Liquidity, Inflation, and Monetary Policy

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Abstract

In standard monetary models nominal interest rates should be decreased in response to a switch to a lower inflation target. This paper considers this interaction between inflation and nominal interest rates in a dynamic model of liquidity. In a repeated Diamond&Dybvig economy a financial intermediation sector provides those agents with money/liquidity who urgently need it and saves for those who do not. I show when a lower inflation target requires a higher nominal interest rate. I then calibrate the model. The model fits the data very well and the response of inflation to a permanent increase in nominal interest rates is negative if nominal interest rates are low ('the market is liquid') and positive if nominal interest rates are high ('the market is illiquid').

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1 Introduction

A cornerstone of all monetary models is the 'Fisher' rule: the nominal interest rate equals the sum of the real interest rate and the inflation rate. This equation suggests that the central bank should lower the nominal interest rate to lower the inflation rate. This is at odds with central banks' conventional wisdom which says that nominal interest rates should be increased to fight inflation.

This conflict is plainest in a specific experiment. Suppose the central bank wants to implement a lower inflation target. Standard New Keynesian models imply that the optimal response is to lower nominal interest rates. This is obvious if prices are flexible or if the central bank does not care about output. In both cases immediate adjustment of the inflation rate to the target level is optimal. The Fisher equation then requires that the nominal interest should be lowered immediately as well. Section 5 shows that sticky prices and a concern for output do not change this conclusion. The nominal interest rates should be lower than what it would be without a drop in the inflation target. But this is the opposite of the strategy a central bank would adopt.[1]

In this paper I explore a different monetary transmission mechanism. The central bank provides the economy with liquidity in a dynamic Diamond & Dybvig (1983) economy (I do not address the issue of bank runs). Households face uncertain liquidity demand and a financial intermediation sector[2] provides some insurance. Money or broadly liquidity then flows from the central bank to commercial banks and finally to households. The more liquidity is provided through commercial banks, the lower

[1]One may wonder why standard models face these difficulties although they have had success in matching the data. The main reason is that different experiments, responses to temporary exogenous shocks, are considered. Section 5 will be more concrete on these issues.

[2]I also call this a commercial banking sector, as it is common practice in regulatory institutions, such as central banks.

is the interest rate households receive on their deposits (the higher is the liquidity premium).

The objective is to use the model to measure how important liquidity is, how liquidity is affected through changes in monetary policy and how monetary policy affects inflation and money demand. I can then answer the question whether liquidity effects may be important enough to confirm central banks' conventional wisdom. Since there are no frictions such as limited participation (Lucas (1990)) or segmented markets (Alvarez, Atkeson & Kehoe (2002)) and nothing dampens the effects of monetary policy even in the long run, I do not address the question what drives the differences between the short-run and the long-run.[3]

The model, which is described in section 2, is closely related to the framework developed by Lagos & Wright (2005). Both papers share the approach to model money but are different in several other dimensions. Money is essential since it enlarges trading opportunities. Here money allows households to respond to liquidity shocks whereas in Lagos &Wright (2005) money allows households to trade if they find a seller in an anonymous decentralized market. It is this modeling approach that turns out to be important for the result as is explained in section 5. But it alone does not resolve the problem. It is the second ingredient, a financial intermediation sector, that generates (strong) liquidity effects. Banks provide households with liquidity when they need it and save when not.[4]

[3] Presumably liquidity effects do not last forever since in U.S. time series nominal interest rates and the inflation rate are positively correlated in the long run. What causes this co-movement is unclear. Either the central bank lowered the nominal interest rate and the Fisher equation dictates that in the long run the inflation rate has to come down. Or there is a trend in inflation (in my model for example because of a trend in costs in the financial sector due to financial reforms), that are simply accommodated by the central bank (to keep real interest rates constant by the Fisher equation).

[4] Recently, Berentsen, Camera and Waller (2004) have added a banking sector to the Lagos & Wright model. In both papers banks serve as insurance against uncertainty about whether this period liquidity is needed or not. The rest is quite different. The issues addressed differ and the linkage between liquidity and savings, that is key to my model, is absent from theirs. Furthermore

A simplified version of the model, considered in section 3, delivers an explicit solution which shows how the monetary transmission mechanism works. A higher nominal interest rate makes providing liquidity more expensive and investing in the asset market more attractive. Profit maximizing banks then provide households with less liquidity and make them hold more illiquid assets through paying a higher deposit rate. This deposit rate that households receive on their savings (with a bank) is (nonlinearly) different from the interest rate paid on bonds or other assets. If this transmission channel is strong enough nominal interest rates should be increased to implement a lower inflation target.

Section 3 leaves two questions unanswered. Is this mechanism present in the data and is it quantitatively strong enough? These questions are tackled in section 4 where I calibrate a more general version of the model. I find that the model accounts well for the data despite a parsimonious parameterization. In particular money demand in the late eighties and nineties is predicted surprisingly well, given that this time period has proven to be troublesome for money demand so far (see e.g. Lucas (2000)). The theoretical interest-rate elasticity of inflation is positive in all periods between 1985 and 1999 but different from one. A one percent permanent increase in nominal interest rates increases inflation by 0.202%. The response of the real side/banks is strong enough to generate a negative inflation response, only if the market is liquid enough (the nominal interest rate is low enough). I will discuss in section 5 why real balance effect in models with money in the utility function (see for example Woodford (2003)) are weak whereas they are strong here.

It is the interaction between households and banks and not firms that is the driving force of the model. This paper is thus complimentary to the macroeconomic literature on financial frictions, such as Bernanke & Gertler (1989) and Kyiotaki & Moore

their approach is not consistent with central banks' conventional wisdom.

(1997), who emphasize the role corporate net worth and retained earnings of firms play in the amplification of shocks,[5] or Phillipon (2003) and Dow, Gorton &Krishnamurthy (2003) who integrate empire-building managers into a dynamic equilibrium model. There are no further frictions, such as sticky prices, so that all effects of monetary policy materialize immediately. Section 5 discusses the relationship with the results of the New Keynesian literature.

All proofs and derivations of results are delegated to the appendix.

2 The Economy

Consider a discrete-time economy, populated by a continuum of infinitely lived agents of measure one. Each period $t \geq 0$ is divided into two distinct and successive subperiods t_1, t_2.

At t_1 a centralized market is available where agents can pay by credit.

At t_2 the market is decentralized and agents can trade goods for money only.

At date t, the expected utility of an agent is (written recursively because of the shock θ explained below, evaluated at t_1)

$$V_t = E_t(c_t^c + u(c_t^d) + \beta\theta_t V_{t+1}) \tag{1}$$

where c_t^c is the consumption level in the centralized market at date t, c_t^d is the consumption level in the decentralized market at date t and E_t denotes expectation formed at t_1. u is continuously differentiable, concave and $u(0)$ is normalized to 0.

[5]Other macroeconomic papers who consider costly external financing include Carlstrom & Fuerst (1997), den Haan, Ramey & Watson (1999), Gomes, Yaron & Zhang (2002a,b) and a survey by Bernanke, Gertler and Gilchrist (2000).

The discount factor β lies strictly between zero and one. Households do not value leisure. The total time endowment available for production is assumed to equal one.[6] The only source of uncertainty is a liquidity shock Θ_t, that changes every agents' personal rate of time preference.[7] With probability $\overline{p}$ a high shock $\theta_t = \overline{\theta} > 1$ realizes and makes one unit of date t goods worth $\frac{1}{\beta \cdot \overline{\theta}} < \frac{1}{\beta}$ of date $t+1$ goods to the agent. With probability $\underline{p} = 1 - \overline{p}$ a low shock $\theta_t = \underline{\theta} < 1$ realizes and makes one unit of date t goods worth $\frac{1}{\beta \cdot \underline{\theta}} > \frac{1}{\beta}$ of date $t+1$ goods to the agent. Θ has expectation one: $\overline{p} \cdot \overline{\theta} + \underline{p} \cdot \underline{\theta} = 1$. In every period t every agent learns her individual realization of Θ_t not until t_2.

At t_1 households have to take several decisions. First they decide how much to consume in the centralized market (c_t^c). Second they choose how many real government bonds they want to buy ($B_{t+1}^H \geq 0$). And third, they can sign a contract with a bank. The contract stipulates that agents transfer A_{t+1} units of goods from their labor income in period t to the bank in exchange for $r_{t+1}A_{t+1}$ units of goods at $t+1$. In addition the agent can withdraw up to M_t units of money at t_2 for consumption in the decentralized market (c_t^d). Any withdrawal of money decreases the household's bank deposit and the repayment at $t+1$ one for one.

A bank engages in two distinct types of activities, one on each side of the balance sheet. Nothing special happens on the asset side. Banks just buy assets. Since all assets pay the same return (there is no aggregate uncertainty and no liquidity effects on the asset side), I ignore capital and loans to firms. The bank buys government bonds only. On the liability side it provides agents with money, to trade in the de-

[6] What is crucial for tractability is linearity in either consumption or labor. Nothing would change with quasilinear preferences as in Hansen (1985), Rogerson (1988) or Lagos & Wright (2005). Moreover, discounting between the centralized and the decentralized market wouldn't add anything.

[7] This way of modelling liquidity follows Diamond and Dybvig (1983) and Diamond and Rajan (2001).

centralized market at t_2. Banks thus funnel resources from households to debtors and money from the central bank to households.

In each period t_1 a bank offers a one period contract to households (Linearity of preferences implies (shown below) that decisions at t do not affect decisions at $t+1$). This contract is a portfolio, that comprises an illiquid and a liquid asset (money). The contract specifies the overall amount of household investment A_{t+1} and the real rate of return r_{t+1}. In addition it is stipulated that up to M_t units of money can be withdrawn at t_2 on short notice. The household's portfolio behaves like a demand deposit with an upper bound. The customer can show up any time and withdraw $\tilde{M}_t \leq M_t$ funds in the form of cash. A portfolio then consists of $m_t := M_t/P_t$ liquid assets and of $A_{t+1} - m_t$ illiquid assets, where P_t denotes the price level at date t. Once the contract is signed, the bank is obliged to fulfill the contract in any case. Thus the bank is supposed to acquire M_t units of money. In equilibrium low shock agents will withdraw $\tilde{M}_t(\underline{\theta})$ units of money and high shock agents will withdraw $\tilde{M}_t(\overline{\theta})$ units of money. $\underline{p} \cdot \tilde{M}_t(\underline{\theta}) + \overline{p} \cdot \tilde{M}_t(\underline{\theta})$ units of money then suffice to meet all requirements. Money is provided by the central bank through open market operations, where bonds which pay an interest rate R^{CB} are traded for non-interest bearing money.[8] Following the literature, I assume that the central bank can implement and control its interest rate target R^{CB} for the bond market through adjusting money supply.[9] The rest of the government only plays a minor role. B_{t+1} bonds are issued every period t and lump-sum taxes $(R_t^{CB}/\pi_t - 1) \cdot B_{t+1}$ are levied to balance the budget. The number of bonds B_{t+1} is exogenous for everyone and bonds are

[8] An equivalent way is to allow commercial banks to lend money at a nominal interest rate of R^{CB}.

