence of licensing.

This particular functional assumption for  $f(\cdot)$  is useful because it allows the probability of success for a firm to be a function of the sum of ideas that are accessible to the firm. This implies that ideas are non-rival in terms of the role they play in the product development process. In the presence of this kind of non-

rivalry, it is expected that a stronger patent system will be useful as an instrument for exclusion and, hence, will raise the individual firm's incentive to invest in R&D. What this model demonstrates is that even when ideas are non-rival at the product development stage, under certain circumstances, the strengthening of the patent system may not always positively influence an individual firm's incentive to do R&D.

The parameter  $\lambda$  can be interpreted as follows :  $\lambda$  is the probability of success that a firm with access to only one new idea faces in this industry.  $\lambda$  is a constant for each industry. I call  $(1 - \lambda)$  the *complexity parameter* of the industry — for a firm that has access to only one idea in an industry parameterized by  $\lambda$  the chance of success is given by  $\lambda$ . In other words, firms in a *complex* product industry have a smaller value of  $\lambda$  than those in a *simple* product industry.<sup>5</sup> For firms to succeed in complex product industries it is, therefore, more important for them to have access to as many ideas as possible (compared to firms in industries with smaller values of  $(1 - \lambda)$ ).

The following functional form for  $g(\cdot)$  is chosen for this analysis:

$$g_i = g(R_i) = \begin{cases} 0 & \text{for } 0 \leq R_i < \frac{1}{\beta} \\ \ln(\beta R_i) & \text{for } R_i \geq \frac{1}{\beta} \end{cases}$$

where  $\beta > 0$  is a constant and  $i = 1, 2$ .

The following functional form for  $C(\cdot)$  is chosen:  $C(\phi) = \frac{1}{S}(\phi^2 + \phi)$  where  $S$  is a strictly positive parameter that represents the strength of the patent regime. A larger value of  $S$  implies a stronger patent regime in the sense that firms can obtain the same IP protection ( $\phi$ ) at a lower cost. The value of  $S$  is determined by the existing patent policy environment and is given in this model.

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<sup>5</sup>The semiconductor industry is an example of a *complex* product industry, where new products typically embody many new ideas.

The model is solved for three different cases : 1) the 3-stage problem without licensing, 2) the planner's problem, where the planner makes R&D and patenting decisions, and 3) the 3-stage problem with licensing.

The no-licensing case is used as a benchmark. This case approximates an industry environment where licensing is not common due to either the history of development of the industry, or technological factors. The planner's problem gives the optimal level of R&D and patenting. The case with licensing is introduced to study complex product industries, like semiconductors, where there are instances of cross-licensing and other multilateral arrangements to share technical knowledge. Cross-licensing, in particular, is very common among semiconductor firms and has become more important in the recent years. Rival firms competing for the same market often share their technical know-how or simply give broad right-of-use over a bunch of patented and non-patented ideas.<sup>6</sup> After talking to the IP managers of some of the firms in complex industries, Cohen, Nelson and Walsh (2000) suggest that the recent surge in patenting (after 1982) may be the result of strategic consideration by firms who want to have a better bargaining position in their cross-licensing arrangements. The reason for studying the licensing case is to understand how, in the presence of ex-post licensing, firms in complex product industries choose their R&D and patenting strategies and how the strategies change when the patent regime becomes more strong.

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<sup>6</sup>For example, competing firms like Intel and AMD engage in cross-licensing arrangements regularly.



### 3 Equilibrium With No Licensing

In this case there are only two stages — the stage where the firms choose R&D expenditures and patenting strategies, and the product development stage. There is no licensing stage. To solve this model I start from the last stage with firm 1's problem. Firm 2 has identical problems at each stage.

In the last stage the profits are realized. Firms do not take any decisions at this stage.

In the first stage firm 1 chooses its R&D expenditure  $R_1$  and patenting strategy  $\phi_1$ . Since there is no licensing in this case firm 1 has access to only those ideas which it can develop,  $N_1$ , and those which it can imitate  $(1 - \phi_2)N_2$ . At this stage, however, both  $N_1$  and  $N_2$  are random variables. The distribution of  $N_1$  depends on the R&D expenditure  $R_1$ , while the distribution of  $N_2$  depends on firm 2's R&D expenditure,  $R_2$ . The number of imitated ideas firm 1 can get depends on firm 2's patenting strategy,  $\phi_2$ . Firm 1 similarly affects the number of imitated ideas firm 2 can have by choosing patenting strategy,  $\phi_1$ .

Firm 1 chooses  $R_1$  and  $\phi_1$  by maximizing expected profit.

$$\max_{R_1, \phi_1} E\{R\} - C(\phi_1) - R_1, \quad (4)$$

where  $R$  stands for revenue.

Since the revenue is a function of  $N_1$  and  $N_2$ , which are random variables at this stage, firm 1 calculates expected revenue. A firm gets positive revenue  $\pi_M > 0$  only when it successfully develops the new product and the other firm fails to develop the product. The probability that firm 1 succeeds with  $(N_1 + (1 - \phi_2)N_2)$  ideas and firm 2 fails with  $(N_2 + (1 - \phi_1)N_1)$  ideas is  $f(N_1 + (1 - \phi_2)N_2)[1 -$

$f(N_2 + (1 - \phi_1)N_1)$ . Therefore, expected revenue of firm 1 is

$$\pi_M E\{\pi_{1nl}|R_1, R_2, \phi_1, \phi_2\}, \quad (5)$$

where  $\pi_{1nl}$  is given by (1). The above expectation is over the number of ideas for each firm conditional on R&D and patent protection.

Given the assumptions about the functional form of  $f(\cdot)$  and distribution of  $N_1, N_2$ , the expected revenue becomes:

$$\begin{aligned} & \pi_M E\{[1 - (1 - \lambda)^{N_1 + (1 - \phi_2)N_2}](1 - \lambda)^{N_2 + (1 - \phi_1)N_1}|R_1, R_2, \phi_1, \phi_2\} \quad (6) \\ &= \pi_M E\{(1 - \lambda)^{N_2 + (1 - \phi_1)N_1} - (1 - \lambda)^{(2 - \phi_1)N_1 + (2 - \phi_2)N_2}|R_1, R_2, \phi_1, \phi_2\}. \end{aligned}$$

Now given the distributional assumptions

$$\begin{aligned} E\{(1 - \lambda)^{N_2 + (1 - \phi_1)N_1}\} &= E\{(1 - \lambda)^{N_2}\}E\{(1 - \lambda)^{(1 - \phi_1)N_1}\} \quad (7) \\ &= \left[ \sum_{N_2=0}^{\infty} \frac{(1 - \lambda)^{N_2} e^{-g_2} g_2^{N_2}}{N_2!} \right] \left[ \sum_{N_1=0}^{\infty} \frac{(1 - \lambda)^{(1 - \phi_1)N_1} e^{-g_1} g_1^{N_1}}{N_1!} \right] \\ &= e^{-g_2 \lambda} e^{-g_1 a(\phi_1)}, \end{aligned}$$

where  $a(\phi_1) = 1 - (1 - \lambda)^{(1 - \phi_1)}$ .

