Fiscal Policy Under Weak Political Institutions*

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VERY PRELIMINARY AND INCOMPLETE VERSION

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Abstract

The purpose of this paper is to determine the normative and positive implications for fiscal policy in a weakly institutionalized economy which is not managed by a benevolent government, but is managed by a selfish dictator. We examine an economy with no capital, with fully state contingent financial instruments, and with exogenous stochastic government purchases. The dictator can use taxes and debt to extract rents, but dissatisfied households can threaten to replace him after his tenure with an equally selfish dictator. In contrast to the optimal tax rate under a benevolent government which is flat along the equilibrium path, we find that the optimal tax rate is history dependent and increasing along the equilibrium path. The reason is that the tax rate reflects the history of incentive compatibility constraints on the dictator. Providing the dictator with incentives to not steal imposes a limit on the size of government assets under his control and puts upward pressure on future tax rates, and in the long run, the tax rate reaches a maximum. Moreover, if we allow households to replace the dictator with a benevolent government at a cost, the tax rate can increase or decrease along the equilibrium path and can experience history dependence even in the long run. The reason is that providing incentives for the households to support the dictator imposes a limit on the size of government debt and puts downward pressure on future tax rates.

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1 Introduction

A central question in macroeconomics dating back to Ramsey (1927) concerns the optimal dynamics of fiscal policy. Specifically, how should the government structure taxes and debt to reallocate resources between households and the government? Current analyses assume the existence of a benevolent social planner with enough commitment devices to implement the optimal policy.[1] While this framework may be suitable for understanding optimal fiscal policy in a democracy, in many emerging economies with weak political institutions, taxes and debt are managed by a small elite who lack commitment power and who maximize their personal welfare as opposed to the welfare of the masses. This elite can use the fiscal apparatus of the state to extract rents, and the experiences of Pinochet in Chile and Mobutu in Zaire, for instance, are a testament to the fact that dictators often use the national treasury as their personal bank account.[2]

The purpose of this paper is to understand optimal fiscal policy under weak political institutions. From a positive perspective, this may help to explain the dynamics of fiscal policy in some emerging economies, and from a normative perspective, this will provide the best policy prescriptions for these economies. We consider the baseline model of Lucas and Stokey (1983) who examine an economy with no capital, with fully state contingent financial instruments, and with exogenous stochastic government purchases. We depart from their model by assuming that a selfish dictator controls taxes and debt, implements government projects, and extracts rents. Households can threaten the dictator by replacing him after his tenure with another identical dictator if they are dissatisfied, so that the best deviation by a dictator is to extract as many rents as possible prior to being kicked out of power.

Previous analyses have shown that under a benevolent government, the income tax rate should experience no history dependence (i.e., it should reflect current economic fundamentals) and should be almost flat along the equilibrium path.[3] The reason is that the government can smooth the risk associated with government purchase shocks via the optimal management of its bonds. Specifically, the government can accumulate assets

[1] For instance, see Barro (1979), Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1995), Aiyagari, Marcet, Sargent, and Seppala (2002), Angeletos (2002), and Werning (2006), among others.

[2] For a discussion of the consequences of the self-interested behavior of governments, see Buchanan and Tullock (1962), North (1981), Dixit (2004), Acemoglu, Johnson, and Robinson (2004), Persson and Tabellini (2000), among others. For a discussion of corruption and its required structure for the implementation of public goods, see Acemoglu and Verdier (2000), Banerjee(1997), Becker (1968), Becker and Stigler (1974), Shleifer and Vishny (1993), and Rose-Ackerman (1978).

[3] See Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1995), and Werning (2006). The quasi-linear utility function which we use is a special case of those discussed in Werning (2006).

in anticipation of shocks associated with high government liabilities and can accumulate debt in anticipation of shocks associated with low government liabilities.

In contrast, our study finds that under political constraints, the income tax rate experiences history dependence and is weakly increasing along the equilibrium path. The reason is that the tax rate must reflect the history of incentive compatibility constraints on the dictator, where these constraints impose an endogenous upper bound on the government's holdings of assets. This is tied to the fact that, off the equilibrium path, the dictator can extract the maximal level of revenue for himself and can abscond with all government assets, so that the provision of incentives for the dictator requires a limitation on the size of the assets under his control. This impedes the extent to which the economy can hedge against adverse shocks, which means that the economy must respond to a tightening of the incentive compatibility constraint on the dictator with a permanent increase in future revenues to offset government liabilities.

Moreover, in the long run, the tax rate is fixed, much like in an economy managed by a benevolent government. The reason is that the economy eventually adjusts to every single possible shock to the dictator's incentive compatibility constraint, and the stochastic level of assets under his control is no longer history dependent and is low enough to induce his cooperation.

We show using a numerical simulation that the movement in the tax rate along the equilibrium path is large. Furthermore, we show that the generation of rents as opposed to the limitations on government asset holdings represents the primary channel through which the political economy constraint affects social welfare. Nonetheless, we also show that choosing a fixed tax rate under a dictatorship as opposed to the optimally increasing tax rate will generate excess rents for the dictator and can produce a severe reductions in welfare.

Underlying our analysis is a commitment by households to support the dictator along the equilibrium path because households believe that the replacement dictator will proceed with the same or worse policies. This allows tax rates to increase along the equilibrium path so that the government accumulates more and more debt and households become worse off in the long run relative to the short run.

In order to consider a more realistic environment where households may actually be tempted to replace the current dictator, we expand the model to a setting where the dictatorship is under the threat of democratization. In every period, households can transition the economy into democracy at a cost, where a democracy is an absorbing state associated with a benevolent government with full commitment power. We show that support for the dictator in every period generates endogenous upper bounds on government debt, since high levels of debt are associated with high future tax rates and a low continuation utility to the household. As in our previous analysis, the tax rate continues to increase whenever the dictator must be induced to to stay in power. In contrast, however, the tax rate must decrease whenever households must be induced to keep the dictator in power. As a consequence, the tax rate can fluctuate and experience history dependence even in the long run.

Our model suggests that weakly institutionalized economies experience endogenous limitations on the size of the debt and assets which their governments can hold. Providing incentives for the dictator to not steal biases the government towards holding less assets, and providing incentives for the households to not democratize biases the government towards holding less debt. Furthermore, fiscal policy in such economies should be more volatile, and also more persistent since policies should not only reflect current economic fundamentals, but the history of economic fundamentals, since the history of economic fundamentals is tied to the history of incentive compatibility constraints underlying the political conflict in society. Moreover, our numerical simulations illustrate the fact that it is important to allow these policies to be more volatile and more persistent, since less volatile and less persistent policies would generate excess rents for the dictator.