[9] I do not assume that banks are subject to any reserve requirements. Monetary policy is thus not transmitted to the economy through a (mechanical) relationship between money, deposits and the amount of credit by any kind of reserve requirements. Adding reserve requirements would increase the effect of R^{CB} on the private banks' costs of providing liquidity since reserve requirements are higher for liquid assets.

held by either the central bank, commercial banks or households. Central and private banks' profits are transferred to households directly.

W_t denotes the amount of wealth available to households in period t. Let $\chi_t = 1$ if the agent accepts the bank's offer in period t and $\chi_t = 0$ otherwise. Wealth then evolves according to

$$W_t = \chi_{t-1} r_t (A_t - \tilde{m}_{t-1}) + R_t^{CB}/\pi_t B_t^H + w_t + T_t \tag{2}$$

where $\tilde{m}_t = \tilde{M}_t / P_t$ is the amount of real money withdrawn in period t, w_t is wage income and $\pi_{t+1} = P_{t+1}/P_t$ is the inflation rate between t and $t+1$. T_t denotes the sum of taxes and profits (of the central and private banks).

A household, endowed with W_t units of wealth, consumes $c_t^c = W_t - \chi_t \cdot A_{t+1} - B_{t+1}^H$ in the centralized market and $c_t^d = \chi_t \cdot \tilde{m}_t$ in the decentralized market. At t_1 he derives expected lifetime utility

$$\begin{aligned} V_t(W_t) &:= c_t^c + E_t u(c_t^d) + \beta E_t \theta_{t+1} V_{t+1}(W_{t+1}) && (3) \\ &= W_t - \chi_t (A_{t+1} - E_t u(\tilde{m}_t)) - B_{t+1}^H + \beta E_t \theta_{t+1} V_{t+1}(W_{t+1}), && (4) \end{aligned}$$

where W_t evolves according to (2) and $E_t u(c_t^d) = E_t u(\tilde{m}_t) = \underline{p} \cdot u(\tilde{m}_t(\underline{\theta})) + \overline{p} \cdot u(\tilde{m}_t(\overline{\theta}))$.

The flow of money during a time period is as follows. In the decentralized market, money starts in the central bank who trades it for bonds to commercial banks. In the decentralized market households who want to consume ($c_t^d > 0$) withdraw money from their account. Finally firms collect the money they obtain for selling goods in the decentralized market and transfer it, at the end of the period, to their workers' bank accounts to earn an interest rate r. Commercial banks then carry the money to the next period. All other transactions are conducted without money and are basically accounting-exercises for the commercial banks.

The household's problem at date t is as follows:

At t_2 it is decided how much money $\tilde{m}_t(\theta_t)$ is withdrawn from the account to spend on consumption in the decentralized market.

At t_1 the choice problem, taking into account the decision at t_2 and at all future dates, reads as follows:

$$\begin{aligned}
&\max_{\chi, B^H \leq B} V_t(W_t) = \qquad (5)\\
&W_t - B^H_{t+1} + \beta(R_{t+1}/\pi_{t+1} B^H_{t+1} + w_{t+1} + T_{t+1})\\
&+ \chi_t\{-A_{t+1} + \underline{p}\{u(\tilde{m}_t(\underline{\theta})) + \beta\underline{\theta}(r_{t+1}(A_{t+1} - \tilde{m}_t(\underline{\theta}))\} + \overline{p}\{u(\tilde{m}_t(\overline{\theta})) + \beta\overline{\theta}(r_{t+1}(A_{t+1} - \tilde{m}_t(\overline{\theta}))\}\}\\
&+ \beta\{-B^H_{t+2} - \chi_{t+1}(A_{t+2} - E_{t+1}u(\tilde{m}_{t+1})) + \beta V_{t+2}(W_{t+2})\},
\end{aligned}$$

where the second line is the change in utility from signing a contract with the bank at t_1 (if $\chi_t = 1$). Line 1 and 3 describe household's utility without signing (at t_1) a contract with a bank.

Output is produced by labor only. Firms hire L workers to produce $F(L) = z \cdot L$ units of output. There is no disutility of labor, so that $L = 1$.

Banks face a fixed cost of $F > 0$ and competition is of the Bertrand type. In the quantitative part (section 4) I allowed for Cournot competition but the data turn out to be best described by a monopolistic banking sector. This does not rule out the existence of many banks. It just says that every bank treats its customers like a monopolist does. Every bank first decides whether to enter the market and pay a fixed cost F. In the second stage every bank offers a contract (A, M, r). A household signs the contract that gives her the highest utility. This leads to the conclusion that, with two or more banks, all banks will earn zero profits. Since $F > 0$ only one bank can be active. It offers a contract that grants the household not more than her reservation utility V_t^o in all periods. The reservation utility equals V_t, from equation 4, when no contract is signed with the bank ($\chi_t = 0$):

$$V_t^o(W_t) \quad = \quad W_t - B_{t+1}^H + \beta E_t \theta_{t+1} V_{t+1}(R_{t+1}/\pi_{t+1} B_{t+1}^H + w_{t+1} + T_{t+1}) \qquad (6)$$

At date t the household knows that at date $t+1$ he will not receive more than her reservation utility. V_t can now be rewritten by plugging in V_{t+1}^o for V_{t+1} into (4):

$$\begin{aligned} V_t(W_t) \quad &= \quad c_t^c + E_t u(c_t^d) + \beta E_t \theta_{t+1}\{W_{t+1}(\chi_t = 1) - B_{t+2}^H\} \\ &+ \quad \beta^2 E_t \theta_{t+1}\theta_{t+2} V_{t+2}(R_{t+2}^{CB}/\pi_{t+2} B_{t+2}^H + w_{t+2} + T_{t+2})\}, \end{aligned}$$

where $W_{t+1}(\chi_t)$ is the wealth level at $t+1$ conditional on the choice χ_t to accept the bank's contract in period t.

V_t^o can also be rewritten by substituting V_{t+1}^o for V_{t+1}:

$$\begin{aligned} V_t^o(W_t) \quad &= \quad W_t - B_{t+1}^H + \beta E_t \theta_{t+1}\{W_{t+1}(\chi_t = 0) - B_{t+2}^H\} \\ &+ \quad \beta^2 E_t \theta_{t+1}\theta_{t+2} V_{t+2}(R_{t+2}^{CB}/\pi_{t+2} B_{t+2}^H + w_{t+2} + T_{t+2})\} \end{aligned}$$

That V_t is linear in W_{t+1} simplifies the analysis substantially. Any two contracts with different stipulated repayments in $t+1$ offered by the bank in t, change V_{t+1} by exactly this difference in repayments. In particular, periods higher than $t+1$ do not affect $V_t - V_t^0$. Therefore the bank's problem at date t has to take into account periods t and $t+1$ only. A profit-maximizing portfolio (A_{t+1}, m_t, r_{t+1}) gives the household her reservation utility if

$$\begin{aligned}
& V_t(W_t) - V_t^o(W_t) = 0 \\
\Leftrightarrow \quad & -A_{t+1} + \underline{p}\{u(\tilde{m}_t(\underline{\theta})) + \beta\underline{\theta}(r_{t+1}(A_{t+1} - \tilde{m}_t(\underline{\theta}))\} + \overline{p}\{u(\tilde{m}_t(\overline{\theta})) + \beta\overline{\theta}(r_{t+1}(A_{t+1} - \tilde{m}_t(\overline{\theta}))\} = 0
\end{aligned} \tag{7}$$

If all households accept the same contract (A_{t+1}, m_t, r_{t+1}) at t_1 then the bank buys $B^B_{t+1} = A_{t+1} - \underline{p} \cdot \tilde{m}_t(\underline{\theta}) - \overline{p} \cdot \tilde{m}_t(\overline{\theta})$ bonds and $\underline{p} \cdot \tilde{m}_t(\underline{\theta}) + \overline{p} \cdot \tilde{m}_t(\overline{\theta})$ units of (real) money are withdrawn at t_2. The function $r(A, m)$ is defined as solving (7) for r. $r(A, m)$ is the lowest interest such that the household saves $A_{t+1} - m_t$ in illiquid assets and m_t in liquid assets. The repayment in $t + 1$ then equals $C(A_{t+1}, m_t) = r(A_{t+1}, m_t) \cdot (A_{t+1} - \tilde{m}_t)$.

Given the cost function, the bank's problem can now be formulated. The bank has to choose a profit-maximizing contract (A_{t+1}, m_t). The bank gets revenue from buying government bonds at an interest rate of R^{CB}_{t+1}. Money is acquired by renting money at a rate R^{CB}_{t+1} from the central bank. Finally the bank has to pay $P_{t+1}C(A_{t+1}, m_t)$ to its customers. The bank chooses $(A_{t+1} = B^B_{t+1} + \underline{p} \cdot \tilde{m}_t(\underline{\theta}) + \overline{p} \cdot \tilde{m}_t(\overline{\theta}), B^B_{t+1}, m_t)$ to maximize (nominal) profits:

$$R^{CB}_{t+1} P_t B^B_{t+1} - (R^{CB}_{t+1} - 1) P_t(\underline{p} \cdot \tilde{m}_t(\underline{\theta}) + \overline{p} \cdot \tilde{m}_t(\overline{\theta})) - P_{t+1} C(A_{t+1}, m_t) - P_t F, \tag{8}$$

such that the household can afford it:

$$A_{t+1} \leq W_t - B^H_{t+1}. \tag{9}$$

I adjust exogenous labor income to ensure that this constraint is never binding. This preserves the linear structure of the model, needed for tractability. Households then sign the same contract, independent of their individual shock in the preceding decentralized market. I can thus identify all variables with their aggregate counterparts.

An *equilibrium* is a sequence of prices r, R^{CB} and P together with bond holdings B^H and B^B, allocations c^c, c^d ,m and T and decisions χ and $\tilde{m}$ such that the households maximize utility, banks maximize profits, the government and the household budget constraint hold and the bond market and the money market clearing conditions are satisfied.

3 Interest Rate, Money and Inflation: The Linear Case

3.1 Characterizing Equilibrium

In this section I assume that preferences in the decentralized market are linear:

$$u(c^d) = c^d. \tag{10}$$

This allows me to explicitly solve for the equilibrium. I first solve for the optimal portfolio offered by the bank. A necessary condition for an equilibrium is $R^{CB}/\pi\beta \leq 1$ since $R^{CB}/\pi\beta > 1$ would imply an infinite demand for bonds. Thus I assume and verify later that $R^{CB}/\pi\beta < 1$ holds in equilibrium, i.e that it is consistent with the bank's problem. In particular, households do not hold bonds themselves $B^H = 0$. [10]

[10] That households do not buy bonds by themselves is not a special feature of a model with linear utility in the centralized and or decentralized market. Private banks always find it advantageous that households save with banks only. In case households are active in the bond market, banks will bid up the real price of bonds and may improve the conditions of agents' contracts until no agent is active in the bond market anymore.