Similarly it can be shown that

$$E\{(1 - \lambda)^{(2 - \phi_1)N_1 + (2 - \phi_2)N_2}\} = e^{-g_1 b(\phi_1)} e^{-g_2 b(\phi_2)}, \quad (8)$$

where  $b(\phi) = 1 - (1 - \lambda)^{(2 - \phi)}$ .

Therefore, firm 1's maximization problem is

$$\begin{aligned} & \max_{R_1, \phi_1} \pi_M \{e^{-g_2 \lambda} e^{-g_1 a(\phi_1)} - e^{-g_1 b(\phi_1)} e^{-g_2 b(\phi_2)}\} - C(\phi_1) - R_1 \quad (9) \\ & \text{s.t. } 0 \leq \phi_1 \leq 1, \\ & \quad \& R_1 \geq 1/\beta. \end{aligned}$$

The Lagrangian for the above problem is:

$$L = \pi_M \{ e^{-g_2 \lambda} e^{-g_1 a(\phi_1)} - e^{-g_1 b(\phi_1)} e^{-g_2 b(\phi_2)} \} - C(\phi_1) - R_1 \quad (10)$$

$$+ \mu_1 \phi_1 + \mu_2 (1 - \phi_1) + \theta_1 (R_1 - \frac{1}{\beta}),$$

where  $\mu_1 \geq 0$ ,  $\mu_2 \geq 0$ ,  $\theta_1 \geq 0$  are Lagrange multipliers.

It can be shown that a unique global maxima exists only for small values of  $\lambda$ . Therefore for this analysis I choose a relatively more complex industry such that the global maxima exists for the above problem.

The first order conditions of the above problem are:

$$\frac{\partial L}{\partial \phi_1} : \quad \pi_M g_1 \left[ -e^{-(g_2 \lambda + g_1 a(\phi_1))} \frac{\partial a}{\partial \phi_1} + e^{-(g_1 b(\phi_1) + g_2 b(\phi_2))} \frac{\partial b}{\partial \phi_1} \right] \quad (11)$$

$$- \frac{\partial C}{\partial \phi_1} + \mu_1 - \mu_2 = 0$$

$$\frac{\partial L}{\partial R_1} : \quad \pi_M \frac{\partial g_1}{\partial R_1} \{ -e^{-(g_2 \lambda + g_1 a(\phi_1))} a(\phi_1) + e^{-(g_1 b(\phi_1) + g_2 b(\phi_2))} b(\phi_1) \} \quad (12)$$

$$- 1 + \theta_1 = 0,$$

where  $\mu_1 \geq 0$ ,  $\mu_2 \geq 0$ ,  $\theta_1 \geq 0$ .

Firm 1 solves these two equations to obtain  $\phi_1(\phi_2, R_2)$  and  $R_1(\phi_2, R_2)$ . Firm 2 solves a similar problem to obtain  $\phi_2(\phi_1, R_1)$  and  $R_2(\phi_1, R_1)$ . In Nash equilibrium

$$\phi_1^* = \phi_1(\phi_2^*, R_2^*), \quad \phi_2^* = \phi_2(\phi_1^*, R_1^*)$$

$$R_1^* = R_1(\phi_2^*, R_2^*), \quad R_2^* = R_2(\phi_1^*, R_1^*).$$

The functional forms of  $a(\cdot)$  and  $b(\cdot)$  are

$$a(\phi) = 1 - (1 - \lambda)^{(1-\phi)}, \quad (13)$$

$$b(\phi) = 1 - (1 - \lambda)^{(2-\phi)}. \quad (14)$$

For a symmetric Nash equilibrium the first order conditions, (11) and (12), give

$$\begin{aligned} & \pi_M g(1 - \lambda)^{(1-\phi)} \ln(1 - \lambda) [-e^{-g(\lambda+1-(1-\lambda)^{(1-\phi)})} \\ & + (1 - \lambda)e^{-2g(1-(1-\lambda)^{(2-\phi)})}] - \frac{\partial C}{\partial \phi} + \mu_1 - \mu_2 = 0, \end{aligned} \quad (15)$$

$$\begin{aligned} & \pi_M \frac{1}{R} [-e^{-g(\lambda+1-(1-\lambda)^{(1-\phi)})} (1 - (1 - \lambda)^{(1-\phi)}) \\ & + e^{-2g(1-(1-\lambda)^{(2-\phi)})} (1 - (1 - \lambda)^{(2-\phi)})] - 1 = 0, \end{aligned} \quad (16)$$

where  $\mu_1 \geq 0, \mu_2 \geq 0$ .

The first order condition identifies three distinct regions of  $S$ , separated by two cut-off points  $\underline{S}^{nl}$  and  $\bar{S}^{nl}$ , with  $\bar{S} > \underline{S}$  and the superscript  $nl$  denoting the no-license case. If the regime parameter  $S < \underline{S}^{nl}$ , then firm 1 will choose  $\phi_1 = 0$ . For  $\underline{S}^{nl} < S < \bar{S}^{nl}$ , firm 1 will choose  $0 < \phi_1 < 1$ . For  $S > \bar{S}^{nl}$ , firm 1 will choose  $\phi_1 = 1$ . Similar results are obtained for firm 2. These results are summarized in the following proposition:

**Proposition 1.** For each industry  $\lambda$  and for each firm  $i$ , there are two cut-off points  $\underline{S}^{nl}$  and  $\bar{S}^{nl}$ , with  $\bar{S} > \underline{S}$  such that

- i) for all  $S < \underline{S}^{nl}$ , firm  $i$  will choose  $\phi_i = 0$ ,
- ii) for all  $\underline{S}^{nl} < S < \bar{S}^{nl}$ , firm  $i$  will choose  $0 < \phi_i < 1$ , and
- iii) for all  $S > \bar{S}^{nl}$ , firm  $i$  will choose  $\phi_i = 1$ .

**Proof:** From equation (15) for  $\phi_1 = \phi_2 = 0$  (i.e.,  $\mu_2 = 0$ ) I get

$$\pi_M g(1 - \lambda) \ln(1 - \lambda) [-e^{-2g\lambda} + (1 - \lambda)e^{-2g\lambda(2-\lambda)}] - \frac{\partial C}{\partial \phi_1} + \mu_1 = 0, \quad (17)$$

where  $\mu_1 > 0$ .