Our paper is related to a number of different strands of research. This includes the large literature which cannot be summarized here on dynamic optimal taxation in the Ramsey setting with and without commitment.[4] The major difference between our work and all of these papers is that they assume the existence of a benevolent government and abstract from any political conflict. Second, our paper is related the literature examining the political economy of debt, though we depart from this work by focusing on the time path of policies as opposed to the level of debt and by focusing on the most efficient repeated game between citizens and a dictator as opposed to the Markovian interaction between different political parties.[5] Third, our paper is related to the political economy literature on the relationship between constraints on politicians and economic outcomes, but differs from current work by focusing on the government's role in managing aggregate risk and financing government purchases.[6] Finally, our paper is in the spirit of the approach to public finance taken by Acemoglu, Golosov, and Tsyvinski (2005) who study the role of political economy constraints in the dynamic Mirrleesian economy.

[4] In addition to previously mentioned papers, see Chari and Kehoe (1990,1993a,1993b), Phelan and Stacchetti (2001), and Sleet and Yeltekin (2004).

[5] For work regarding the political economy of debt, see Aghion and Bolton (1990), Persson and Svensson (1989), Battaglini and Coate (2006), Lizzeri (1999), and Alesina (1990).

[6] See Acemoglu (2005) and Persson and Tabellini (2000) for models relating constraints on politicians to economic outcomes.

The paper is organized as follows. Section 2 describes the baseline model. Section 3 defines and provides necessary and sufficient conditions for a sustainable competitive equilibrium. Section 4 characterizes the best sustainable competitive equilibrium and provides a numerical example. Section 5 extends the model to a setting where the dictator can be threatened by democratization. Section 6 concludes and the Appendix contains additional proofs not included in the text.

2 Model

Consider an economy where in every period, nature chooses the optimal size of government projects. Households choose one of two possible dictators to run the government. The dictator in power chooses policies which consist of a tax rate, state prices, a default decision, the size of government projects, and the size of rents. Markets open and households choose an allocation of consumption, labor, and state contingent claims subject to current prices and policies and their expectations of future prices and policies.

2.1 Economic Environment

The economy is infinitely lived with time periods $t = \{0, ..., \infty\}$ and with a stochastic state $s_t \in S \equiv \{1, ..., N\}$ with $N \geq 2$. The state determines the exogenous optimal size of government projects $g(s_t)$, where $0 \leq g(1) \leq ... \leq g(N)$.[7] s_t follows a first order Markov process, where $\Pr\{s_{t+1}|s_t\} = \pi(s_{t+1}|s_t)$ and where s_0 is degenerate without loss of generality. Let $s^t = \{s_0, ..., s_t\} \in S^t$ represent a history, and let $\pi\left(s^k|s^t\right)$ represent the probability of s^k conditional on s^t for $k \geq t$, where $\pi\left(s^k\right) = \pi\left(s^k|s^0\right)$.

There is a continuum of mass 1 of households that like sequences of consumption $\{c_t\}_{t=0}^{\infty}$, labor $\{n_t\}_{t=0}^{\infty}$, and government projects $\{g_t\}_{t=0}^{\infty}$ which increase

$$\mathbf{E}_0\left(\sum_{t=0}^{\infty}\beta^t\left(c_t - \eta\frac{n_t^{\gamma}}{\gamma} + z\left(g_t, s_t\right)\right)\right), \text{ where } \beta \in (0,1)\,, \eta > 0, \text{ and } \gamma > 1. \tag{1}$$

The assumption of risk neutrality and separability in consumption is important for the tractability of the result, since this will pin down state prices along the equilibrium path. Moreover, the utility function is isoelastic in labor since this will correspond to the case where the tax rate is fixed across all states of the world in an economy managed by a benevolent government (see Werning (2006)). All of our results generalize to any disutility

[7] None of our results change if we introduce productivity shocks.

of labor $v(n)$ which is increasing and convex.[8] We assume that

$$z(g_t, s_t) = \begin{cases} 0 & \text{if } g_t = g(s_t) \\ -Z & \text{otherwise} \end{cases},$$

where $Z > 0$ so that households benefit from government spending g_t being equal to the optimal spending $g(s_t)$. The fact that $z(g(s_t), s_t) = 0$ is simply a normalization which will allow us to ignore this term from hereon, since we will examine equilibria where optimal government projects are implemented, as is implied for the following assumption.

Assumption 1 *Z is arbitrarily large.*

At every date t, the household chooses an allocation $\omega_t = \left\{c_t, n_t, \{b_t(s_{t+1})\}_{s_{t+1}\in S}\right\}$ subject to

$$c_t + \mathbf{q}_t \cdot \mathbf{b}_t = (1-\tau_t)\, n_t + ((1-D_t)\max\{0, b_{t-1}(s_t)\} + \min\{0, b_{t-1}(s_t)\}), \tag{2}$$

$$b_t(s_{t+1}) \in \left[\underline{\mathbf{b}}, \overline{\mathbf{b}}\right], \; c_t \geq \underline{\mathbf{c}}, \text{ and } n_t \geq 0,$$

$$\text{where } \mathbf{q}_t \cdot \mathbf{b}_t = \sum_{s_{t+1}\in S} q_t(s_{t+1})\, b_t(s_{t+1}).$$

Equation (2) is the household's dynamic budget constraint. It means that consumption and the household's current portfolio (the left hand side of (2)) are financed by wages net of taxation, plus coupon payments to or from the government conditional on default (the right hand side (2)). τ_t is the tax rate, $b_t(s_{t+1})$ is the household's period t holding of a government bond which pays 1 unit of consumption at $t+1$ conditional on the realization of s_{t+1}, and $q_t(s_{t+1})$ is the price of such a bond.[9] In contrast to the standard economy, we have introduced $D_t = \{0, 1\}$ which is the default decision by the government with 1 denoting default. This means that if households expect default for all s_{t+1}, they will never lend to the government at t. We have excluded the possibility of trading claims with a richer maturity structure without any loss of generality and only to simplify notation.[10] We take b_{-1} as exogenous, and merely to reduce notation and without any bearing on our results, we assume that $b_{-1} = 0$.[11]

For technical reasons, there are bounds on asset holdings, which ensure that neither

[8] In this circumstance, Theorems 1 and 2 are with respect λ which is a state variable which parameterizes the tax rate as a *function* of government purchases.

[9] Even with non-contingent debt, there are other ways of replicating a complete market portfolio using riskless bonds of different maturities, as discussed in Angeletos (2002) and Buera and Niccolini (2004)

[10] Details available upon request.