A simple withdrawal rule $\tilde{m}$ at t_2 simplifies the analysis. In case of a low shock the maximum amount of money is withdrawn $\tilde{m}_t(\underline{\theta}) = m_t$ and $\beta r_{t+1}\underline{\theta}$ is smaller than one. Assumption 1 given below ensures that for the optimal contract $\beta r_{t+1}\overline{\theta} > 1$ and thus $\tilde{m}(\overline{\theta}) = 0$ holds.

This all-or-nothing withdrawal rule allows me to solve equation (7) for $r(B, m) := r(A - \underline{p}m, m)$ and $C(B, m) = r(B, m) \cdot B$. Define $\gamma = \underline{p} \cdot (1 - \underline{\theta})$.[11]

$$r(B, m) = \frac{1}{\beta} \frac{1}{1 + \gamma \cdot m/B} \tag{11}$$

$$C(B, m) = \frac{1}{\beta} \frac{B}{1 + \gamma \cdot m/B} \tag{12}$$

The bank's optimal plan is given as a solution to first order conditions since the cost function is (weakly) convex.

Proposition 1 $C(B, m) = \frac{B}{1+\gamma m/B}$ *is a convex function.*

Households follow a simple strategy: They always accept the contract ($\chi = 1$) and do not buy bonds ($B_t^H = 0$).

Given a sequence of (rationally expected) R^{CB}, an equilibrium is determined once the amount of money m_t and the inflation rate π_t are known.[12] An equilibrium (m_t, π_t) solves the bank's first order conditions:

$$C_B(B, m_t) \cdot \pi_{t+1} = R_{t+1}^{CB} \tag{13}$$

$$C_m(B, m_t) \cdot \pi_{t+1} = \underline{p}(1 - R_{t+1}^{CB}). \tag{14}$$

[11] If $\beta = 1$, γ is the percentage utility gain (in consumption equivalents) of making consumption state-contingent.

[12] The issue of indeterminacy (due to an interest-rate policy) is discussed in section 5.

I denote by $m_t(R^{CB}, \ldots), \pi_t(R^{CB}, \ldots)$ the unique solution to (13) and (14). Obtaining an explicit solution for the inflation rate is the crucial step in solving for an equilibrium. The next proposition accomplishes that.

Proposition 2 *The inflation rate equals*

$$\pi_t = \beta \frac{(R_t^{CB}\gamma + \underline{p} \cdot (R_t^{CB} - 1))^2}{4\underline{p}\gamma(R_t^{CB} - 1)} \tag{15}$$

Given this explicit expression of the inflation rate, m_t can be determined:

$$m_t = \frac{1}{2}\left(\frac{R^{CB}}{\underline{p}(R^{CB} - 1)} - \frac{1}{\gamma}\right) \cdot B_{t+1} \tag{16}$$

Money will be held in equilibrium ($m_t > 0$) if the marginal value $-C_m(K+B, m = 0)$ exceeds the marginal costs $\underline{p} \cdot (R^{CB} - 1)/(\pi(m = 0))$ for the bank. This holds if

$$\gamma > \frac{\beta \underline{p}(R^{CB} - 1)}{\pi(m = 0)} \tag{17}$$

$$\Leftrightarrow \quad \gamma > \frac{\underline{p} \cdot (R^{CB} - 1)}{R^{CB}} \tag{18}$$

I need an assumption that liquidity is too expensive to be given to both low and high types. R^{CB} has to be high enough so that liquidity is low and therefore r is high enough.

Assumption 1

$$R^{CB} > \frac{\underline{p}(1 - \underline{p} + 2\gamma)}{\underline{p} - \underline{p}^2 - \gamma + 3\underline{p}\gamma}, \tag{19}$$

which is equivalent to $\beta r_{t+1}\overline{\theta} = \frac{2\underline{p}(R^{CB}-1)}{\gamma R^{CB}+\underline{p}(R^{CB}-1)} \cdot \frac{1+\gamma-\underline{p}}{1-\underline{p}} > 1$.

Finally, I get:

Proposition 3 *The sequence* $(m_t(R^{CB}, \ldots), \pi_t(R^{CB}, \ldots))$ *is the unique equilibrium.* $R_t^{CB}/\pi_t\beta < 1$ *for all* t.

3.2 Results

I can now develop the links between monetary policy and inflation in the simplified version of the model with linear utility. But the basic mechanism will work in the general case in section 4 as well. Monetary policy is represented as a rationally expected sequence of nominal interest rates R_t^{CB}. Increasing interest rates is termed a contractive monetary policy whereas decreasing interest rates is termed an expansive monetary policy. How a change in monetary policy affects inflation and money is explored in this section.

Banks provide households with liquidity since a more liquid portfolio makes agents willing to accept a lower return r on their savings. The optimal provision of liquidity balances the benefit from lowering repayments $(r \cdot A)$ and the costs of lending money $(\propto (R^{CB} - 1))$.

A tightening of monetary policy increases the bank's costs of lending money from the central bank but makes it more attractive to invest in the asset market. This leads to a swap of liquid for illiquid assets. Households then hold a less liquid portfolio but receive a higher real deposit rate, which reflects a higher liquidity premium $(\frac{1}{1+\gamma \cdot m/B})$. This increase in bank's costs then prevents nominal interest rates and inflation from moving one-for-one.

This linkage between money and interest rates overcomes the basic problem of standard models, where assets are priced by the consumption Euler equation. In models with money in the utility function (and linear utility in consumption but without real balance effects) $r = \frac{1}{\beta}$ and thus $\pi = R^{CB}\beta$, what is the case in this paper only if no liquidity is provided $(m = 0)$. $\pi = R^{CB}\beta$ then implies that inflation moves one-to-one with R^{CB}.[13] This argument ignores real balance effects, that is non-zero effects of changes in m on real interest rates. In a model where money enters the utility function, real balance effects are present if consumption and money holdings are non-separable. I discuss this possibility in section 5.

The perfect parallel movement of inflation and nominal interest rates breaks down in this paper. The inflation rate can even decrease in response to an increase in R^{CB}. For this to be the case the market has to be sufficiently liquid and households' willingness to substitute returns on their savings for liquidity has to be sufficiently high. Both criteria are fulfilled if the value of liquidity ($\propto \gamma$) is sufficiently high relative to its costs ($\propto \underline{p}(R^{CB}-1)$). The following assumption makes precise what is meant by sufficiently high.

Assumption 2

$$R^{CB} \quad < \quad \frac{\underline{p}+2\gamma}{\underline{p}+\gamma} \tag{20}$$

As expected, the right-hand side is increasing in γ and decreasing in $\underline{p}$ and the left-hand side is increasing in R^{CB}.[14] An immediate implication is that inflation decreases if nominal interest rates are increased only if nominal interest rates are low enough, that is the market is liquid enough. More precisely:

Proposition 4 *The inflation rate π is a convex function of R^{CB}. The response of π is the larger the higher is R^{CB}.*

[13]Even adding banks to a standard model, with e.g. a utility function $c+v(m)$ (with finite v(0)), does not change this conclusion. In that case the bank's real repayment to the household, who saves A units of assets and m units of money, equals $\frac{A-m+v(0)-v(m)}{\beta}$. Although the return on A is lower due to a liquidity premium, the marginal cost equals $\frac{1}{\beta}$.

[14]The appendix shows that if $\underline{p} > \frac{1}{2}$ or $\underline{p} < \frac{1}{2}(3\gamma+1+(9\gamma^2+2\gamma+1)^{1/2})$ then an open interval of R^{CB} exists that simultaneously fulfills assumptions 1 and 2.

I can now prove the following propositions, finally showing that inflation decreases if monetary policy becomes tighter. Assumption 2 is not needed in propositions 5 and 6.

A first step is to show a liquidity effect, i.e. that the portfolio becomes less liquid.

Proposition 5 *An increase in* R_t^{CB} *decreases* m_{t-1}/B.

As argued above, a less liquid portfolio makes a higher repayment to the agents necessary. This is indeed what happens.

Proposition 6 *An increase in* R_t^{CB} *increases* r_t.

Having established a liquidity effect and the consequences for r, the main result on the response of the inflation rate can be shown.

Proposition 7 *An increase in* R_t^{CB} *decreases* π_t.

The last proposition shows a more specific result. Not only the real interest rate r but also the nominal interest rate paid to households increases. Keeping the nominal interest rate on agents' deposits constant and let the decrease in inflation do the job is not sufficient. Nominal deposit rates have to increase.

Proposition 8 *An increase in* R_t^{CB} *increases the nominal return* $r_t \cdot \pi_t$ *on households' savings.*

4 Interest Rate, Money and Inflation: The General Case

In this section I quantitatively assess the change in inflation in response to a change in nominal interest rates. I calibrate a more general version since the linear version of section 3 is to simplified to match the data well.

Linear utility implies an interest rate elasticity of money that is far too high for a realistic ratio of liquid to illiquid assets. I therefore assume utility in the decentralized market $u(c^d)$ to be strictly concave.

The non-linearity in the decentralized market does not affect the linearity in the centralized market. The heterogeneity in wealth does not matter, banks at t take into account periods t and $t+1$ only and offer a contract (A, m) that makes agents indifferent between acceptance and rejection. $R/\pi \cdot \beta > 1$ is again no equilibrium and households do not hold any bonds ($B^H = 0$). What changes is that the withdrawal rule in the decentralized market is not of the all-or-nothing variety. In equilibrium low types still withdraw all money but high types withdraw a positive amount of money. There is an implicit solution[15] for the real interest rate paid on deposits:

$$r(B,m) = \frac{1}{\beta}\frac{B + \underline{p} \cdot [\tilde{m}(\underline{\theta}) - u(\tilde{m}(\underline{\theta}))] + \overline{p} \cdot [\tilde{m}(\overline{\theta}) - u(\tilde{m}(\overline{\theta}))]}{B + \overline{p} \cdot (\overline{\theta} - 1) \cdot (m - \tilde{m}(\overline{\theta}))} \tag{21}$$

The cost function then equals

$$C(B,m) = r(B,m) \cdot B \tag{22}$$

Furthermore, I take into account that there are substantial noninterest incomes and expenses, such as for example managing costs, fee income and valuation gains.[16] For the bank's decision it is only relevant what fraction of these incomes and expenses is marginal, that means changes with the number of assets. But this fraction is

[15] It is an implicit solution because $\tilde{m}$ depends on r.

[16] In the year 2000 all FDIC-insured commercial banks had noninterest income of 154.2 billion dollars and noninterest expenses of 216.8 billion dollars. Noninterest income and expenses are thus a large fraction of total income since interest income equaled 428.1 billion dollars and interest expenses equaled 224.6 billion dollars.

unobservable. I therefore capture marginal incomes and expenses through a function $c(B)$. I first allowed for a cost to manage m as well, but the cost function for m was virtually zero in any calibration.

Finally, I allow the velocity in the decentralized market to be different from one.[17] Households spend $\underline{p}\tilde{m}(\underline{\theta}) + \overline{p}\tilde{m}(\overline{\theta})$ in the decentralized market but banks have to hold $v(\underline{p}\tilde{m}(\underline{\theta}) + \overline{p}\tilde{m}(\overline{\theta}))$ only, where v is smaller than 1 and describes how fast money circulates in the decentralized market.