Therefore,

$$\frac{\partial C}{\partial \phi_1} \Big|_{\phi_1=0} = \frac{1}{S} > \pi_M g^0 (1 - \lambda) \ln(1 - \lambda) [-e^{-2g^0\lambda} + (1 - \lambda)e^{-2g^0\lambda(2-\lambda)}], \quad (18)$$

or,

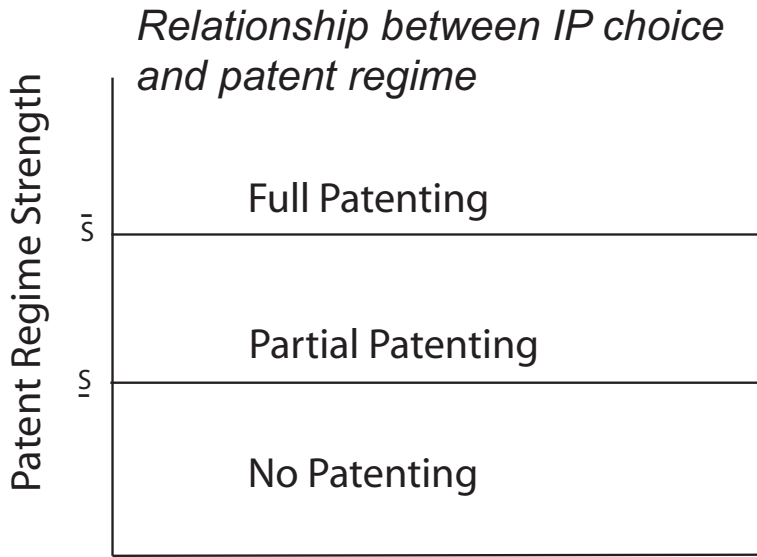
$$S < [\pi_M g^0 (1 - \lambda) \ln(1 - \lambda) (-e^{-2g^0\lambda} + (1 - \lambda)e^{-2g^0\lambda(2-\lambda)})]^{-1}, \quad (19)$$

where  $g^0 = g|_{\phi=0}$ . The right hand side of the above expression gives  $\underline{S}^{nl}$ .

Similarly, for  $S > \bar{S}^{nl}$  given by  $\bar{S}^{nl} = 3[\pi_M g^1 \ln(1 - \lambda) (-e^{-g^1\lambda} + (1 - \lambda)e^{-2g^1\lambda})]^{-1}$ , where  $g^1 = g|_{\phi=1}$ , firms will choose  $\phi_1 = \phi_2 = 1$ . ■

Expressions for  $g^1$  and  $g^0$  are solved in the next two subsections in cases 1 and 2 respectively.

The above results are intuitive. If the IP regime is very weak ( $S$  below a certain cut-off value) then for each firm in a particular industry the marginal cost of enforcing IP will exceed the marginal gain from enforcing IP rights. For those low values of  $S$ , firms will choose not to enforce patents at all. Similarly, in a very strong IP regime the marginal gains exceed marginal cost and hence firms choose the highest IP protection possible.



The above problem is now solved for three different patent regimes — strong patent regime ( $S > \bar{S}^{nl}$ ), weak patent regime ( $S < \underline{S}^{nl}$ ) and moderate patent regime ( $\underline{S}^{nl} < S < \bar{S}^{nl}$ ).

### 3.1 Case 1 : Strong patent regime

When firms choose  $\phi_1 = \phi_2 = 1$  from (16) it follows that

$$\pi_M \lambda \frac{1}{R} e^{-2g\lambda} = 1, \quad (20)$$

or, (given the functional assumption on  $g(\cdot)$ )

$$\pi_M \lambda \frac{1}{R} (\beta R)^{-\lambda} (\beta R)^{-\lambda} = 1. \quad (21)$$

Therefore, the R&D expenditure of each firm under a very strong patent regime in the no-licensing case is given by:

$$R_{\{\phi=1\}}^{nl} = [\lambda\pi_M\beta^{-2\lambda}]^{\frac{1}{1+2\lambda}} \quad (22)$$

### 3.2 Case 2 : Weak patent protection

When firms choose  $\phi_1 = \phi_2 = 0$  equation (16) gives:

$$\pi_M\lambda\frac{1}{R}e^{-2g\lambda}[-1 + (2 - \lambda)e^{-2g\lambda(1-\lambda)}] = 1. \quad (23)$$

The expression  $[-1 + (2 - \lambda)e^{-2g\lambda(1-\lambda)}]$  is denoted by  $\delta$ . Since  $(2 - \lambda)e^{-2g\lambda(1-\lambda)} < 2$ , it must be that  $\delta < 1$ .

Therefore, using the functional assumption on  $g(\cdot)$ ,

$$\pi_M\lambda(\beta R)^{-2\lambda}\delta = 1, \quad (24)$$

or,

$$R_{\{\phi=0\}}^{nl} = [\lambda\pi_M\beta^{-2\lambda}\delta]^{\frac{1}{1+2\lambda}} \text{ where } \delta < 1. \quad (25)$$

Therefore  $R_{\{\phi=0\}}^{nl} < R_{\{\phi=1\}}^{nl}$ .

### 3.3 Case 3 : Moderate patent regime

I discuss the most general case here. For this part the following approximations for  $a$  and  $b$  are made: for small  $\lambda$

$$a = 1 - (1 - \lambda)^{(1-\phi)} \approx 1 - (1 - \lambda(1 - \phi)) = \lambda(1 - \phi), \quad (26)$$

$$b = 1 - (1 - \lambda)^{(2-\phi)} \approx 1 - (1 - \lambda(2 - \phi)) = \lambda(2 - \phi). \quad (27)$$

Using the above approximations the first order conditions, (15) and (16), give:

$$\pi_M g \lambda e^{-(2-\phi)g\lambda} [1 - e^{-(2-\phi)g\lambda}] = \frac{\partial C}{\partial \phi} = \frac{2\phi + 1}{S}, \quad (28)$$

$$\pi_M \lambda \frac{1}{R} e^{-(2-\phi)g\lambda} [-(1 - \phi) + (2 - \phi)e^{-(2-\phi)g\lambda}] = 1. \quad (29)$$

From these first order conditions the following results are obtained:

**Proposition 2.** For small values of  $\lambda$ ,

- i)  $\frac{\partial R_1^*}{\partial \phi_1^*} > 0$ ,
- ii)  $\frac{\partial \phi_1^*}{\partial S} > 0$ .