[11] If b_{-1} is positive, it is always optimal for the government to default in the first period, so that the equilibrium is equivalent to $b_{-1} = 0$.

consumers nor the government will use Ponzi schemes. We will examine equilibria where the range $[\underline{\mathbf{b}}, \overline{\mathbf{b}}]$ for $\underline{\mathbf{b}} < 0 < \overline{\mathbf{b}}$ is arbitrarily large so that these limits do not bind along the equilibrium path. Consumption must also be above some lower bound $\underline{\mathbf{c}}$. We add the following assumption which ensures that a sustainable competitive equilibrium exists following *every* possible policy choice.

Assumption 2 $\underline{\mathbf{b}} \geq \underline{\mathbf{c}}$.

This assumption implies that if a households expect zero wages net of taxation and cannot borrow, that it can always consume a low enough level of resources in order to pay off its debt.[12]

2.2 Political Environment

After s_t is realized, households choose the dictator labeled by i_t, where $i_t = \{1, 2\}$ and where our results depend on the existence of at least one replacement dictator with whom to threaten yesterday's dictator.[13] We impose sincere voting by all households. Let $P_t^j = \{0, 1\}$ equal 1 if $i_t = j$, so that j is in power at t. Dictator j likes a sequence of power $\{P_t^j\}_{t=0}^{\infty}$ and a sequence of rents $\{x_t\}_{t=0}^{\infty}$ which increase

$$\mathbf{E}_0 \left(\sum_{t=0}^{\infty} \beta^t \left(P_t^j x_t + \left(1 - P_t^j\right) \underline{\mathbf{x}} \right) \right),$$

where we have set the flow utility of being out of power to $\underline{\mathbf{x}}$. Since we are interested in a setting where the income tax rate is positive, we assume that $g(1) + \underline{\mathbf{x}} \geq 0$.

The dictator in power sets policies $\rho_t = \left\{ \tau_t, D_t, \{q_t(s_{t+1})\}_{s_{t+1} \in S}, g_t, x_t \right\}$ which must satisfy

$$g_t + x_t - \mathbf{q}_t \cdot \mathbf{b}_t = \tau_t n_t - \left((1 - D_t) \max\{0, b_{t-1}(s_t)\} + \min\{0, b_{t-1}(s_t)\} \right), \tag{3}$$
$$\text{and } g_t \geq 0.$$

Equation (3) means that government projects, rents, and the current government portfolio (the left hand side of (3)) are financed by tax revenue and coupon payments to or from the households conditional on default (the right hand side of (3)). Because we consider

[12]Without this assumption, it becomes important to consider punishments for policy choices under which sustainable competitive continuation equilibria exist, which significantly complicates the current analysis.

[13]Frictions into the selection process can be introduced without loss of generality. For instance, it is possible that dictator 1 is accidentally selected when households choose 2 and vice versa. Details available upon request.

a sustainable equilibrium as defined by Chari and Kehoe (1990,1993a,1993b), the government chooses state prices as opposed to the size of government bonds. After policies are chosen, markets open and clear.[14] As an aside, note that if a dictator j in power at t expects never to return to power in the future ($P_k^j = 0$ for all $k > t$), then he will choose policies so as to maximize the rents x_t which he can take with him before being kicked out. Note that without loss of generality, we could allow for various frictions which hinder the efficiency of rent generation.[15]

2.3 Repeated Game Interaction

As in the analysis Chari and Kehoe (1990,1993a,1993b), the interaction between households and the two possible dictators resembles a game. Moreover, households are small in their private economic behavior, but large in their public choice of dictator. The interaction takes the following form:

1. Nature chooses the state s_t.
2. Households choose the dictator i_t.
3. Dictator i_t chooses policies ρ_t.
4. Markets open and households choose labor, consumption, and savings.

3 Sustainable Competitive Equilibria

In this section, we provide a formal definition of a sustainable competitive equilibrium which builds on the work of Chari and Kehoe (1990,1993a,1993b). Using the primal approach, we reduce the dimensionality of the problem so as to focus on a sequence of consumption, labor, government purchases, and rents which constitute a competitive equilibrium. To check if such a sequence is sustainable, we begin by describing a sustainable and competitive continuation equilibrium where the current dictator is permanently thrown out of power starting from tomorrow and where future governments permanently default. We argue that this continuation equilibrium constitutes the optimal punishment, and this provides us with an incentive compatibility constraint on the dictator which must hold in every period. We discuss how this constraint imposes an endogenous upper bound on the size of government assets.

[14] One can easily make x_t unobservable since households can always infer its value by observing c_t, g_t, and n_t.

[15] For instance, one can let x_t on the left hand side of (3) be replaced by θx_t for $\theta > 1$.

3.1 Definition

At the beginning of the period t, households choose the dictator as a function of the history $h_t^0 = \{s^t, \rho^{t-1}, i^{t-1}\}$ together with a contingency plan for choosing future dictators for all possible future histories. Let $\Sigma_t(h_t^0)$ represent the replacement decision. Following this decision, the dictator of identity $j = i_t$ chooses a policy as a function of the history together with a contingency plan for setting future policies for all possible future histories. Let $h_t^1 = \{h_t^0, i_t\}$, and let $\sigma_t^j(h_t^1)$ denote a dictator j's time t choice of policy ρ_t which consists of a tax rate, state contingent prices, a default decision, size of government projects, and size of rents, where it is clear that $\sigma_t^j(h_t^1)$ is empty if $j \neq i_t$, so that j is out of power. After the dictator sets current policy, households make their private market decision. Faced with a history $h_t^2 = \{h_t^1, \rho_t\}$, households privately choose time t allocation together with a contingency plan for choosing future allocations. Let $f_t(h_t^2)$ denote the household's choice of ω_t, which consists of a choice over consumption, labor, and holdings of state contingent claims. To understand why public actions are not conditioned on the household's private decisions but only on policies and replacement decisions which are public, see Chari and Kehoe (1990).

In order to define a sustainable equilibrium, we need to explain how the replacement rule and policy plan induce future histories. Given h_t^0, the replacement rule Σ induces a history $h_t^1 = \{h_t^0, \Sigma_t(h_t^0)\}$, and given h_t^1, the policy plan σ^j for $i_t = j$ induces a history $h_t^2 = \{h_t^1, \sigma_t^j(h_t^1)\}$ and a history $h_{t+1}^0 = \{h_t^1, \sigma_t^j(h_t^1), s_{t+1}\}$, and so on. Given a history h_t^0 and policy plans σ^1 and σ^2, a continuation policy of Σ is $\left\{\Sigma_t(h_t^0), \Sigma_{t+1}\left(h_t^0, \sigma_t^{i_t(h_t^0)}(h_t^0, \Sigma_t(h_t^0)), s_{t+1}\right), ...\right\}$ where $\sigma_t^{i_t(h_t^0)}(h_t^1)$ represents the politicies implemented by the dictator $i_t(h_t^0)$. Given a history h_t^1, a replacement rule Σ, and a policy plan σ^{-j}, a continuation policy of σ^j is $\{\sigma_t^j(h_t^1), \sigma_{t+1}^j(h_t^1, s_{t+1}, \Sigma_{t+1}(h_t^1, \sigma_t^{i_t}(h_t^1), s_{t+1})), ...\}$. Finally, given a history h_t^2, a replacement rule Σ, and policy plans σ^1 and σ^2, a continuation allocation f must be $\left\{f_t(h_t^2), f_{t+1}\left(h_t^2, s_{t+1}, \Sigma_{t+1}(h_t^2, s_{t+1}), \sigma_{t+1}^{i_{t+1}(h_t^2)}(h_t^2, s_{t+1}, \Sigma_{t+1}(h_t^2, s_{t+1}))\right), ...\right\}$.