Another feature that could be added is more competition in the banking sector. Some degree of monopoly power is useful since otherwise the whole difference between real deposit returns and banks' real return on assets is attributed to liquidity preferences only.[18] I allowed for Cournot competition but it turns out that a monopoly describes the data best.

With all the added features the first order conditions read as:

$$R^{CB} = \pi \cdot r_B(B,m)B + \pi \cdot r(B,m) + \pi(c'(B)) \quad (23)$$

$$\underline{p}(1 - R^{CB})v = \pi r_m(B,m)B. \quad (24)$$

4.1 Calibration

The calibration strategy is as follows. I take the evolution of the nominal interest rate and the size of the banks' balance sheet, the sum of liquid and illiquid assets, as exogenously given in the model. Feeding this into the model generates predictions for the share of liquid assets and the real interest rate paid on deposits. I then choose

[17] Velocity in the whole economy, $P_t GNP_t / m_t$, depends on the nominal interest rate.

[18] How households value the liquidity provided by banks is basically measured through agents' willingness to accept an interest rate below the market return. Other types of insurance (unemployment, car, etc.) do not matter. Banks here provide a certain type of insurance and households are paying a certain price that reflects their valuation of this specific insurance.

parameter values to minimize a weighted sum of the residuals for these two target time series. This involves a search for parameters that provide the smallest deviation of the model prediction from the data and ensure that all equilibrium conditions are satisfied. Both time series are real. The goal of this exercise is to explore how the inflation rate changes in response to a change in nominal interest rate. I therefore do not target the time series for inflation but only the mean, to match the real return on bonds. A consequence is that adding a policy rule, as suggested by Taylor (1993), does not change the results. I discuss this and the reasons for having two exogenous variable, nominal interest rates and banks's assets, later.

There is a well documented structural change in the U.S. economy in the first half of the eighties.[19] Stock and Watson (2002) suggest that financial market reforms embodied in the Monetary Control Act of 1980 and the Garn-St. Germain Act of 1982 could be causal. Since I want to avoid too much overlap with this period of substantial changes in the financial sector, my sample starts in 1985 and then comprises the years 1985-1999.

The data on commercial banks and savings institutions are taken from the Federal Reserve Database[20] and the FDIC (Federal Deposit Insurance Corporation)[21]. As liquid assets I include $M1$, the monetary aggregate usually used as highly liquid assets, e.g. in Lucas (2000). I add saving deposits, which both have zero maturity and have the feature that they pay some interest and cash can be withdrawn without prior notice. As illiquid assets I choose small and large time deposits, that is assets with non-zero maturity. Figure 1 shows both time series.

[19] Kim & Nelson (1999) and McConnell & Perez-Quiros (2000) were the first to point this out. A detailed discussion and further references can be found in Blanchard & Simon (2001) and Stock & Watson (2002).

[20] URL: http://research.stlouisfed.org/fred2 and http://www.federalreserve.gov/releases.

[21] URL: http://www.fdic.gov.

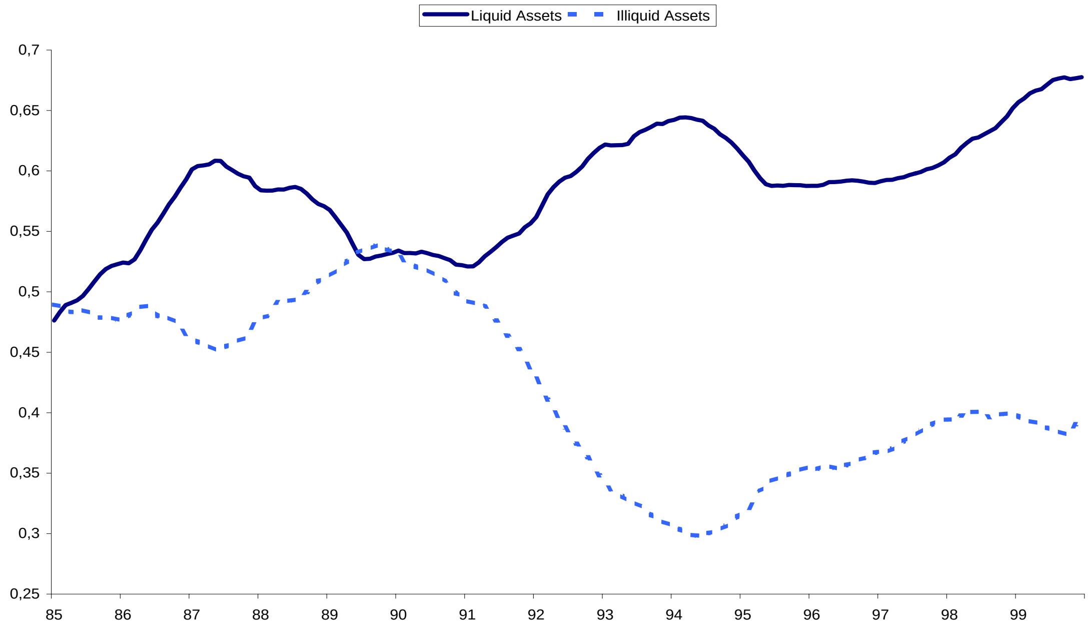

Figure 1: Liquid (m) and Illiquid Assets ($A - m$)

The central bank controls a short term nominal interest rate, which in this model then equals the return on all assets. Thus R^{CB} should equal the nominal return on banks' assets. These data are available, but only at an annual frequency, from the FDIC. I choose a slightly different approach. I follow the literature on money demand (for e.g. Lucas(2000)) and consider a short term interest rate.[22] I then adjust the mean of this short term rate to match the nominal return on banks' assets, which is 1.84% per quarter (the real return is 1.04%). The number I add is 0.00462. The correlation between my measure (annualized) and the correct measure from the FDIC is 0.859. Thus the procedure seems to be pretty reliable. The only substantial difference is that the FDIC measure has no spike in 1995.

The interest rate paid on deposits is the weighted average of interest rate paid on assets in $M2$, called M2 Own Rate. This is, to my knowledge, the most reliable measure of deposit returns available.The (ex-post) inflation rate is computed from

[22]Secondary Market Rate of a 6-month Treasury bill.

Figure 2: Nominal Return on Banks' Assets (My Measure).

the CPI.[23] Figure 2 shows the nominal quarterly return on banks' assets and figure 3 shows the real quarterly return both on banks' assets and on deposits.

The model time period is a quarter.[24] The utility derived from consumption in the decentralized market is assumed to be:

$$u(c^d) = \kappa \cdot (c^d)^\delta. \tag{25}$$

The income/cost function $c(B)$ is assumed to be linear:

$$c(B) = \alpha B. \tag{26}$$

[23]The CPI is for all urban consumers and all items. The results are insensitive to the choice of the (ex-post) deflator.

[24]I use monthly observations, but only to triple the number of observations.

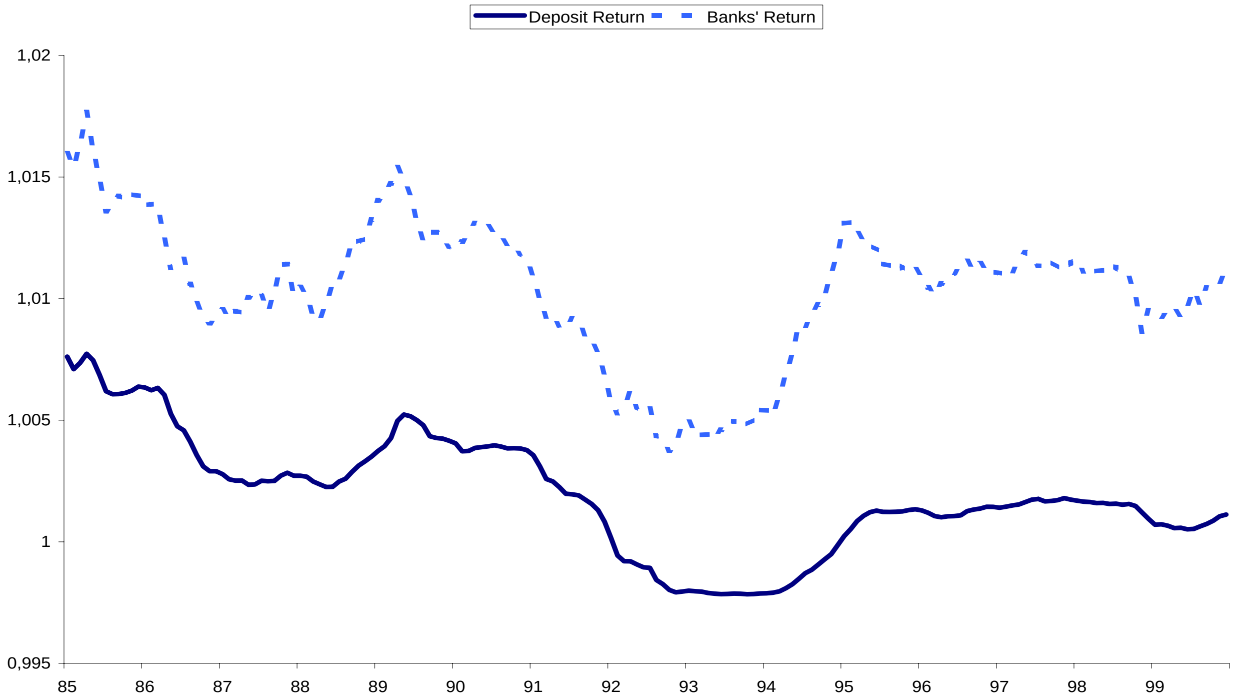

Figure 3: Real Interest Rates on Bonds and Deposits

Let r_t^*, m_t^* and A_t^* denote the data and $\check{r}_t$ and $\check{m}_t$ model predictions. In addition to minimizing the distance of the model and the data, I also target the mean of the inflation rate π.

The parameters are then chosen to minimize the following objective function:

$$\omega_1 \sum_t (m_t^*/A_t^* - \check{m}_t/A_t^*)^2 + \omega_2 \sum_t (r_t^* - \check{r})^2 + \omega_3 \text{mean}(\pi^* - \check{\pi})$$

Seven parameters have to be determined: the time preference rate β, the probability of a low shock $\underline{p}$, the value of liquidity γ, the number of banks N (assumed to be continuous), the two utility parameters ν and δ and the income/cost function parameter α. Table 4.1 shows the results. It turns out that a household who consumes 1.012$ in the centralized market is as well off as an agent who receives a 1$ transfer to his bank account.

Table 1: Parameter Estimates

p	γ	β	υ	κ	δ	α
0.866	0.858	0.974	0.572	0.055	0.935	-0.004

Figure 4 compares the data and the model predictions for liquidity and figure 5 does the same for the real return on deposits. The mean inflation rate in the model and in the data both equal 1.0079.