**Proof:** i) Equation (29) gives

$$\pi_M \lambda e^{-(2-\phi)g\lambda} \{(2 - \phi)e^{-(2-\phi)g\lambda} - 1 + \phi\} \frac{\partial g}{\partial R} = 1. \quad (30)$$

Differentiating w.r.t.  $\phi$ , the following result is obtained for small values of  $\lambda$ :

$$-\frac{\partial R}{\partial \phi} \frac{1}{R} [1 + (2 - \phi)\lambda + (2 - \phi)^2\lambda] + g\lambda [-(1 - \phi) + 2(2 - \phi)] = 0 \quad (31)$$

or,

$$\frac{\partial R}{\partial \phi} \Big|_{nl} = \frac{(3 - \phi)g\lambda R}{1 + \lambda(2 - \phi)(3 - \phi)}. \quad (32)$$

Since  $\phi \leq 1$  it must be that  $\frac{\partial R}{\partial \phi} > 0$ .

ii) Differentiating equation (28) w.r.t.  $S$ :

$$\begin{aligned} & \pi_M \lambda e^{-(2-\phi)g\lambda} \frac{\partial g}{\partial \phi} \frac{\partial \phi}{\partial S} [(1 - e^{-(2-\phi)g\lambda})(1 - g\lambda(2 - \phi)) + g\lambda e^{-(2-\phi)g\lambda}(2 - \phi)] \\ & + \frac{\partial \phi}{\partial S} [\pi_M (g\lambda)^2 e^{-(2-\phi)g\lambda} (1 - 2e^{-(2-\phi)g\lambda}) - \frac{2}{S}] = -\frac{(2\phi + 1)}{S^2}. \end{aligned}$$



For small values of  $\lambda$ , the following result is obtained from the above equation:

$$\frac{\partial \phi}{\partial S} [\pi_M g \lambda ((2 - \phi) \lambda \frac{\partial g}{\partial \phi} - g \lambda) - \frac{2}{S}] = -\frac{2\phi + 1}{S^2} \quad (33)$$

As  $\lambda \rightarrow 0$ , the negative term on the l.h.s. dominates the positive term. Since the function is continuous, for small values of  $\lambda$  the bracketed term on the l.h.s. is negative. Since the r.h.s. is also negative, it must be that  $\frac{\partial \phi}{\partial S} > 0$ . ■

Proposition 2 implies that patenting strategies and R&D strategies are strategic complements. When firms choose strong IP protection they also have less access to imitated ideas and, hence, must also choose higher R&D to obtain a greater number of in-house ideas. Also firms choose higher levels of patent protection when the external patent policy regime changes in favor of stronger IP rights. These results can be combined to get the following proposition:

**Corollary 1:** A stronger patent regime in an industry with no licensing will lead to more patent protection and more R&D.

The above corollary directly from Proposition 2. This result is in tune with what economists have presented as the main reason for having the institution of patent. When ideas are non-rival and can be imitated at a small cost, the resulting positive externality provides incentives for individual firms to invest less in R&D to produce new ideas. The institution of patents takes care of this by awarding ownership rights. Patents, therefore, take care of the free rider problem and boost investment in R&D.

In addition the following results are also obtained:

**Proposition 3.**

i)  $\frac{\partial S}{\partial \lambda} < 0$  for all  $\lambda$ , i.e., firms in more complex industries will choose  $\phi = 0$  for a

larger range of S-values, and

ii)  $\frac{\partial \bar{S}}{\partial \lambda} < 0$  for all  $\lambda$ , i.e., firms in more complex industries will choose  $\phi = 1$  for a smaller range of S-values.

**Proof:** See Appendix A.

The intuition is that firms in smaller  $\lambda$  industries produce complex products that require using a large number of ideas. These firms would, therefore, choose the weakest IP protection ( $\phi = 0$ ) for a wider range of IP regimes as compared to firms in higher  $\lambda$  industries.

In the case with no licensing it is observed that firms in complex industries are unable to capture all of the positive externality of the knowledge pool created at the industry level. Next I consider the social planner's problem.

## 4 The Planner's Problem

The consumers are represented by a linear demand :  $P(Q) = a - bQ$ , with  $a, b > 0$ . The planner maximizes the sum of the producer surplus and the consumer surplus.

Three distinct cases can arise : i) only one firm successfully develops the product, ii) both firms successfully develop the product, and iii) none of the firms are successful. The total surplus will be different in each of these three cases.

Case i) With only one firm succeeding in developing the final product, there will be a monopoly. The probability that only firm will successfully develop the final product is  $\{f(N_1, (1 - \phi_2)N_2)[1 - f(N_2, (1 - \phi_1)N_1)] + f(N_2, (1 - \phi_1)N_1)[1 - f(N_1, (1 - \phi_2)N_2)]\}$ .

The monopoly profit is obtained by

$$\max_Q (a - bQ)Q. \quad (34)$$

Using the functional assumptions, the above maximization gives a producer surplus of  $\frac{a^2}{4b}$  and a consumer surplus of  $\frac{a^2}{8b}$ . The producer surplus is denoted by  $\pi_M$ , which also denotes the monopoly revenue following the notation introduced in the previous section. Then the producer surplus is  $\pi_M$ , consumer surplus is  $\frac{1}{2}\pi_M$  and total surplus is  $\frac{3}{2}\pi_M$ .

Therefore the total expected revenue when only one firm has successfully developed a product is

$$\begin{aligned} \pi_i^{SP} &= \frac{3}{2}\pi_M \{f(N_1, (1 - \phi_2)N_2)[1 - f(N_2, (1 - \phi_1)N_1)] \\ &+ f(N_2, (1 - \phi_1)N_1)[1 - f(N_1, (1 - \phi_2)N_2)]\}. \end{aligned} \quad (35)$$

Case ii) When both firms successfully develop the final product, the firms will engage in Bertrand type competition, producing the minimum producer surplus and maximum consumer surplus. The probability that both firms will successfully develop the final product is given by  $f(N_1, (1 - \phi_2)N_2)f(N_2, (1 - \phi_1)N_1)$ .

In this case producer surplus is 0, consumer surplus is  $2\pi_M$ , and the total surplus is  $2\pi_M$ .

Therefore the total expected revenue when both firms have successfully developed a product is

$$\pi_{ii}^{SP} = 2\pi_M f(N_1, (1 - \phi_2)N_2)f(N_2, (1 - \phi_1)N_1). \quad (36)$$

Case iii) If no firm is successful then both producer surplus and consumer surplus, and hence total surplus equal 0. This will happen with probability  $[1 - f(N_1, (1 - \phi_2)N_2)][1 - f(N_2, (1 - \phi_1)N_1)]$ .

Therefore the planner chooses :

$$\begin{aligned} \{R_1^{SP}, R_2^{SP}, \phi_1^{SP}, \phi_2^{SP}\} &= \operatorname{argmax} E[\pi_i^{SP} + \pi_{ii}^{SP}] \\ &- C(\phi_1) - C(\phi_2) - R_1 - R_2 \end{aligned} \quad (37)$$

s.t.  $0 \leq \phi_1 \leq 1$ ,  $0 \leq \phi_2 \leq 1$ ,  $R_1 \geq 1/\beta$ ,  $R_2 \geq 1/\beta$ ,

where  $\pi_i^{SP}$  and  $\pi_{ii}^{SP}$  are given by (35) and (36).