Consider the situation of the dictator in power at t. Given some history h_t^1 and given that future replacement decisions and allocations evolve according to Σ and f, respectively, the dictator j in power chooses a continuation policy that maximizes:

$$V_t^j\left(h_t^1;\Sigma,\sigma^1,\sigma^2,f\right)=\mathbf{E}\left\{\sum_{k=t}^{\infty}\beta^{k-t}\left(P^j\left(h_k^0\right)x\left(h_k^1\right)+\left(1-P^j\left(h_k^0\right)\right)\underline{\mathbf{x}}\right)|h_t^1,\Sigma,\sigma^1,\sigma^2,f\right\},$$

s.t.

$$g_k\left(h_k^1\right)+x_k\left(h_k^1\right)-\mathbf{q}_k\left(h_k^1\right)\cdot\mathbf{b}_k\left(h_k^2\right)=$$
$$\tau_k\left(h_k^1\right)n_k\left(h_k^2\right)-\left(\left(1-D\left(h_k^1\right)\right)\max\left\{0,b_{k-1}\left(s_k\right)\left(h_{k-1}^2\right)\right\}+\min\left\{0,b_{k-1}\left(s_k\right)\left(h_{k-1}^2\right)\right\}\right),$$
$$\text{and } g_k\left(h_k^1\right)\geq 0.$$

where for all $k\geq t$, future history are induced by Σ,σ^1, and σ^2 from h_t^1.

Next consider a private agent in period t. Given some history h_t^2, and given that future policies evolve according to σ^1 and σ^2 and replacement rules evolve according to Σ, a household chooses a continuation allocation to maximize:

$$W\left(h_t^2;\Sigma,\sigma^1,\sigma^2,f\right)=\mathbf{E}\left\{\sum_{k=t}^{\infty}\beta^{k-t}\left[c_k\left(h_k^2\right)-\eta\frac{n_k\left(h_k^2\right)^{\gamma}}{\gamma}+z\left(g_k\left(h_k^1\right),s_k\right)\right]|h_t^1,\Sigma,\sigma^1,\sigma^2,f\right\}$$

s.t.

$$c_t\left(h_t^2\right)+\mathbf{q}_t\cdot\mathbf{b}_t\left(h_t^2\right)=\left(1-\tau_t\right)n_t\left(h_t^2\right)+\left(\left(1-D_t\right)\max\left\{0,b_{t-1}\left(s_t\right)\left(h_{t-1}^2\right)\right\}+\min\left\{0,b_{t-1}\left(s_t\right)\left(h_{t-1}^2\right)\right\}\right)$$
$$\text{and } b_t\left(s_{t+1}\right)\left(h_t^2\right)\in\left[\underline{\mathbf{b}},\overline{\mathbf{b}}\right],\ c_t\left(h_t^2\right)\geq\underline{\mathbf{c}},\text{ and } n_t\left(h_t^2\right)\geq 0,$$
$$\text{and for } k>t,$$
$$c_k\left(h_k^2\right)+\mathbf{q}_k\left(h_k^1\right)\cdot\mathbf{b}_k\left(h_k^2\right)=$$
$$\left(1-\tau_k\left(h_k^1\right)\right)n_k\left(h_k^2\right)+\left(\left(1-D\left(h_k^1\right)\right)\max\left\{0,b_{k-1}\left(s_k\right)\left(h_{k-1}^2\right)\right\}+\min\left\{0,b_{k-1}\left(s_k\right)\left(h_{k-1}^2\right)\right\}\right),$$
$$\text{and } b_k\left(s_{k+1}\right)\left(h_k^2\right)\in\left[\underline{\mathbf{b}},\overline{\mathbf{b}}\right],\ c_k\left(h_k^2\right)\geq\underline{\mathbf{c}},\text{ and } n_k\left(h_k^2\right)\geq 0,$$

where ρ_t is given in h_t^1 and for $k>t$ all future histories are induced by Σ, σ^1, and σ^2 from h_t^2.

Finally, consider the public decision by households to replace the dictator. Given some history h_t^0 and given that future policies and allocations evolve according to σ^1, σ^2, and f, households collectively choose dictator i_t versus dictator $-i_t$ if

$$W\left(h_t^0,i_t,\sigma_t^{i_t}\left(h_t^0,i_t\right);\Sigma,\sigma^1,\sigma^2,f\right)\geq W\left(h_t^0,-i_t,\sigma_t^{-i_t}\left(h_t^0,-i_t\right);\Sigma,\sigma^1,\sigma^2,f\right)\qquad(4)$$

where all future histories are induced by Σ, σ^1, and σ^2 from h_t^0. Note that (4) will trivially hold if $\sigma_t^{i_t}\left(h_t^0,i_t\right)=\sigma_t^{-i_t}\left(h_t^0,-i_t\right)$, so that we can easily ignore this constraint if the dictators have symmetric strategies. This will not be true however in Section 5 since households will be able to replace the dictator with a benevolent social planner at a cost.

Definition 1 *A sustainable competitive equilibrium is a quadruplet* $\{\Sigma, \sigma^1, \sigma^2, f\}$ *that satisfies the following conditions:*

1. *Given* $\{\sigma^1, \sigma^2, f\}$, Σ *solves the household's political problem for every history* h_t^0.
2. *Given* $\{\sigma^j, f, \Sigma\}$, σ^j *solves dictator* j*'s problem for every history* h_t^1 *for* $j = 1, 2$.
3. *Given* $\{\sigma^1, \sigma^2, \Sigma\}$, f *solves the household's market problem for every history* h_t^2.

Notice that as in Chari and Kehoe (1990,1993a,1993b), we require that dictators and households act optimally for every history of policies, even those which are not induced by any strategy or those which have violated feasibility.