The model replicates the data very well, in particular given the small number of parameters. The non-linearities of the model are helpful in matching the data. It does much better than a linear model alone. There remain two problems. The data on money lag the model predictions what suggests that some form of portfolio adjustment costs are helpful to explain the data. Second, the largest deviation between the data and the model occurs in the year 1995, which is exactly the year where my measure of banks' return differ from the correct FDIC measure.

I can now use the model estimates to compute the nominal interest rate elasticity of inflation. Figure 6 shows the evolution of this elasticity, computed using the implicit function theorem. The values range between 0.075 and 0.301. They are positive in all periods and closer to zero than to one. But it is not constant. As shown in proposition 4 the elasticity is higher(lower) if nominal interest rates are higher(lower). This argument can even render the elasticity negative. Figure 7 shows the elasticity of inflation for nominal interest rates between 0% and 3% (quarterly rates), evaluated at the mean asset level. The elasticity is negative if the nominal interest rate is below one and is positive otherwise.

One purpose of targeting only the real variables, the real interest rate on deposits and the share of liquid assets, and not inflation is that the model can be estimated

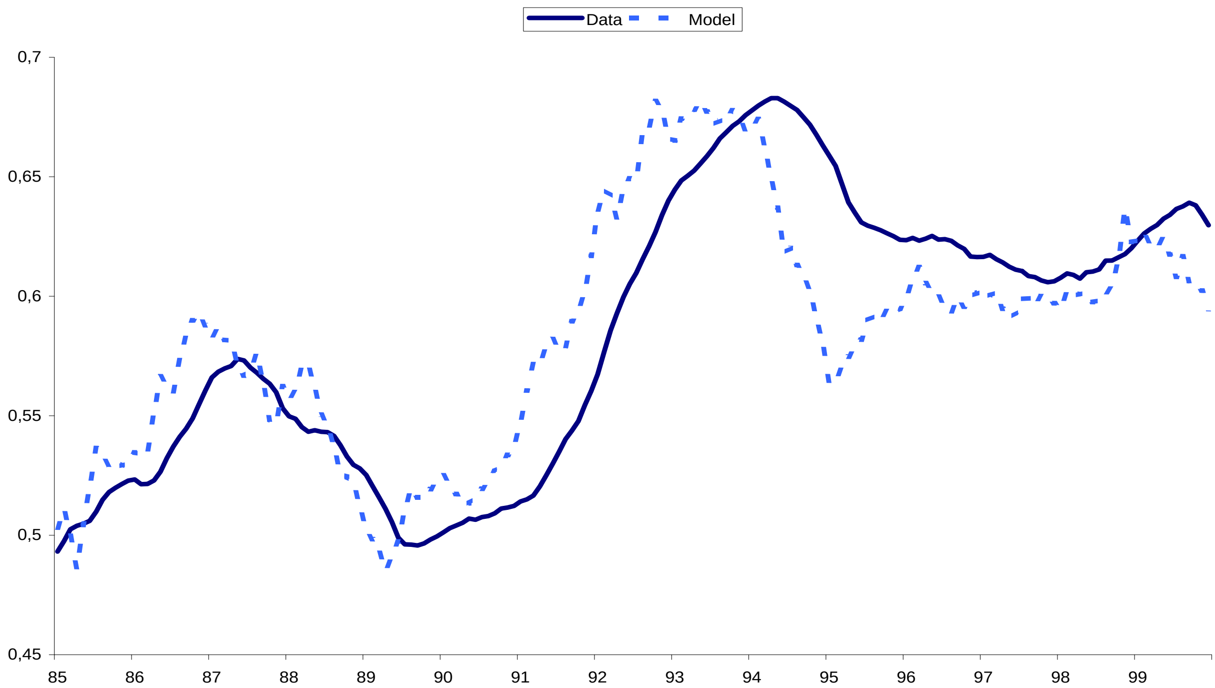

Figure 4: Ratio of Liquid Assets m/A.

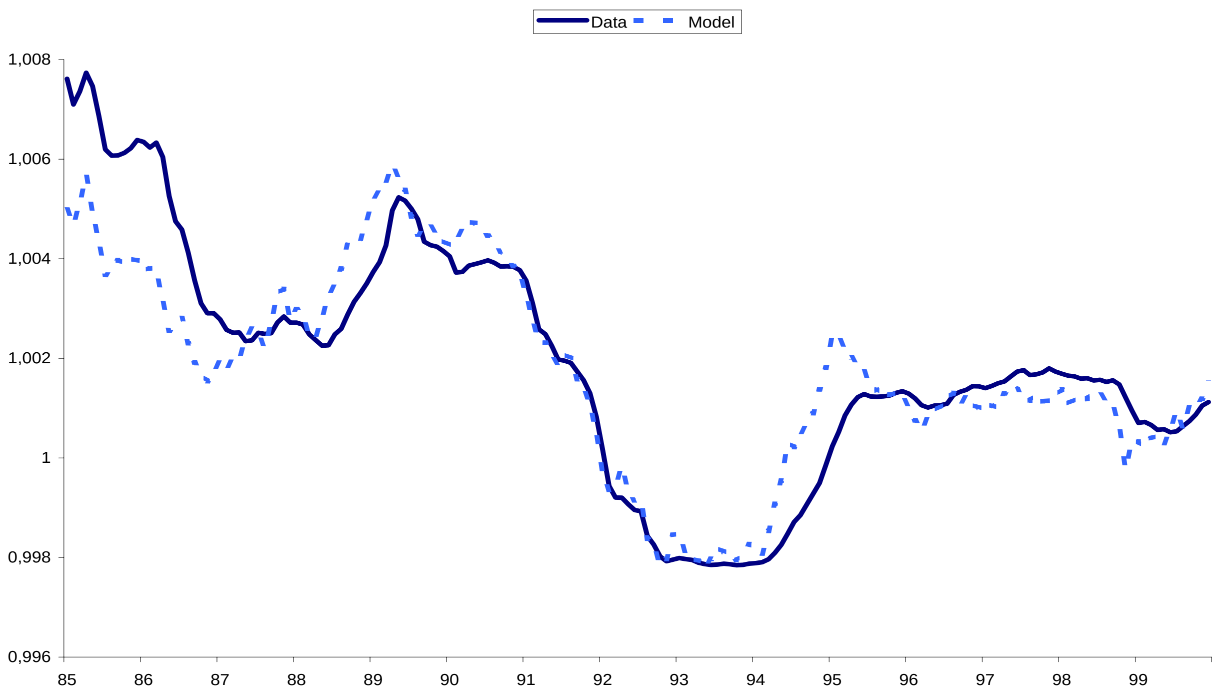

Figure 5: Real Deposit Returns r.

Figure 6: Nominal Interest Rate Elasticity of Inflation over time

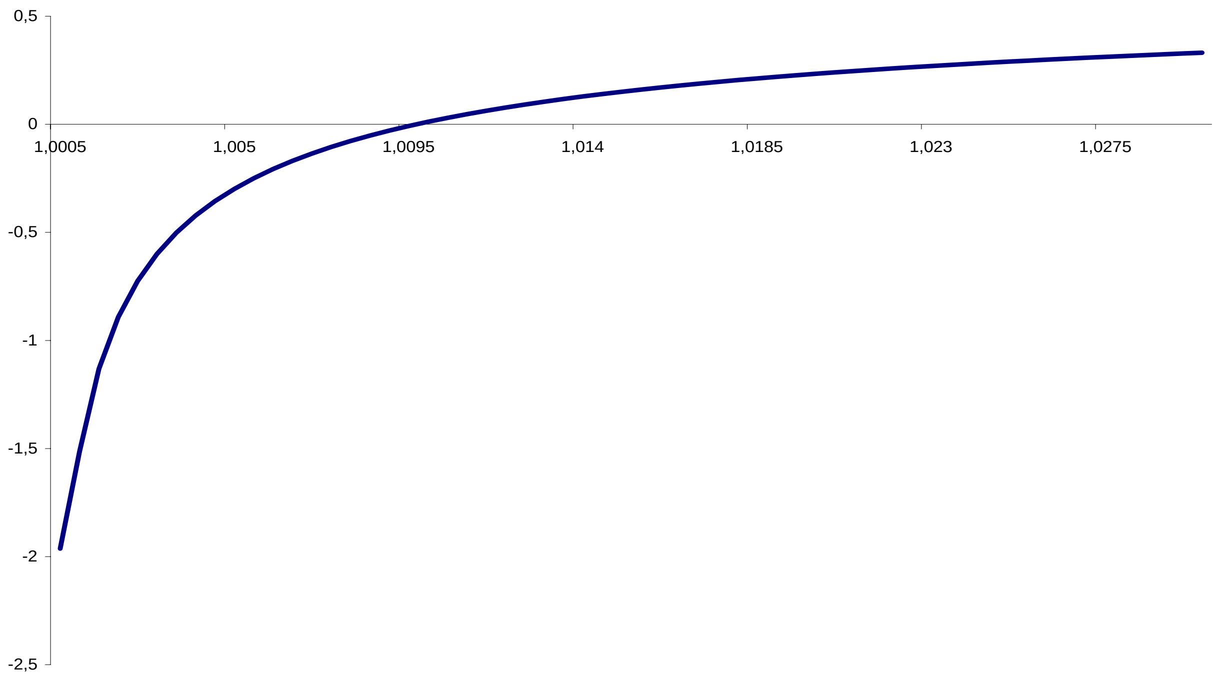

Figure 7: Nominal Interest Rate Elasticity of Inflation as a function of R

separately from a policy rule. The Taylor policy rule (Taylor (1993)) specifies the nominal interest rate target as a (affine-linear) function of inflation and the output gap; one variable, inflation, is not targeted in the estimation and the output gap is zero. The estimation of the model and the policy rule are thus orthogonal. This justifies taking R as exogenous. The evolution of the other variable A, again taken as exogenous, mainly captures changes in the financial sector and in the size of the banking sector. These features are not modeled here. Therefore I have to take A as exogenous as well.

5 Relationship to the Literature

To get a better understanding of this paper's contribution, I discuss how my results are related to several issues and results of the New Keynesian (NK) literature (Woodford (2003)).

Real Balance Effects

A central part of the mechanism in this paper is that changes in money (liquidity) holdings change the real interest rate (on deposits). This mechanism is also present in models where money enters the utility function. Real balance effects are present if the utility function is non-separable between consumption and money. The non-separability implies that changes in money holdings affect the marginal utility of consumption and by the consumption Euler equation real interest rates.

In my model the elasticity of r w.r.t. m equals -0.010. Woodford (2003) shows that even a value of -0.02 does not change the quantitative implications in standard New Keynesian models substantially. The reason is that in a NK model a change in money changes the marginal utility of consumption (because of non-separability) and thus affects real interest rates (through the Euler equation). But if money holdings go up

10% today and next period, the marginal utility today and next period change by the same amount and thus the real interest rate today remains unaffected. Although it would change if only today's money holdings change.

In my model, in contrast, the real rate on deposits decreases today and next period if money holdings go up 10% today and next period.