The maximization problem can be rewritten as

$$\begin{aligned} \max_{R_1, R_2, \phi_1, \phi_2} \pi_M E \{ & \frac{3}{2} [1 - (1 - \lambda)^{N_1 + (1 - \phi_2)N_2}] (1 - \lambda)^{N_2 + (1 - \phi_1)N_1} \\ & + \frac{3}{2} [1 - (1 - \lambda)^{N_2 + (1 - \phi_1)N_1}] (1 - \lambda)^{N_1 + (1 - \phi_2)N_2} \\ & + 2 [1 - (1 - \lambda)^{N_1 + (1 - \phi_2)N_2}] [1 - (1 - \lambda)^{N_2 + (1 - \phi_1)N_1}] \} \\ & - C(\phi_1) - C(\phi_2) - R_1 - R_2. \end{aligned} \quad (38)$$

Now the expected revenue part can be simplified as follows:

$$\begin{aligned} & \pi_M \left[ -\frac{1}{2} (1 - \lambda)^{N_2 + (1 - \phi_1)N_1} - \frac{1}{2} (1 - \lambda)^{N_1 + (1 - \phi_2)N_2} - (1 - \lambda)^{(2 - \phi_1)N_1 + (2 - \phi_2)N_2} \right] \\ = & \pi_M \left[ -\frac{1}{2} e^{-(g_2\lambda + g_1a(\phi_1))} - \frac{1}{2} e^{-(g_1\lambda + g_2a(\phi_2))} - e^{-(g_1b(\phi_1) + g_2b(\phi_2))} \right]. \end{aligned}$$

The Lagrangian for this problem is

$$\begin{aligned} L &= \pi_M \left[ -\frac{1}{2} e^{-(\lambda g_2 + g_1a(\phi_1))} - \frac{1}{2} e^{-(\lambda g_1 + g_2a(\phi_2))} - e^{-(g_1b(\phi_1) + g_2b(\phi_2))} \right] \\ &- C(\phi_1) - C(\phi_1) - R_1 - R_2 \\ &+ \eta_1 \phi_1 + \eta_2 (1 - \phi_1) + \delta_1 \phi_2 + \delta_2 (1 - \phi_2) + \gamma_1 \left( R_1 - \frac{1}{\beta} \right) + \gamma_2 \left( R_2 - \frac{1}{\beta} \right), \end{aligned}$$

where  $\eta_1 \geq 0$ ,  $\eta_2 \geq 0$ ,  $\delta_1 \geq 0$ ,  $\delta_2 \geq 0$ ,  $\gamma_1 \geq 0$ ,  $\gamma_2 \geq 0$  are Lagrange multipliers.

The results of the planner's problem is summarized in the following proposition.

**Proposition 4.** The planner will choose

i)  $\phi_1^{SP} = \phi_2^{SP} = 0$ , for all values of  $S$  and

ii)  $R^{SP} > R_{\phi=1}^{nl}$ .

**Proof:** i) Note that the above objective function is a monotonically decreasing function of  $\phi_1, \phi_2$ . Hence the planner will choose  $\phi_1^{SP} = \phi_2^{SP} = 0$ .

ii) The first order conditions for  $R_1$  and  $R_2$  are given by

$$R_1 : \quad \pi_M \frac{\partial g_1}{\partial R_1} \left[ \frac{1}{2} e^{-(\lambda g_2 + g_1 a(\phi_1))} a(\phi_1) + \frac{1}{2} e^{-(\lambda g_1 + g_2 a(\phi_2))} \lambda \right. \\ \left. + e^{-(g_1 b(\phi_1) + g_2 b(\phi_2))} b(\phi_1) \right] - 1 = 0, \quad (39)$$

$$R_2 : \quad \pi_M \frac{\partial g_2}{\partial R_2} \left[ \frac{1}{2} e^{-(\lambda g_2 + g_1 a(\phi_1))} \lambda + \frac{1}{2} e^{-(\lambda g_1 + g_2 a(\phi_2))} a(\phi_2) \right] \\ + e^{-(g_1 b(\phi_1) + g_2 b(\phi_2))} b(\phi_2) - 1 = 0. \quad (40)$$

Therefore, in the symmetric equilibrium

$$\pi_M \lambda \frac{1}{R} e^{-2g\lambda} [1 + (2 - \lambda)e^{-2g\lambda(1-\lambda)}] = 1. \quad (41)$$

I define  $\psi = [1 + (2 - \lambda)e^{-2g\lambda(1-\lambda)}]$ . Now  $\psi > 1$  for all values of  $g$ .

Therefore, using the functional assumptions, the R&D expenditure chosen by the planner for each firm is given by:

$$R^{SP} = [\lambda \pi_M \beta^{-2\lambda} \psi]^{\frac{1}{1+2\lambda}}, \text{ where } \psi > 1. \quad (42)$$

Therefore,  $R^{SP} > R_{\phi=1}^{nl}$ . ■

The above results are intuitive. The planner gets the maximum social surplus when there is competition, i.e., when both firms successfully develop the product. Therefore, the planner would favor the minimum IP protection that would allow firms to tap into each others' knowledge pool and increase the probability of successful product development. R&D costs money, but in deciding the level of optimal R&D the planner takes into account both the producer surplus and the consumer surplus. The consumer surplus is an increasing function of R&D expenditure. The consumers derive surplus from R&D, but they do not incur the R&D expenditure. Therefore, the planner would want more R&D than what each firm would choose on their own because the planner problem has this additional consumer-surplus component.<sup>7</sup>

Next I discuss the case where ex-post licensing of ideas is allowed.

## **5 Case 2 : Equilibrium with Licensing**

In this case there are three non-trivial stages. In stage one firms decide IP and R&D strategies. Ideas are realized and imitated at the end of the first stage. In stage 2 one firm offers to be the licensee of the un-imitated ideas owned by other firms, provided that licensing is profitable for the buyer. Licensing gives a firm the opportunity to access the others firm's patented ideas that improve the chances of successful product development. In stage three, firms develop products and the profits are realized. The firms do not make any decision at this stage.

A very specific form of licensing is considered here. After the invention and

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<sup>7</sup>Note that the planner's decision is independent of the patent regime. Hence a change in the patent regime would not change anything in the planner's problem.

imitation stage the firms go to the licensing stage. Each firm is a buyer of ideas with probability  $\frac{1}{2}$  and a seller of ideas with probability  $\frac{1}{2}$ . The buyer firm makes a take-it-or-leave-it offer to the other firm to acquire all of the other firm's un-imitated ideas. The offer includes a payment that makes the seller firm indifferent between selling and not selling. This licensing structure is considered for its simplicity. I do not address the issue of firms' choice of licensing mechanism since that is not the focus of this paper.