3.2 Competitive Equilibria

Along the equilibrium path of any sustainable competitive equilibrium, the household's allocation will be a function of the equilibrium policy sequence $\rho = \{\rho(s^t)\}_{t=0}^{\infty}$. We therefore begin by characterizing a competitive equilibrium under ρ, where $q(s^{t+1}|s^t)$ represents the price of a security indexed to history s^{t+1} traded at history s^t, and $b(s^{t+1}|s^t)$ is defined analogously. Let $\tau(s^t)$ represent the tax rate chosen at s^t and define other variables analogously. First order conditions for the household yields:

$$\eta n\left(s^t\right)^{\gamma-1} = \left(1-\tau\left(s^t\right)\right) \tag{5}$$

$$b\left(s^{t+1}|s^t\right) = \begin{cases} \underline{\mathbf{b}} & \text{if } q\left(s^{t+1}|s^t\right) > \beta\pi\left(s^{t+1}|s^t\right) \\ \widetilde{b} \in \left[\underline{\mathbf{b}}, \overline{\mathbf{b}}\left(1-D\left(s^{t+1}\right)\right)\right] & \text{if } q\left(s^{t+1}|s^t\right) = \beta\pi\left(s^{t+1}|s^t\right) \\ \overline{\mathbf{b}}\left(1-D\left(s^{t+1}\right)\right) & \text{if } q\left(s^{t+1}|s^t\right) < \beta\pi\left(s^{t+1}|s^t\right) \end{cases} \tag{6}$$

Equation (5) is the standard intratemporal condition, and equation (6) is an intertemporal condition which takes the government's default decisions into account. Specifically, if the government is expected to default in the future, households will never lend to the government today.

Now consider an equilibrium where the debt limits do not bind, and imagine if default never occurs. The debt limits imply the following transversality condition:

$$\lim_{k\to\infty} q\left(s^k|s^t\right) b\left(s^k|s^{k-1}\right) = 0. \tag{7}$$

where $q\left(s^k|s^t\right) = q\left(s^k|s^{k-1}\right) \times ... \times q\left(s^{t+1}|s^t\right)$. One can multiply both sides of (3) by $q\left(s^k|s^t\right)$ for $k \geq t$ and take the sum of all sequential budget constraints (3) for $k \geq t$ subject to (7) to achieve the present value budget constraint for the government at every node s^t:

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} q\left(s^k|s^t\right) \left[\tau\left(s^k\right) n\left(s^k\right) - g\left(s^k\right) - x\left(s^k\right)\right] = b\left(s^t|s^{t-1}\right).$$

This expression means that the government's debt must equal the total stream of primary surpluses run by the government. The substitution of (5) and (6) yields:

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k|s^t\right) \left(R\left(n\left(s^k\right)\right) - g\left(s^k\right) - x\left(s^k\right)\right) = b\left(s^t|s^{t-1}\right), \tag{8}$$

$$\text{where } R\left(n\left(s^k\right)\right) = n\left(s^k\right) - \eta n\left(s^k\right)^{\gamma}.$$

$R\left(n\left(s^k\right)\right)$ is a function which determines total revenue as a function of the allocation of labor given (5). (8) can be written for $t = 0$ as:

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) \left[R\left(n\left(s^t\right)\right) - g\left(s^t\right) - x\left(s^t\right)\right] = 0. \tag{9}$$

In the proof of Proposition 1, we show that (8) and (9) are also necessary in an economy which experiences some default. The resource constraint of the economy can be derived by taking the sum of (2) and (3):

$$c\left(s^t\right) + g\left(s^t\right) + x\left(s^t\right) = n\left(s^t\right) \;\; \forall s^t. \tag{10}$$

Expressions (9) and (10) have the advantage of being in terms of allocations as opposed to prices and asset levels which is why they are useful in constructing a competitive equilibrium. Let $c = \{c\left(s^t\right)\}_{t=0}^{\infty}$ and define other sequences of variables analogously. We can prove that (9) and (10) are necessary and sufficient for a competitive equilibrium sequence $\{c, n, g, x\}$.

Proposition 1 *An allocation* $\{c, n, g, x\}$ *constitutes a competitive equilibrium without binding debt limits if and only if it satisfies* (9) *and* (10) *and if* $\underline{\mathbf{b}}$ *is sufficiently low and* $\overline{\mathbf{b}}$ *is sufficiently large.*

Proof. *See Appendix.* ■

3.3 Worst Sustainable Competitive Equilibrium

For a competitive equilibrium sequence ρ to be sustainable, it is necessary that the dictator does not wish to deviate from it given the households' replacement rule. In this section, we describe a particular punishment which can be used, and we will later show that this punishment is optimal. Specifically, we can show that following any current policy ρ_t, there exists a continuation equilibrium which is sustainable and competitive where a dictator is permanently thrown out of power and where households expect permanent default by all future governments.

To this end, define the most extractive tax rate $\tau^{\max}$ associated with the most extractive labor allocation $n^{\max}$ which solves:

$$R^{\max} = \max_{n} R\left(n\right).$$

For a given outstanding asset position b, define a policy $\widehat{\rho}$ as

$$\widehat{\rho} = \left\{\widehat{\tau}, \widehat{D}, \left\{\widehat{q}\left(s\right)\right\}_{s\in S}, \widehat{g}, \widehat{x}\right\} = \left\{\tau^{\max}, 1, \left\{0\right\}_{s\in S}, 0, R^{\max} + \max\left\{0, -b\right\}\right\}.$$

This policy consists of the dictator extracting as much tax revenue as possible, defaulting, not implementing government projects, and using all of the government's resources as rents. Let $-i_t$ represent the identity of the dictator who is not in power at t.

Proposition 2 *For any $\left\{s^t, b_{t-1}\left(s_t\right), \rho_t, i_t\right\}$, there exists a sustainable competitive continuation equilibrium where $\rho\left(s^k\right) = \widehat{\rho}$ and $i\left(s^k\right) = -i_t$ for all $k > t$ and $\pi\left(s^k|s^t\right) > 0$.*
Proof. *See Appendix.* ■

Corollary 1 *For any sustainable competitive equilibrium where $\rho\left(s^k\right) = \widehat{\rho}$ and $i\left(s^k\right) = -i_t$ for all $k > t$ and $\pi\left(s^k|s^t\right) > 0$, it is necessary that $\rho\left(s^t\right) = \widehat{\rho}$, yielding a continuation value to i_t at s^t equal to*

$$R^{\max} + \max\left\{0, -b\left(s^t|s^{t-1}\right)\right\} + \beta\frac{\mathbf{x}}{1-\beta}. \tag{11}$$

Proof. *See Appendix.* ■

Proposition 2 establishes the existence of a sustainable competitive equilibrium where the current dictator is permanently replaced, and the replacement dictator is is maximally extractive forever. Households accept this replacement dictator, because they are indifferent between maintaining him and replacing him, since any dictator in power is always maximally extractive off the equilibrium path. Moreover, off the equilibrium path, the dictator is limited by the household's expectation of future default, so that he cannot extract resources by borrowing, but he can only do so by extracting revenue, collecting household claims, and not implementing government projects. This explains Corollary 1 which establishes that $\widehat{\rho}$ represents the best response of a dictator expecting to be permanently kicked out of power and replaced with a government which permanently defaults.