Indeterminacy

Sargent and Wallace (1975) and Woodford (2003) point out that interest-rate rules, such as an exogenous sequence of interest rates, can lead to (local) indeterminacy. Local indeterminacy means that there is an infinite number of other equilibria which are arbitrarily close to a given equilibrium. This is a priori also problem in this paper when an exogenous interest sequence of interest rates is implemented. One possible solution is to formulate policy in terms of monetary targets. The other possible solution is to implement an interest-rate rule that leads to determinacy, defined as follows. Let $\overline{\pi}_t$ and $\overline{m}_t$ be the equilibrium sequences as characterized in the main text for a sequence of nominal interest rates $\overline{R}_t$. This equilibrium is *locally determinate* if there exists $\epsilon > 0$ such that there is no other equilibrium $\hat{\pi}_t$ and $\hat{m}_t$ with $\mid \overline{\pi}_t - \hat{\pi}_t \mid < \epsilon$ and $\mid \overline{m}_t - \hat{m}_t \mid < \epsilon$.

The following interest rate rule renders $\overline{\pi}_t$ and $\overline{m}_t$ a determinate equilibrium:

$$R_t = \overline{R}_t + \alpha \mid \pi_t - \overline{\pi}_t \mid, \tag{27}$$

for a large α. I use absolute values to ensure that R is always positive.

Suppose there is another equilibrium $\hat{\pi}_t$ and $\hat{m}_t$ with $\overline{\pi}_t \neq \hat{\pi}_t$ (this is wlog since $\overline{m}_t \neq \hat{m}_t$ implies $\overline{\pi}_t \neq \hat{\pi}_t$). Since α is large, R_t deviates a lot from $\overline{R}_t$ and as a consequence π_t deviates a lot from $\overline{\pi}_t$. And this process continues and will drive π outside the ϵ bounds. This is the same logic used in NK models to show that an active policy leads to determinacy. In both models, the coefficient α has to be large enough such that the candidate equilibrium deviates enough.

A Regime Change: Implementing a new inflation target

Consider the following situation. At time $t = 0$ the central bank is told to implement an inflation target $\hat{\pi}^*$ that is lower than current inflation. I now compute what the optimal path for inflation, output and nominal interest rates are.

I consider a simple deterministic NK economy that can be described by two equations (see e.g. Clarida, Gali and Gertler (1999) (CGG) or Woodford (2003). An aggregate-supply relation

$$\pi_t = \kappa x_t + \beta \pi_{t+1} \tag{28}$$

and an intertemporal IS-equation

$$x_t = x_{t+1} - \sigma(i_t - \pi_{t+1} - r^n), \tag{29}$$

where x_t is the output gap, i_t is the nominal interest rate, π_t is the inflation rate and r^n is the constant natural rate of interest.

At $t = 0$, the time of the regime change, the central bank commits to a sequence π_t and x_t to maximize

$$-\sum_{t=0}^{\infty} \beta^t (\pi_t - \pi^*)^2 + \lambda_x (x_t - x^*)^2 \tag{30}$$

I want to compute now how the regime change affects inflation, the output gap and in particular the nominal interest rate. For that purpose I compare the path of the nominal interest when the inflation target is not changed (it is still $\overline{\pi}^*$) to the path when there is a new target $\hat{\pi}^* < \overline{\pi}^*$. With flexible prices the result is obvious. The inflation rate is always set equal to its target level since the output gap is zero. The nominal interest then equals $i_t = r^n + \overline{\pi}^*$ without a target change and $i_t = r^n + \hat{\pi}^*$ with the new target. Thus the central bank immediately reduces the nominal interest by $\overline{\pi}^* - \hat{\pi}^* > 0$ to implement the lower inflation rate. I will now show that i is also lowered in a model with pricing frictions.

The IS-equation must not be included since it determines i once the optimal π and x are known. I solve the aggregate-supply relation for x_t

$$x_t = (\pi_t - \beta\pi_{t+1})/\kappa \tag{31}$$

and plug it into the objective function

$$max_{\pi_t, t\geq 1} - \sum_{t=0}^{\infty} \beta^t(\pi_t - \pi^*)^2 + \lambda_x((\pi_t - \beta\pi_{t+1})/\kappa - x^*)^2, \tag{32}$$

where $\pi^* = \overline{\pi}^*$ without a regime change and $\pi^* = \hat{\pi}^*$ with a regime change. The inflation rate in period $t = 0$ is predetermined at a level $\overline{\pi}$, so that there are no gains from a surprise inflation. The first order necessary and sufficient conditions yield:

$$\pi_t - \pi^* + \lambda_x/\kappa(x_t - x_{t-1}) \quad = \quad 0 \qquad \text{for } t \geq 1 \tag{33}$$

The Appendix shows that the solution is

$$\pi_t \quad = \quad c\delta^t + k \tag{34}$$

$$x_t \quad = \quad c/\kappa\delta^t(1 - \beta\delta) + k/\kappa(1 - \beta) \tag{35}$$

where

$$\delta = b/2 - \frac{\sqrt{b^2 - 4/\beta}}{2} \in (0,1) \tag{36}$$
$$b = \frac{\kappa^2}{\beta\lambda_x} + \frac{1+\beta}{\beta} > 2 \tag{37}$$
$$k = \pi^* \tag{38}$$
$$c = \overline{\pi} - k > 0 \tag{39}$$

Inflation will converge to k. Since the experiment is a decrease in the inflation target, $\hat{\pi}^* < \overline{\pi}$ holds. I can now compare the two regimes. I start with inflation, where $\pi_t(\overline{\pi}^*)$ and $\pi_t(\hat{\pi}^*)$ denote the paths under the two different regimes. The same notation is used for k, c, x and i. For $t \geq 1$ it holds that

$$\begin{aligned}
\pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*) &= (c(\hat{\pi}^*) - c(\overline{\pi}^*))\delta^t + k(\hat{\pi}^*) - k(\overline{\pi}^*) && (40)\\
&= (\hat{\pi}^* - \overline{\pi}^*)(1 - \delta^t) && (41)\\
&< 0
\end{aligned}$$

The second component that determines i is the growth rate of x, $(\Delta x)_t = x_t - x_{t-1}$.

$$\begin{aligned}
(\Delta x)_t(\hat{\pi}^*) - (\Delta x)_t(\overline{\pi}^*) &= (c(\hat{\pi}^*) - c(\overline{\pi}^*))(1 - \beta\delta)/\kappa(\delta^t - \delta^{t-1})\\
&= -(k(\hat{\pi}^*) - k(\overline{\pi}^*))(1 - \beta\delta)/\kappa(\delta^t - \delta^{t-1}) && (42)\\
&< 0
\end{aligned}$$

Plugging everything into the IS-equation (29) yields:

$$\begin{aligned}
i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) &= \frac{(\Delta x)_{t+1}(\hat{\pi}^*) - (\Delta x)_{t+1}(\overline{\pi}^*)}{\sigma} + \pi_{t+1}(\hat{\pi}^*) - \pi_{t+1}(\overline{\pi}^*)\\
&< 0
\end{aligned}$$

Thus the nominal interest rate has to be lower to implement a lower inflation target at all points of time. This conclusion does not change if for example decision lags (in expenditures) are added. The path is then basically shifted to the right.

Conventional Central Bank Wisdom and Monetary Models

Central banks increase nominal interest rates to decrease inflation, putting up with a possible slowdown in economic growth, a strategy which is at adds with standard monetary models (CIA or money-in-the-utility function models with or without pricing frictions). I have already computed the response to a regime change, where nominal interest rates are lowered to lower inflation. I now more generally explain what drives the predictions of standard monetary models. Two optimality conditions turn out to be accountable. The consumption Euler equation, which relates consumption/output growth to real interest rates and the Fisher identity, which relates nominal interest rates, real interest rates and the inflation rate. The Fisher relation implies that an increase in nominal interest rates either increases real interest rates or increases inflation. Since output growth slows down, real interest rate decrease by the consumption Euler equation. Thus the inflation rate rises. An empirical episode that illustrates these problems are the years 1979-1982 (Volcker-deflation). This time period is known for high nominal interest rates but consumption and output growth rates were low and not high.

Responses to Shocks

How are the above arguments related to the empirical success of NK models which has been established numerous time (see Woodford (2003)). As explained above, either the growth rate of output or the inflation rate has to increase if nominal interest are raised. In NK models output grows at a higher rate so that the inflation rate can decrease.[25]

The main reason why inflation indeed falls is that a different experiment is conducted, the (impulse) response to exogenous temporary shocks. Another reason, that I do not want to dwell on, is that only locally determinate equilibrium are considered (see Benhabib, Schmitt-Grohé and Uribe (2001)).[26]

Two types of shocks are considered. Monetary shocks, which I consider first, and nonmonetary shocks to the real side of the economy.

Figure 8 qualitatively shows the response of consumption (=output, since there is no capital) and inflation to a short-lived increase in nominal interest rates. The important observation is that output drops immediately and then slowly returns to its steady state level. The nominal interest rate is higher and the inflation rate is lower than their steady state levels. For this policy experiment the consumption Euler equation is satisfied since output and consumption are growing faster and real interest rates are higher after a monetary policy shock. For output to grow faster and to be lower than in a steady state a large immediate fall in output is necessary. This conclusion, which is consistent with conventional wisdom, already changes if a more persistent shock to nominal interest rates is considered (see Figure 9 and also Woodford (2003), Chapter 4, Figure 4.8). As explained above, an increase in nominal interest implies either a higher inflation rate or a higher growth rate of output or both. Since nominal interest are still above trend after a couple of periods and output growth is already pretty flat, the inflation rate has to be higher as well. Adding various decisions lags to price and quantity choices does not change this reasoning, except that the degree of persistence of the monetary policy shock, needed

[25]With habit persistence output can fall in some periods but inflation still decreases.

[26]Benhabib, Schmitt-Grohé and Uribe (2001) show that there is at least a second finite, stable equilibrium. This equilibrium not only exhibits different responses to shocks but nothing (except an assumption) prevents the economy from converging to this equilibrium.

for inflation to increase, increases. If for example expenditures are assumed to be determined with a lag of two periods, output drops in period two. Only for these two periods the consumption Euler equation is not valid.

A second type of experiments are the responses to nonmonetary shocks, that is shocks that affect inflation π and real interest rates r. A look at the Fisher equation $i = \pi + r$ shows that these shocks then mechanically change nominal interest rates i.

If the central bank's strategy is just to accommodate whatever happens to π and r, the policy function has a positive coefficient for π and r.[27] But the coefficients are positive not because the central bank wants to fight inflation but because it accommodates. This means that causality is reversed.

But the central bank may want to take an active role and not just accommodate. Consider for example a positive 'cost-push'-shock (a shock to the aggregate-supply relation).

To flesh out how it can affect inflation and output consider first a central bank that fixes the nominal interest rates no matter what.[28] As a result the output gap drops and then reverts back to trend. For this to be an equilibrium, real interest rates have to be higher than in a steady state. Since nominal interest rates are constant, the inflation rate has to drop (below its target value).