It is important to note that the specific structure of licensing arrangement is such that the seller always accept an offer and the buyer is better off licensing as long as the payoff to the buyer under the no-licensing regime is positive. It has been already discussed that in the no-licensing case the expected payoff must be positive for firms in industries with small values of  $\lambda$ . Hence the buyers and sellers in sufficiently complex industries will choose to engage in licensing when they enter the licensing stage, regardless of how many total ideas they have acquired (even for  $N_1 = 0$  or  $N_2 = 0$ ).

The model is solved backwards, starting from stage 3. In stage 3, firms undertake production and profits are realized. At the beginning of stage 2, either firm 1 or 2 makes a "take-it-or-leave-it" offer to the other firm. The licensing mechanism is such that an offer, once made, is always accepted. A new product is developed (or not) at the end of this period. In stage 1 firms choose their patenting and licensing strategies.

I start with firm 1's problem. In stage 3, profits are realized and the firms do not make any decisions. In stage 2, given  $N_1, N_2$ , firm 1 is the buyer of ideas with probability  $\frac{1}{2}$  and seller with probability  $\frac{1}{2}$ . As a buyer, firm 1 offers  $P_1^L$  to acquire all of firm 2's ideas. Similarly as a seller firm 1 receives an offer of  $P_2^L$  as

a payment for giving all its ideas to firm 2.

When firm 1 is the buyer (which happens with probability  $\frac{1}{2}$ ) the expected revenue conditional on the number of ideas of each firm is

$$\begin{aligned}\pi_{1B} &= \pi_M f(N_1 + N_2)[1 - f(N_2 + (1 - \phi_1)N_1)] - P_1^L & (43) \\ \text{s.t. } P_1^L &\leq \pi_M f(N_2 + (1 - \phi_1)N_1)[f(N_1 + N_2) - f(N_1 + (1 - \phi_2)N_2)].\end{aligned}$$

When firm 1 is the seller (which happens with probability  $\frac{1}{2}$ ) the expected revenue conditional on the number of ideas of each firm is

$$\begin{aligned}\pi_{1S} &\geq \pi_M f(N_1 + (1 - \phi_2)N_2)[1 - f(N_2 + N_1)] + P_2^L & (44) \\ \text{s.t. } P_2^L &= \pi_M f(N_1 + (1 - \phi_2)N_2)(f(N_2 + N_1) - f(N_2 + (1 - \phi_1)N_1)).\end{aligned}$$

Hence in stage 1, given that firms decide to engage in licensing in stage two, firm 1 solves for :

$$\begin{aligned}\{R_1^*, \phi_1^*\} &= \operatorname{argmax} \frac{1}{2} E[\pi_{1B} + \pi_{1S} | R_1, \phi_1, R_2^*, \phi_2^*] - C(\phi_1) - R_1 & (45) \\ \text{s.t. } P_1^L &= f(N_2 + (1 - \phi_1)N_1)[f(N_1 + N_2) - f(N_1 + (1 - \phi_2)N_2)] \\ &\& P_2^L = f(N_1 + (1 - \phi_2)N_2)(f(N_2 + N_1) - f(N_2 + (1 - \phi_1)N_1)) \\ &\& 0 \leq \phi_1 \leq 1, \quad R_1 \geq 1/\beta.\end{aligned}$$

The expectation in (45) is over the number of ideas of each firm conditional on R&D and patenting. The expected revenue part of the above equation gives

$$\frac{1}{2} E[\pi_{1B} + \pi_{1S}] = \frac{\pi_M}{2} E\{2(1 - \lambda)^{(1-\phi_1)N_1+N_2} - 2(1 - \lambda)^{(2-\phi_1)N_1+2N_2}\}$$



$$\begin{aligned}
& -(1-\lambda)^{N_1+(1-\phi_2)N_2} + (1-\lambda)^{N_1+N_2} \} \\
= & \frac{\pi_M}{2} \{ 2e^{-g_2\lambda} e^{-g_1 a(\phi_1)} - 2e^{-g_1 b(\phi_1)} e^{-2g_2\lambda} \\
& - e^{-g_1\lambda} e^{-g_2 a(\phi_2)} + e^{-g_1\lambda} e^{-g_2\lambda} \},
\end{aligned}$$

where  $a(\phi) = 1 - (1-\lambda)^{(1-\phi)}$  and  $b(\phi) = 1 - (1-\lambda)^{(2-\phi)}$ .

The Lagrangian of the above equation is given by:

$$\begin{aligned}
L = & \frac{\pi_M}{2} \{ 2e^{-g_2\lambda} e^{-g_1 a(\phi_1)} - 2e^{-g_1 b(\phi_1)} e^{-2g_2\lambda} - e^{-g_1\lambda} e^{-g_2 a(\phi_2)} + e^{-(g_1+g_2)\lambda} \\
& - C(\phi_1) - R_1 + \mu_3 \phi_1 + \mu_4 (1 - \phi_1) + \theta_2 (R_1 - \frac{1}{\beta}) \},
\end{aligned} \tag{46}$$

where  $\mu_3 \geq 0, \mu_4 \geq 0, \theta_2 \geq 0$  are Lagrange multipliers.

The first order conditions are:

$$\begin{aligned}
\phi_1 : & \frac{\pi_M}{2} 2g_1 \{ -e^{-(g_2\lambda+g_1 a(\phi_1))} \frac{\partial a}{\partial \phi_1} + e^{-(g_1 b(\phi_1)+2g_2\lambda)} \frac{\partial b}{\partial \phi_1} \} \\
& - \frac{\partial C}{\partial \phi_1} + \mu_3 - \mu_4 = 0,
\end{aligned} \tag{47}$$

$$\begin{aligned}
R_1 : & \frac{\pi_M}{2} \frac{\partial g_1}{\partial R_1} \{ -2e^{-(g_2\lambda+g_1 a(\phi_1))} a(\phi_1) + 2e^{-(g_1 b(\phi_1)+2g_2\lambda)} b(\phi_1) \\
& + \lambda e^{-(\lambda g_1+g_2 a(\phi_2))} - \lambda e^{(g_1+g_2)\lambda} \} - 1 + \theta_2 = 0.
\end{aligned} \tag{48}$$

For the symmetric equilibrium equations (47) and (48) give

$$\pi_M g \lambda e^{-(2-\phi)g\lambda} [1 - e^{-2g\lambda}] - \frac{\partial C}{\partial \phi} + \mu_3 - \mu_4 = 0, \tag{49}$$

$$\frac{\pi_M}{2} \frac{1}{R} \lambda [(2\phi - 1)e^{-(2-\phi)g\lambda} + 2(2 - \phi)e^{-(4-\phi)g\lambda} - e^{-2g\lambda}] - 1\theta_2 = 0. \tag{50}$$

The first order conditions again identify three distinct regions of the  $S$ -line, separated by two cut-off points  $\underline{S}^l$  and  $\bar{S}^l$ . For all  $S > \bar{S}^l$ , firms choose the

maximum patent protection ( $\phi_1 = \phi_2 = 1$ ). For all  $S < \bar{S}^l$ , firms choose the minimum patent protection ( $\phi_1 = \phi_2 = 0$ ).