As an aside, note that the described punishment is by no means the unique method of providing the dictator with a continuation value equal to (11). One can instead imagine a scenario where the current dictator is allowed to stay in power forever where he can receive a flow utility equal to $(1-\beta)\, R^{\max} + (1-\beta) \max\{0, -b\,(s^t|s^{t-1})\} + \beta \underline{\mathbf{x}}$ forever. In such a circumstance, extraction of resources from the national treasury would be more gradual and household debt would be rolled over in every period.

3.4 Sustainable Competitive Equilibria

In the above section, we describe a particular sustainable competitive continuation equilibrium which could be potentially used as a punishment for a deviating dictator. Using the methods of Abreu (1988), we now show that this corresponds to the optimal punishment which means that the best deviation by the dictator in power yields (11). The following assumption ensures that the maximal tax revenue which can be extracted weakly exceeds the flow utility achieved by a dictator who is out of power.

Assumption 3 $R^{\max} \geq \underline{\mathbf{x}}$.

This assumption means that a dictator always prefers being in power to being out of power since he can always extract at least $R^{\max}$ for himself.

Proposition 3 *An allocation* $\{c, n, g, x\}$ *constitutes a sustainable competitive equilibrium without binding debt limits if and only if it satisfies* (9) *and* (10)*, if* $\underline{\mathbf{b}}$ *is sufficiently low and* $\overline{\mathbf{b}}$ *is sufficiently large, and if*

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k|s^t\right) x\left(s^k\right) \geq R^{\max} + \max\left\{0, -b\left(s^t|s^{t-1}\right)\right\} + \beta \frac{\underline{\mathbf{x}}}{1-\beta} \;\forall s^t, \tag{12}$$

$$\textit{where } b\left(s^t|s^{t-1}\right) \textit{ satisfies } (8).$$

Proof. *Proposition 1 establishes the necessity and sufficiency of* (9) *and* (10) *under a large enough range* $\left[\underline{\mathbf{b}}, \overline{\mathbf{b}}\right]$ *and it establishes* (8) *under any competitive equilibrium. For the necessity of* (12)*, a dictator's strategy of choosing* $\widehat{\rho}$ *whenever in power achieves a* *payoff weakly larger than the right hand side of* (12) *by Assumption 3, and a dictator can never achieve a payoff strictly larger than the left hand side of* (12)*. For sufficiency, consider the following equilibrium where* $i\left(s^k\right) = 1\ (= 2)\ \forall s^k$*. Given a prescribed sequence of policies* ρ*, any deviation from* ρ *at* t *results in* $\rho\left(s^k\right) = \widehat{\rho}$ *and* $i\left(s^k\right) = 2\ (= 1)$ *for all* $k > t$ *and* $\pi\left(s^k|s^t\right) > 0$*, which is a sustainable competitive continuation equilibrium by Proposition 2. Corollary 1 establishes that the best deviation by the dictator in power is to choose* $\rho_t = \widehat{\rho}$ *which yields a payoff equal to the right hand side of* (12)*, which makes dictator* 1 (2) *weakly worse off than following* ρ*. To ensure sustainability of the equilibrium from the perspective of the households, imagine that if* $i_k \neq 1\ (\neq 2)$*, then the replacement dictator chooses* ρ *forever, which satisfies* (4)*.* ■

The intuition for Proposition 3 is that the dictator should prefer remaining in power in the future to extracting the maximal revenue and being kicked out of power today. Because households expect default by the future government off the equilibrium path, the dictator cannot borrow off the equilibrium path, so that the best he can do prior to being kicked out is to extract as much tax revenue as possible, default, not implement government projects, and use all of the government's resources as rents. In a sustainable competitive equilibrium, the same dictator stays in power forever along the equilibrium path, since this maximizes his value of power (equal to the left hand side of (12)), and households support this dictator since they are indifferent between maintaining him and replacing him.[16] Why does there not exist a punishment which provides a continuation value below the right hand side of (12)? Because choosing $\widehat{\rho}$ is always an option while in power, and this yields a payoff always weakly greater than the right hand side of (12).

To gain an insight into (12), note that it can be combined with (8) to yield two inequalities:

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k|s^t\right) x\left(s^k\right) \geq R^{\max} + \beta \frac{\mathbf{x}}{1-\beta}\ \forall s^t, \tag{13}$$

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k|s^t\right) R\left(n\left(s^k\right)\right) \geq G\left(s^t\right) + R^{\max} + \beta \frac{\mathbf{x}}{1-\beta}\ \forall s^t, \tag{14}$$

$$\text{where } G\left(s^t\right) = \sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k|s^t\right) g\left(s^t\right).$$

Equations (13) and (14) illustrate the fact that there are two ways in which incentives are provided for the dictator. The first way illustrated in (13) is through the provision of rents

[16] In Section 5, we can relax this by allowing households to replace the dictator with a benevolent government.

which strains the economy through the resource constraint. The second way illustrated in (14) is by reducing the asset holdings of the government in order to leave little for the dictator to steal. Note that while increasing the stream of rents increases the value of power, it also increases the amount of assets held by the government to pay for these rents (that is, holding future taxes constant). Under our assumption of risk neutrality, these two effects cancel each other out and yield equation (14) , which provides an lower bound for the stream of primary government surpluses gross of rents–i.e., the sequence of $\tau n - g$. One can see then that equation (14) derived from a political interaction between households and a dictator resembles an ad-hoc asset limit in an economy managed by a benevolent social planner.

4 Best Sustainable Competitive Equilibrium

Proposition 3 provides conditions under which a sequence $\{c, n, g, x\}$ constitutes a sustainable competitive equilibrium. In this section, we characterize the best sustainable competitive equilibrium. We show that the optimum can be constructed by providing the dictator with a constant stream of rents such that (13) binds, and this allows us to ignore constraint (9) which is redundant. We then write the problem recursively, and we show that the optimal policy takes a simple form where today's tax rate is the same as yesterday's, unless it is below some state dependent lower bound, in which event it experiences a discontinuous upward jump. We illustrate the mechanics of the model using numerical example, and we discuss welfare implications.