Now suppose the central bank wants to dampen the drop in inflation. To get a higher inflation rate it has to accept a stronger drop in the output gap and therefore a steeper increase of output back to trend. Again the Fisher and the consumption Euler equation tell the central bank how this goal can be achieved. Since inflation and output growth are higher the nominal interest rate has to be higher. Thus when

[27]The exact numerical value depends on what is included on the right-hand side of the policy function, for example current or future expected values.

[28]The problem of indeterminacy can be solved as described above.

the central bank wants to implement a higher inflation rate than would prevail with a constant nominal interest rate, nominal interest rates have to rise. For two reasons: both the inflation rate and output growth are higher than in the steady state.[29]
For the optimal path of inflation and output (see for example CGG) it turns out that nominal interest rates are lower than in the steady state. Again the optimal policy rule has a positive coefficient on inflation but the causation is reversed. If a higher inflation rate is desired, the nominal interest rate has to be higher.

6 Concluding Remarks

Central banks increase nominal interest rates to decrease inflation, putting up with a possible slowdown in economic growth. Considering such a policy in a standard model implies that central bankers are doing a bad job. Tightening monetary policy implements a higher inflation target. But the Fed, e.g. under chairman Paul Volcker, expressly tightened monetary policy to bring inflation permannently and not e.g. to accommodate higher economic growth (which never materialized).

In this paper I developed a new model of money demand based on liquidity preferences to assess these issues. In the theoretical part I show that it is possible to overcome the predictions of standard models, that is central bankers achieve their goals.
The quantitative evidence is quite promising. On the one hand, the model, although parsimoniously parameterized, matches the data very well. On the other hand an increase in nominal interest rates decreases inflation in the majority of periods. A

[29] The central bank can also implement an inflation rate higher than its target level if it follows the rule that nominal interest rates are increased more than one-for-one with inflation. In this case inflation has to increase because otherwise real interest rates fall, what is inconsistent with the Euler equation and output reverting back to trend.

one percent increase in nominal interest rates results on average in a 0.202% increase in inflation.

Is this bad news for central bankers? Not, necessarily. Adding some short-run frictions to the model could require that nominal interest rates, in response to a decrease in the inflation target, are first increased before they are decreased eventually. But for this to be true a strong liquidity effect, which is absent from New Keynesian models but key in this paper, is necessary.

The experiment I consider, a reduction in the inflation target, allows me to isolate the problems of standard models. It is the way how money is put into the model and not the presence of sticky prices that creates these problems. All the interesting action in this paper happens what is a black-box in a model where money enters the utility function. If anything sticky prices can be a valuable ingredient since any monetary model should not only be confronted with the experiment in this paper but still faces the challenge of matching the responses to exogenous shocks. Other rigidities will also be helpful to explore certain empirical dimensions of the model. One type of adjustment cost, that is more specific to the environment in this paper, are costs to adjust portfolios, both for banks and households. On the household side the model then would inherit some features from the literature on segmented markets (see for example (Alvarez, Atkeson & Kehoe (2002)).

The concern of this paper is with first differences of inflation and nominal interest rates. It answers the question what should be the change in nominal interest rates in response to a change in the inflation rate target. The model leaves unexplained what drives inflation rate in the long run, for example the downward trend during the 1980's and 1990's. It could be either monetary policy through setting nominal interest rate or (cost-)shocks as explained in the introduction. The relative contribution of these two explanations is important to assess the validity of the Fisher equation in the long run. The long-run correlation between nominal interest rates and inflation is positive, but not one. But whether this positive number is due to policy or shocks to the economy is unclear. To address this issue I would have to add cost shocks to the financial sector. Presumably shocks will explain a large part of movements in inflation. A feature this model would share with the New Keynesian literature (Smets and Wouters (2005)), where in addition monetary policy shocks account for only a small fraction of inflation (Christiano, Eichenbaum and Evans (2005)).

Another feature that this model shares with most of the New Keynesian literature (Woodford (2003)) is the absence of capital. This is clearly unsatisfactory in this model since capital is the primary suspect when it comes to dampen liquidity and monetary policy effects in the long run. All these issues are left for future research.

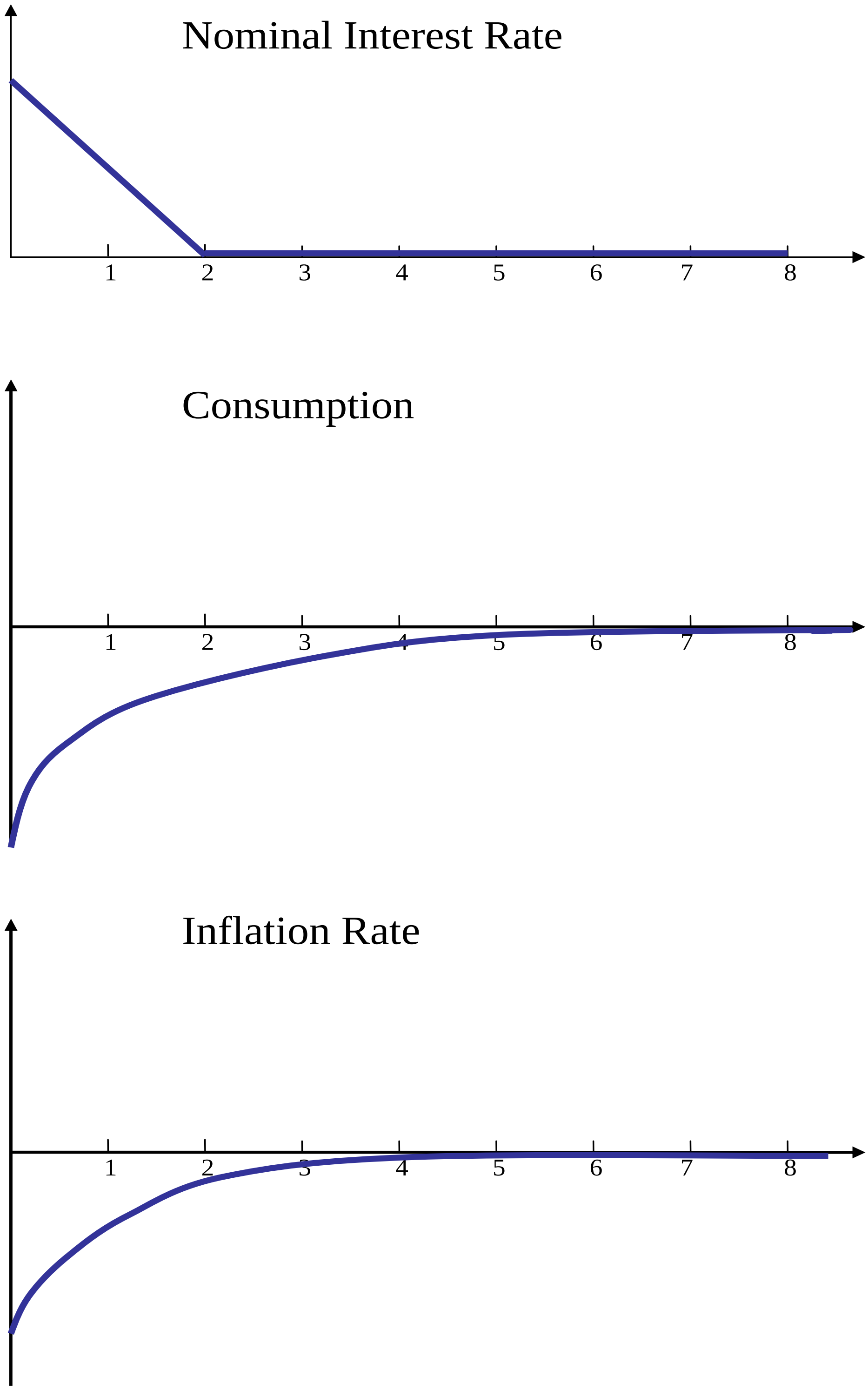

Figure 8: Impulse Responses to a Monetary Policy Shock in Standard Models

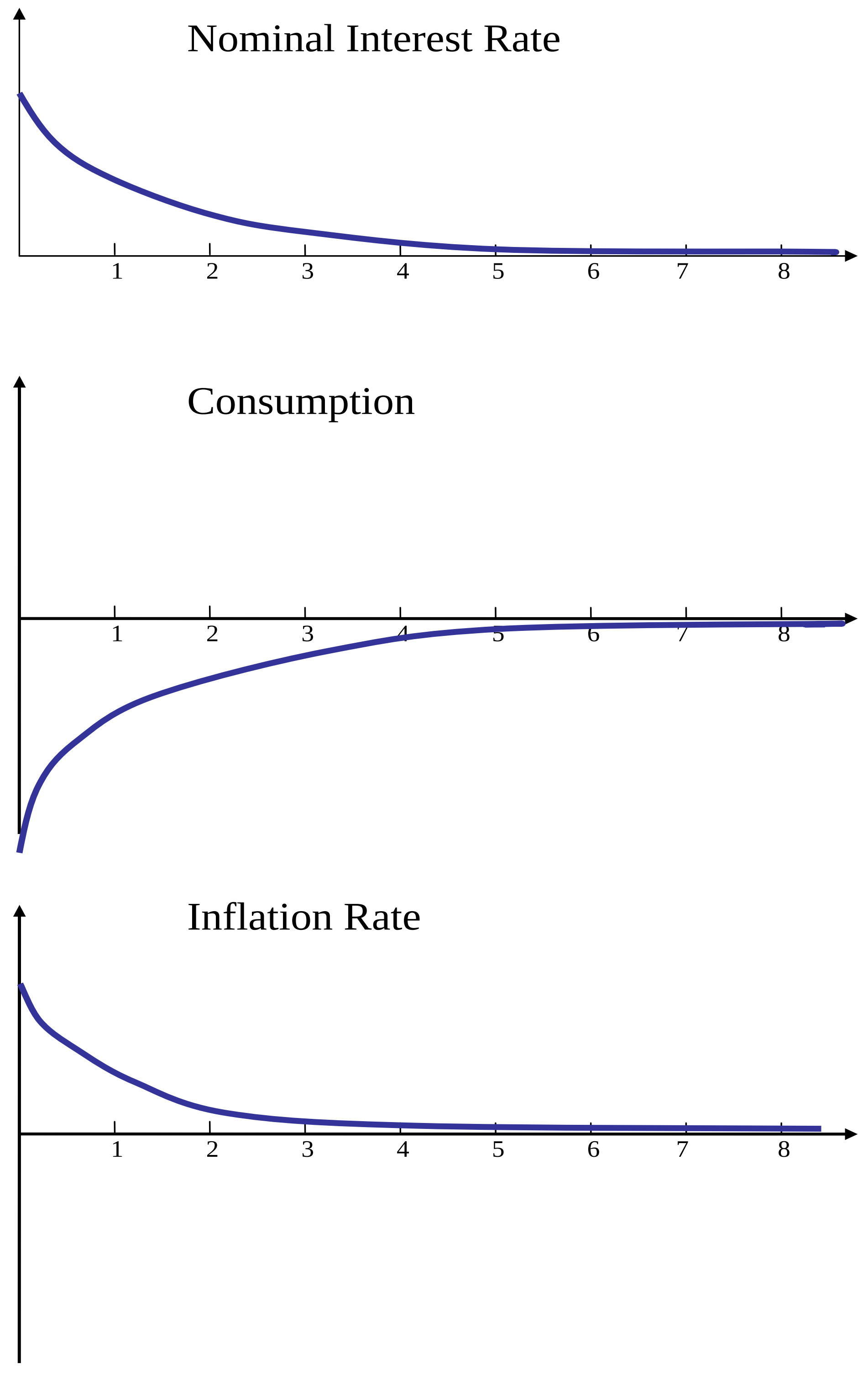

Figure 9: Impulse Responses to a Persistent Monetary Policy Shock in Standard Models