I again start by solving the model for three different patent regimes — a strong patent regime, a weak patent regime and a moderate patent regime.

## 5.1 Case 1 : Strong patent regime

When the patent regime is very strong, firms choose  $\phi_1 = \phi_2 = 1$ .

From the first order condition:

$$\frac{\pi_M}{2} \lambda \frac{1}{R} [e^{-g\lambda} + 2e^{-3g\lambda} - e^{-2g\lambda}] = 1. \quad (51)$$

For small values of  $\lambda$  it can be shown

$$R_{\{\phi=1\}}^l < R_{\{\phi=1\}}^{nl}, \quad (52)$$

i.e., the R&D chosen by firms at the maximum level of IP protection is lower in the presence of licensing as compared to the no-licensing case.

## 5.2 Case 2 : Weak patent regime

When the patent regime is very weak, firms choose  $\phi_1 = \phi_2 = 0$ .

From the first order condition:

$$\pi_M \lambda \frac{\partial g}{\partial R} e^{-2g\lambda} [2e^{-2g\lambda} - 1] = 1. \quad (53)$$

Comparing this with the no-licensing case I find that

$$R_{\{\phi=0\}}^l = R_{\{\phi=1\}}^{nl}, \quad (54)$$

that is, the R&D chosen by firms at the minimum level of IP protection in the presence of bargaining is equal to that of the no-bargaining case.

Thus, so far it has been shown that when firms choose the lowest IP protection ( $\phi = 0$ ), their R&D expenditure choice remains the same in both the no-licensing and licensing equilibrium. This is not surprising, because the licensing stage is trivial when there is nothing to license and, hence, the two cases give identical results.

The most general case is considered next.

### 5.3 Case 3 : Moderate patent regime

For interior solution, the first order conditions for a symmetric equilibrium are

$$\pi_M g \lambda e^{-(2-\phi)g\lambda} [1 - e^{-2g\lambda}] - \frac{\partial C}{\partial \phi_1} = 0, \quad (55)$$

$$\frac{\pi_M \lambda}{2R} [(2\phi - 1)e^{-(2-\phi)g\lambda} + 2(2 - \phi)e^{-(4-\phi)g\lambda} - e^{-2g\lambda}] = 1. \quad (56)$$

Analyzing the above equations the following results are obtained for firms in complex industries:

#### **Proposition 5.**

- i) For small values of  $\lambda$ ,  $\frac{\partial R}{\partial \phi}$  will be positive, but  $\frac{\partial R}{\partial \phi}|_l < \frac{\partial R}{\partial \phi}|_{nl}$ , where  $nl$  stands for the no-licensing case and  $l$  stands for the licensing case, and
- ii)  $\frac{\partial \phi}{\partial S}$  will be positive.

**Proof:** i) Differentiating (55) with respect to  $\phi$

$$\frac{\pi_M \lambda}{2R^2} \frac{\partial R}{\partial \phi} [-(2\phi - 1)e^{-(2-\phi)g\lambda} - 2(2 - \phi)e^{-(4-\phi)g\lambda} + e^{-2g\lambda}] \quad (57)$$

$$\begin{aligned}
& -(2\phi - 1)(2 - \phi)\lambda e^{-(2-\phi)g\lambda} - 2(2 - \phi)(4 - \phi)\lambda e^{-(4-\phi)g\lambda} + 2\lambda e^{-2g\lambda}] \\
& = \frac{\pi_M \lambda}{2R} [-2e^{-(2-\phi)g\lambda} - (2\phi - 1)g\lambda e^{-(2-\phi)g\lambda} + 2e^{-(4-\phi)g\lambda} - 2(2 - \phi)g\lambda e^{-(2-\phi)g\lambda}]
\end{aligned}$$

For small  $\lambda$  the exponential terms can be approximated by 1, which gives

$$\frac{\partial R}{\partial \phi} \Big|_{l=} = \frac{3g\lambda R}{2 + 7\lambda(2 - \phi) - 2\lambda}. \quad (58)$$

Comparing this with the no-licensing case, it can be shown that  $\frac{\partial R}{\partial \phi} \Big|_l < \frac{\partial R}{\partial \phi} \Big|_{nl}$ .

ii) Differentiating equation (56) w.r.t.  $S$  gives

$$\begin{aligned}
& \lambda^2 \pi_M \frac{\partial g}{\partial \phi} \frac{\partial \phi}{\partial S} e^{-(2-\phi)g\lambda} [1 - e^{-2g\lambda} - (2 - \phi)g(1 - e^{-2g\lambda}) + 2ge^{-2g\lambda}] \quad (59) \\
& + \pi_M g^2 \lambda^2 e^{-(2-\phi)g\lambda} (1 - e^{-2g\lambda}) \frac{\partial \phi}{\partial S} = \frac{2}{S} \frac{\partial \phi}{\partial S} - \frac{2\phi + 1}{S^2}
\end{aligned}$$

For small  $\lambda$ ,

$$\frac{\partial \phi}{\partial S} \left\{ 2\pi_M g \lambda^2 \frac{\partial g}{\partial \phi} - \frac{2}{S} \right\} = -\frac{2\phi + 1}{S^2}. \quad (60)$$

For small  $\lambda$  the term inside the parenthesis on the left-hand side will be negative. Therefore, it must be that  $\frac{\partial \phi}{\partial S} > 0$ . ■

This proposition states that although a strengthening of the patent regime will lead to higher patenting and higher R&D both in the absence of licensing and in the presence of licensing, the strategic complementarity between R&D decision and patenting decision of a firm is weakened in the presence of licensing. Even if both kind of licensing environment generate the same increase in patenting in response to a stronger patent regime change, the R&D increment will be smaller in an industry where licensing is widespread.

Firms in complex industries rely heavily on licensing to tap into other firm's ideas. As the surveys mentioned before suggest, licensing has become a very important for firms in the semiconductor industry after the 1982-change in the U.S.

patent regime. The relatively small increase in the aggregate R&D in this industry then may be due to the relatively weak complementarity between a firm's R&D decisions and patenting decision. In presence of licensing, a firm in a complex industry has access to an additional mechanism for obtaining other firm's ideas and will, therefore, rely less on in-house R&D to generate ideas. A stronger patent regime will increase patenting, but the corresponding increase in R&D will be smaller in presence of licensing than what would have been in the absence of licensing.