4.1 Optimal Policy

Proposition 3 implies that the best sustainable competitive equilibrium solves the following program:

$$\max_{\{c,n,g,x\}} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) \left[c\left(s^t\right) - \eta \frac{n\left(s^t\right)^{\gamma}}{\gamma} + z\left(g\left(s^t\right), s_t\right)\right] \tag{15}$$

$$\text{s.t.} \quad (9), (10), (13), \text{and} \quad (14). \tag{16}$$

We ignore the welfare of the dictator only for expositional simplicity. All of our results will continue to hold if we add an additional constraint that the left hand side of (12) exceeds some exogenous amount, so that we are simultaneously maximizing the welfare of the household and the welfare of the dictator.[17]

Assumption 1 means that if it exists, a solution to (15) sets

$$g\left(s^t\right) = g\left(s_t\right), \tag{17}$$

so that optimal government projects are implemented. We assume that such an equilibrium exists, and so we consider equilibria where (17) holds.[18] Note that as a consequence of (17),

$$G\left(s^t\right) = G\left(s_t\right),$$

so that the present discounted value of government liabilities is state dependent.

The next two lemmas drastically simplify the problem. First, define

$$n^{fb} = \gamma^{1/(\gamma-1)} n^{\max},$$

which corresponds to the first-best allocation of labor under zero taxes. Note that $R(n)$ is globally concave and decreasing in the interval $\left[n^{\max}, n^{fb}\right]$.

Lemma 1 $\{c, n, g, x\}$ *is a solution to* $(15)-(16)$ *if and only if it is a solution to* (15) *s.t.* $(10), (13), (14)$, *and*

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) \left[R\left(n\left(s^t\right)\right) - g\left(s_t\right) - x\left(s^t\right)\right] \geq 0, \tag{18}$$

Proof. *See Appendix.* ■

Lemma 2 *There exists a solution to* $(15)-(16)$ *which sets*

$$x\left(s^t\right) = x^* = (1-\beta) R^{\max} + \beta \underline{\mathbf{x}} \ \forall s^t. \tag{19}$$

Proof. *See Appendix* ■

Lemma 1 means that constraint (9)–the present value budget constraint of the government–is equivalent to its relaxed form. This follows from the fact that we are examining equi-

[17] The only difference becomes that Theorem 1 is modified so that $\tau_0 = \tau^* \geq \tau(s_0)$ where τ^* is large enough to promise a particular continuation value to the dictator.

[18] It can easily be shown that a necessary and sufficient condition for existence is:

$$\beta \frac{R^{\max}}{1-\beta} \geq \max_{s \in S} G(s) + \beta \frac{\underline{\mathbf{x}}}{1-\beta},$$

which establishes that the economy can generate enough revenue to satisfy (14).

libria with positive tax rates and that an equilibrium can always be improved by reducing the tax rate, which reduces revenues, so that (18) binds. As a consequence of Lemma 1, the optimal choice of n must be included in the efficient range of allocations $\left[n^{\max}, n^{fb}\right]$, where utility is increasing and revenue is decreasing in n over this interval.

To understand Lemma 2, note that due to the strict concavity of the program in n and the convexity of the constraint set, the optimum will admit a *unique* sequence n, though there may be multiple combinations of c and x which constitute an optimum. Because households are risk neutral in their consumption and the dictator is risk neutral in his rent, we can easily choose a sequence x such that the dictator receives a constant stream of rents, which leads to Lemma 2, which uses the fact that it is optimal to make rents as low as possible so that (13) binds in every period.[19]

Notice that using these two lemmas, one can substitute x^* into (18), which yields a constraint which is identical to constraint (14) at $t = 0$, so that it is redundant. Therefore, we can rewrite the program $(15) - (16)$ using the resource constraint (10) as:

$$\max_{n} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) \left[n\left(s^t\right) - \eta \frac{n\left(s^t\right)^{\gamma}}{\gamma} - x^* - g\left(s_t\right)\right] \text{ s.t. } (14) \text{ where } G\left(s^t\right) = G\left(s_t\right) \tag{20}$$

where we have ignored the subtraction of $x^* + g\left(s_t\right)$ since they enter additively and do not affect the optimal solution. Given our definition of x^*, (14) is now functioning as a constraint which forbids the government from holding any assets.[20]

Define $J\left(s^t\right)$ as a state variable equal to the left hand side of (14), so that it represents the present discounted value of future revenues. Furthermore, let $Q\left(s, J\right)$ represent the solution to (20) subject to $s_0 = s$ and $J\left(s^0\right) = J$. Given these definitions, we can rewrite

[19] If $b_{-1} \neq 0$, we can let $x\left(s^t\right) = (1-\beta)\left(R^{\max} + \max\left\{0, -b_{-1}\right\}\right) + \beta \underline{\mathbf{x}}\ \forall s^t$ without loss of generality.

[20] This should be interpreted carefully since it is due to the fact that $x\left(s^t\right)$ is constant. Given an optimal sequence n, one can always backload x so that the government holds assets along the equilibrium path. Moreover, if $b_{-1} < 0$, it will be the case that $x\left(s^t\right) > x^*$, so that (14) corresponds to positive asset bounds.

(20) recursively where $\pi_{ks} = \Pr\{s_{t+1} = k | s_t = s\}$:

$$Q(s, J) = \max_{n \in \left[n^{\max}, n^{fb}\right], \{J_k\}_{k \in S}} \left\{ n - \eta \frac{n^\gamma}{\gamma} + \beta \sum_{k \in S} \pi_{ks} Q(k, J_k) \right\} \tag{21}$$

s.t.

$$J = R(n) + \beta \sum_{k \in S} \pi_{ks} J_k \tag{22}$$

$$J_k \geq G(k) + R^{\max} + \beta \frac{\mathbf{x}}{1-\beta} \ \forall k \in S \tag{23}$$

where $J_0 = G(s_0) + R^{\max} + \beta \frac{x}{1-\beta}$. J represents the present discounted value of the revenue which needs to be generated in state s. $Q(s, J)$ represents the highest possible value of social welfare achievable in state s conditional on the generation of J, where equation (22) guarantees that J is generated. Note that we have ignored $x^* + g(s)$ in (21) since the sum enters additively and does not affect the optimal solution. Equation (23) is the recursive version of (14), and it ensures that the present discounted value of revenues tomorrow is large enough. We prove the following technical results in the Appendix where we use the techniques of Thomas and Worrall (1988).