Appendix

Derivation of the Cost function

Given that $\tilde{m}_t(\underline{\theta}) = m_t$ and $\tilde{m}_t(\underline{\theta}) = 0$ the contract gives the household her reservation utility (see equation 7) if (ignoring time indices)

$$\begin{aligned} & -A + \underline{p}(m + \beta\underline{\theta} r(A - m) + \overline{p}\beta\overline{\theta} r A = 0 \\ \Leftrightarrow \quad & r = \frac{1}{\beta}\frac{A - \underline{p}m}{A - \underline{p}\underline{\theta}m} = \frac{1}{\beta}\frac{B^B}{B^B + \underline{p}(1 - \underline{\theta})m} \end{aligned}$$

Thus

$$\begin{aligned} r(B, m) &= \frac{1}{\beta}\frac{1}{1 + \gamma \cdot m/B} \quad \text{and} \\ C(B, m) &= \frac{1}{\beta}\frac{B}{1 + \gamma \cdot m/B} \end{aligned}$$

Proof of Proposition 1 I first calculate second derivatives.

$$\begin{aligned} C_{22}(B, m) &= \frac{1}{\beta}\frac{2\gamma^2}{B \cdot (1 + \gamma m/B)^3} \\ C_{12}(B, m) = C_{21}(B, m) &= \frac{1}{\beta}\frac{-2m\gamma^2}{B^2 \cdot (1 + \gamma m/B)^3} \\ C_{11}(B, m) &= \frac{1}{\beta}\frac{2m^2\gamma^2}{B^3 \cdot (1 + \gamma m/B)^3} \end{aligned}$$

Since $C_{22}(B, m) \cdot C_{11}(B, m) - C_{12}(B, m) \cdot C_{21}(B, m) = 0$ and $C_{11}(B, m) > 0$, the Hesse matrix of C has a positive and a zero eigenvalue. Thus C is convex.

Proof of Proposition 2 I first calculate the first derivatives of the cost function.

All time indices will be ignored.

$$
\begin{aligned}
C_B(B,m) &= \frac{1}{\beta}\frac{1+2\gamma m/B}{(1+\gamma m/B)^2} && (43)\\
C_m(B,m) &= \frac{1}{\beta}\frac{-\gamma}{(1+\gamma m/B)^2} && (44)
\end{aligned}
$$

Equilibrium conditions 13 and 14 (the bank's first order conditions) imply that

$$
\begin{aligned}
\frac{R^{CB}}{\underline{p}(R^{CB}-1)} &= \frac{C_K}{-C_m} = \frac{1+2\gamma m/B}{\gamma} \quad \text{and}\\
R^{CB} - \underline{p}(R^{CB}-1)m/B) &= \pi C_K + \pi C_m m/B) = \frac{\pi}{\beta}\frac{1}{1+\gamma m/B}
\end{aligned}
$$

Solving the first equation for m/B and the second for π results in:

$$
\begin{aligned}
\frac{m}{B} &= \frac{1}{2}\left(\frac{R^{CB}}{\underline{p}(R^{CB}-1)} - \frac{1}{\gamma}\right) \quad \text{and}\\
\pi &= \beta(1+\gamma\frac{m}{B})\cdot(R^{CB} - \underline{p}(R^{CB}-1)\frac{m}{B})
\end{aligned}
$$

Plugging in m/B into the last equation proves the claim:

$$
\begin{aligned}
\pi &= \beta\frac{1}{4}\left(\frac{\gamma R^{CB}}{\underline{p}(R^{CB}-1)} + 1\right)\left(R^{CB} + \frac{\underline{p}(R^{CB}-1)}{\gamma}\right) && (45)\\
&= \beta\frac{(R^{CB}\gamma + \underline{p}\cdot(R^{CB}-1))^2}{4\underline{p}\gamma(R^{CB}-1)} && (46)
\end{aligned}
$$

Proof of Proposition 3

For $(m_t(R^{CB},\ldots),\pi_t(R^{CB},\ldots))$ to be an equilibrium, it remains to be shown that $R_t^{CB}/\pi_t\beta < 1$ and $\beta r\overline{\theta} > 1$ (ex-post verification). Equilibrium condition 13 says that

$$
\frac{R^{CB}}{\pi} = C_B(B,m) \qquad (47)
$$

By equation 43 in the proof of proposition 2 $R_t^{CB}/\pi_t\beta < 1$ is equivalent to

$$\frac{1}{\beta}\frac{1+2\gamma m/B}{(1+\gamma m/B)^2} < \frac{1}{\beta} \tag{48}$$

The left hand side is smaller than $1/\beta$ thus $R_t^{CB}/\pi_t\beta < 1$.

Since

$$\begin{aligned} r_{t+1} &= \frac{1}{\beta}\frac{1}{1+\gamma\cdot m_t/B} \quad \text{and} && (49)\\ m_t &= \frac{1}{2}\left(\frac{R^{CB}}{\underline{p}(R^{CB}-1)} - \frac{1}{\gamma}\right)\cdot B && (50) \end{aligned}$$

it follows that

$$r = 2\frac{\underline{p}(R^{CB}-1)}{\beta(\underline{p}R^{CB}-\underline{p}+\gamma R^{CB})} \tag{51}$$

As

$$\overline{\theta} = \frac{1-\underline{p}+\gamma}{1-\underline{p}} \tag{52}$$

$$\beta r\overline{\theta} > 1 \tag{53}$$

is (after some simple algebra) equivalent to

$$R^{CB} > \frac{\underline{p}(1-\underline{p}+2\gamma)}{\underline{p}-\underline{p}^2-\gamma+3\underline{p}\gamma}, \tag{54}$$

what is exactly assumption 1.

Consistency of Assumption 1 and 2. An open interval exists that simultaneously fulfills both assumptions if and only if

$$\frac{\underline{p}(1-\underline{p}+2\gamma)}{\underline{p}-\underline{p}^2-\gamma+3\underline{p}\gamma} < \frac{p+2\gamma}{p+\gamma} \tag{55}$$

If

$$(3\underline{p}\gamma+\underline{p}-\underline{p}^2-\gamma) > 0 \tag{56}$$

holds then (55) is equivalent to

$$\phi(\underline{p}) := (2\gamma+\underline{p})(3\underline{p}\gamma+\underline{p}-\underline{p}^2-\gamma)-(\underline{p}+\gamma)\underline{p}(1+2\gamma-\underline{p}) > 0. \tag{57}$$

Since $\phi(1/2)=0$ and $\frac{\partial\phi}{\partial\underline{p}} = 4\gamma^2 > 0$ the claim for $\underline{p} > 1/2$ follows. Note, that (56) is fulfilled for $\underline{p} > 1/2$.

If (56) does not hold then (55) always holds. This is the case if $\underline{p} < \frac{1}{2}(3\gamma+1+(9\gamma^2+2\gamma+1)^{1/2})$.

Proof of Proposition 4 5, 6, 7 and 8 Since I have derived an explicit solution, the proof is basically calculating derivatives:

$$\begin{aligned}\frac{\partial\pi}{\partial R^{CB}} &= \beta\frac{2(\underline{p}R^{CB}-\underline{p}+R^{CB}\gamma)(\underline{p}+\gamma)(R^{CB}-1)-(\underline{p}R^{CB}-\underline{p}+R^{CB}\gamma)^2}{4\underline{p}\gamma(R^{CB}-1)^2} \\ &= \frac{\beta(\gamma R^{CB}+\underline{p}(R^{CB}-1))(R^{CB}\gamma-2\gamma+\underline{p}(R^{CB}-1))}{4\underline{p}\gamma(R^{CB}-1)^2}\end{aligned}$$

This implies that

$$\begin{aligned}
& \frac{\partial \pi}{\partial R^{CB}} < 0 \\
\Leftrightarrow \quad & R^{CB}\gamma - 2\gamma + \underline{p}(R^{CB} - 1) < 0 \\
\Leftrightarrow \quad & R^{CB} < \frac{\underline{p} + 2\gamma}{\underline{p} + \gamma}.
\end{aligned}$$

Thus assumption 2 makes sure that $\frac{\partial \pi}{\partial R^{CB}} < 0$.

Convexity of inflation (proposition 4) follows from taking second derivatives:

$$\frac{\partial^2 \pi}{\partial^2 R^{CB}} = \frac{\beta\gamma}{2\underline{p}(R^{CB} - 1)^3} > 0.$$

m_t/B_{t+1} is decreasing in R^{CB} as well:

$$\begin{aligned}
\frac{\partial m_t/B_{t+1}}{\partial R^{CB}} &= \frac{1}{2}\frac{\partial(\frac{R^{CB}}{\underline{p}(R^{CB}-1)} - \frac{1}{\gamma})}{\partial R^{CB}} \\
&= \frac{-1}{4(R^{CB} - 1)^2\underline{p}} \\
&< 0.
\end{aligned}$$

Since r_{t+1} is a decreasing function of m_t/B by equation 11, r_{t+1} increases. Concerning the nominal return to households it holds that (by plugging in the equilibrium expression for $r, m/B$ and π:

$$r_{t+1} \cdot \pi_{t+1} = \frac{1}{2}(R^{CB}_{t+1} + \frac{\underline{p}}{\gamma}(R^{CB}_{t+1} - 1)) \tag{58}$$

This expression is increasing in R^{CB}_{t+1} with derivative $\frac{\gamma+\underline{p}}{2\gamma} > 0$.

Derivations of Results for Implementing a new Inflation Target

The first order condition 33 yields a difference equation for π:

$$\pi_{t+1} = \frac{\kappa^2}{\lambda_x \beta}(\pi_t - \pi^*) + \frac{1+\beta}{\beta}\pi_t - \frac{1}{\beta}\pi_{t-1} \qquad \text{for } t \geq 1. \tag{59}$$

The solution to this equation is:

$$\pi_t = c\delta^t + k, \tag{60}$$

where $k = \pi^*$, $\delta = b/2 - \frac{\sqrt{b^2 - 4/\beta}}{2}$, $b = \frac{\kappa^2}{\beta \lambda_x} + \frac{1+\beta}{\beta}$ and c is determined to solve the initial condition $\pi_0 = \overline{\pi} = c + k$. This gives $c = \overline{\pi} - k$. Plugging the solution for π_t into the aggregate supply equation and solving for x_t completes the proof.

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