#### **5.4 A Numerical Experiment**

As noted earlier, the U.S. patent regime change had an impact on the licensing environment of the semiconductor industry. The change in the patent regime seems to have increased licensing and cross-licensing activities in the semiconductor industry. In the light of the above observation, the pre-1982 licensing environment in this industry can be compared to the no-licensing case described in this paper and the post-1982 licensing environment to the with-licensing case. In the context of this model the patent regime has changed more things than just the regime parameter; it has also changed the licensing environment.

In this section, I report a numerical exercise to understand the impact on firm-level patenting and research variables due a joint change in the patent regime and the licensing environment. The parameter values chosen are reported below.

The complexity parameter is chosen to be  $\lambda = 0.005$  so as to represent complex industries, like semiconductor, electronics, etc.<sup>8</sup> The profit ( $\pi_M$ ) is chosen to be

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<sup>8</sup>The MPEG4 Visual Patent Portfolio contains approximately 180 patents (<http://www.mpeg4.com/m4v/index.cfm>). This is not a final product for the consumers but

10 million.<sup>9</sup> To obtain values for the  $\beta$  parameter, the R&D is taken to be a fifth of the total profit. The average number of patents in the semiconductor industry for 2002 is about 55. This gives a parameter value of  $\beta$  in the order of  $10^{17}$ . For this exercise,  $\beta_1 = \beta_2 = 3 \times 10^{17}$  are chosen. The parameter  $S$  relates to the cost of patenting. A parameter value of  $S = 2 \times 10^{-6}$  signifies that the cost of complete patent protection ( $\phi = 1$ ) for a firm runs in the order of a million.  $S = 2 \times 10^{-6}$  is considered a weak patent regime. A 50% increase in the regime parameter constitutes a stronger patent regime. The results obtained are summarized in the following table.

	Weak Patent Regime	Stronger Patent Regime
No Licensing	$\phi$ (patenting) = 0.15 g (research) = 54.15	
With Licensing		$\phi$ (patenting) = 0.69 g (research) = 54.08

The above exercise shows that a strong patent regime change that also alters the licensing environment will have a large positive impact on firm-level patenting decision, but may have only a small (and negative) impact on the research decision. This tallies well with the data from the semiconductor industry where the post-1982 large increase in patenting has not been matched by a similar large compression technology that allows developers of web streaming and videophone to develop their final products. A firm in an industry with  $\lambda = 0.005$  that acquires all of these 180 patents will have 60% chance of developing a final product.

<sup>9</sup>The general conclusion of this experiment remains same for higher or lower values of profit. However, keeping all other parameters the same, much lower profit drives the patenting parameter to zero, while much higher profit makes the patenting parameter equal to 1.

increase in research. The *patent paradox* might be a product of the changed licensing environment that has followed the patent regime change. A strong licensing environment coupled with a strong intellectual property regime has enhanced the importance of patenting for firms by making the size of the patent portfolio an important determinant of the licensing process, but has not changed the incentives to conduct R&D significantly.

## **6 Conclusion**

The strengthening of the U.S. patent regime after 1982 was followed by a large increase in the number of patents and by an unchanging R&D expenditure trend, particularly in the complex product industries. Whether these observations can be explained by studying the effects of a stronger patent regime on an industry environment where bilateral licensing of technologies is common, is that main focus of this paper. The model presented here shows that for complex product industries, where bilateral licensing is common, a stronger patent-regime change will have a smaller positive effect on the firm-level R&D decision compared to that in other industries where licensing is less important.

The impact of the licensing environment on firm-level R&D decision is probably also a function of the size of the firm in terms of the stock of patents. Larger firms with a already large patent portfolio might always enjoy a better bargaining position and, hence, their firm-level R&D decision might be less sensitive to a change in the patent regime. The patenting and R&D data for the semiconductor industry shows that the four largest firms in the semiconductor industry have increased both their patenting activity as well as their R&D activity substan-

tially after the 1982-change in the U.S. patent regime.<sup>10</sup> The patent paradox does not seem to be holding for this group of firms, while it definitely holds for the medium and small-sized firms. Since this model considers homogenous firms, the size-effect is not captured in the model. This remains a project for the future.

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## Appendix A

### Proof of Proposition 3

i)  $\underline{S} = [\pi_M g^0 \lambda (1 - e^{-2g^0 \lambda}) e^{-2g^0 \lambda}]^{-1}$ .

Now,  $R^0 = [\lambda \pi_M \beta^{-2\lambda} \delta]^{\frac{1}{1+2\lambda}}$ , where  $\delta < 1$ . Therefore

$$\begin{aligned} g^0 &= \ln(\beta R^0) \\ &= \ln[\lambda \pi_M \beta \delta]^{\frac{1}{1+2\lambda}} \\ &= \frac{1}{1+2\lambda} \ln(\lambda \pi_M \beta \delta) \end{aligned} \tag{61}$$

Therefore,

$$\frac{\partial g^0}{\partial \lambda} = -\frac{\ln(\lambda \pi_M \beta \delta)}{(1+2\lambda)^2} + \frac{1}{(1+2\lambda)\lambda} \tag{62}$$

$$= \frac{1}{(1+2\lambda)} \left[ \frac{1}{\lambda} - g^0 \right]$$

Differentiating  $\underline{S}$  w.r.t.  $\lambda$  gives

$$\begin{aligned} \frac{\partial \underline{S}}{\partial \lambda} &= -\pi_M e^{-2g^0 \lambda} \left( \frac{\partial g^0}{\partial \lambda} \lambda + g^0 \right) [1 - e^{-2g^0 \lambda} + 2g^0 \lambda (2e^{-2g^0 \lambda} - 1)] \quad (63) \\ &= -\pi_M e^{-2g^0 \lambda} \left( \frac{1}{1+2\lambda} \left( \frac{1}{\lambda} - g^0 \right) \lambda + g^0 \right) [1 - e^{-2g^0 \lambda} + 2g^0 \lambda (2e^{-2g^0 \lambda} - 1)] \\ &= -\pi_M e^{-2g^0 \lambda} \left( \frac{1}{1+2\lambda} \left( \frac{1}{\lambda} - g^0 \right) \lambda + g^0 \right) [1 - e^{-2g^0 \lambda} + 2g^0 \lambda (2e^{-2g^0 \lambda} - 1)] \end{aligned}$$

Now,  $\left( \frac{1}{1+2\lambda} \left( \frac{1}{\lambda} - g^0 \right) \lambda + g^0 \right) = 1 + g^0 + g^0 \lambda > 0$ . For  $\lambda \rightarrow 0$ , it can be shown that  $[1 - e^{-2g^0 \lambda} + 2g^0 \lambda (2e^{-2g^0 \lambda} - 1)] > 0$ . Hence for small values of  $\lambda$ ,  $\frac{\partial \underline{S}}{\partial \lambda} < 0$ . ■