Lemma 3 *(i) The set of values J for which a solution for $Q(s, J)$ exists is a compact interval $\left[G(s) + R^{\max} + \beta \frac{x}{1-\beta}, \frac{R^{\max}}{1-\beta}\right]$, and (ii) the frontier $Q(s, J)$ is decreasing, strictly concave, and continuously differentiable in J on $\left[G(s) + R^{\max} + \beta \frac{x}{1-\beta}, \frac{R^{\max}}{1-\beta}\right]$.*
Proof. *See Appendix.* ■

Let λ and $\beta \pi_{ks} \phi_k$ represent the Lagrange multiplier on (22) and (23), respectively. Taking first order conditions, we achieve:

$$n \ : \ 1 - \tau = \eta n^{\gamma - 1} = \frac{1 + \lambda}{1 + \lambda \gamma} \tag{24}$$

$$J_k \ : \ -Q_J(k, J_k) = \lambda + \phi_k \tag{25}$$

and the envelope condition yields:

$$J : -Q_J(s, J) = \lambda. \tag{26}$$

λ represents the shadow cost of the incentive compatibility constraint on the dictator. From (24), an increase in this shadow cost increases the tax rate. To understand the intuition for this, note that in the economy managed by a benevolent social planner, we can ignore (23) which means that the tax rate is constant across time. As such, $R(n(s^t))$ is the same in every period, so that the government holds assets when government liabilities $G(s_t)$ are relatively high, and the government holds debt when government liabilities $G(s_t)$ are relatively low. This is the optimal form of hedging and it allows the economy to smooth distortions due to taxation.

In our setting, political constraints impose a cost on the government's holding of assets, since the dictator is tempted to steal these assets. Therefore, the government can no longer prepare for certain shocks to government purchases by holding many assets, which implies that the government must respond to such shocks with an increase in the tax rate. Furthermore, this increase in the tax rate stretches into the future because of the optimality of smoothing tax distortions into the future. Formally, $-Q_J(k, J_k) = \lambda_k$, the shadow cost of the incentive compatibility constraint on the dictator after the realization of k, where $\lambda_k = \lambda + \phi_k$, so that this shadow cost is weakly increasing along the equilibrium path. This means that the tax rate is weakly increasing along the equilibrium path. We can in fact explicitly characterize its law of motion.

Theorem 1 *The solution to* (15) *admits an optimal tax policy characterized by*

$$\tau\left(s^t\right) = \max\left\{\tau\left(s^{t-1}\right), \underline{\tau}(s_t)\right\} \text{ and } \tau_0 = \underline{\tau}(s_0) \tag{27}$$

Proof. Define $\underline{\lambda}_k = -Q_J\left(k, G(k) + R^{\max} + \beta\frac{x}{1-\beta}\right)$. We establish that

$$\lambda\left(s^t\right) = \max\left\{\lambda\left(s^{t-1}\right), \underline{\lambda}(s_t)\right\} \text{ and } \lambda_0 = \underline{\lambda}(s_0)$$

which by (24) will imply (27). Imagine if $\lambda(s^t) < \underline{\lambda}(s_t)$. Equation (26) and the concavity of $Q(\cdot)$ imply that $J(s^t) < G(s_t) + R^{\max} + \beta\frac{x}{1-\beta}$, which violates (14). Imagine if $\lambda(s^t) > \lambda(s^{t-1}) \geq \underline{\lambda}(s_t)$. Equation (25) implies that $\phi(s^t) > 0$, so that $J(s^t) = G(s_t) + R^{\max} + \beta\frac{x}{1-\beta}$, but this contradicts the definition of $\underline{\lambda}(s_t)$. The fact that $\lambda_0 = \underline{\lambda}(s_0)$ follows from the fact that $J_0 = G(s_0) + R^{\max} + \beta\frac{x}{1-\beta}$. ■

4.2 Dynamics of the Tax Rate

Theorem 1 establishes the law of motion of the tax rate and shows that the tax rate is weakly increasing along the equilibrium path, so that whenever the dictator is going to be tempted to steal government asset holdings, the tax rate must increase into the future so as to reduce these asset holdings. Therefore, in contrast to an economy managed by a benevolent social planner with a constant tax rate, the tax rate is weakly increasing along the equilibrium path.

When does the tax rate increase? This will generally depend on the particular process for g. Nonetheless, we can know a few facts about the dynamics of the tax rate. First, we show that there is a range of values for the lower bound $\underline{\tau}(s)$, so that there is always a sequence of shocks s^{∞} under which the tax rate strictly increases. This follows from the fact that there is variation in $G(s)$, the size of total government liabilities, which induces variation in the incentive compatibility constraint on the dictator (14). Second, we show that the minimal tax rate is strictly increasing in government purchases if shocks to government purchases are i.i.d. The intuition for this second result is that, under i.i.d. shocks, relatively higher shocks to government purchases are met with relatively higher tax rates, since this produces a relative increase in the sequence of primary surpluses expected tomorrow when government purchases are relatively lower.

Proposition 4 *(i)* $\underline{\tau}(s_{\min}) < \underline{\tau}(s_{\max})$ *where* $s_{\min} = \arg\min_{s\in S} G(s)$ *and* $s_{\max} = \arg\max_{s\in S} G(s)$ *and (ii) if* $\pi(k|s) = \pi(k)$ $\forall k, s \in S$ *then* $\underline{\tau}(s)$ *is strictly increasing in* s.
Proof. *See Appendix.* ■

Why is the tax rate not constant? Imagine if it were, then satisfaction of (14) would inevitably set the tax rate to $\underline{\tau}(s_{\max})$, so that the government is prepared to hold a low enough level of assets when $G(s_{\max})$, the highest liability state, occurs. If the constant tax rate were below this level, the dictator would extract everything from the government treasury when $s_{\max}$ occurs. Assuming the initial state is not $s_{\max}$, satisfaction of the present value constraint for the government (9) would thus require $x(s^t) > x^*$ for some s^t, meaning that a fixed tax rate would induce excessive rents to the dictator. Therefore, keeping rents as low as possible while continuing to limit the government's ability to hold assets means that it is optimal for the tax rate to increase only when it is constrained to do so.

4.3 Numerical Example

We now present a numerical simulation of our model our model. Let

$$(\eta, \gamma, \beta, \underline{x}) = (.75, 2, .95, 0).$$

As a normalization, let the resource constraint of the economy be

$$c + g + x = 10n,$$

where we assume that $x = \underline{x}$ under a benevolent government. Assume that there are two shocks to government purchases with values of 10 and 20 where $\Pr\{g_t = g_{t-1}\} = .99$ so that the shocks are very persistent and let $g_0 = 10$.

Figure 1 displays the dynamics of the tax rate, government assets, and output subject to the government purchase shock in an economy managed by a benevolent social planner. Output and the tax rate are flat for all shocks, and the government hedges against high government purchase shocks by holding assets whenever these shocks occur, so that it is able to run a primary deficit.

Figure 1: Benevolent Social Planner

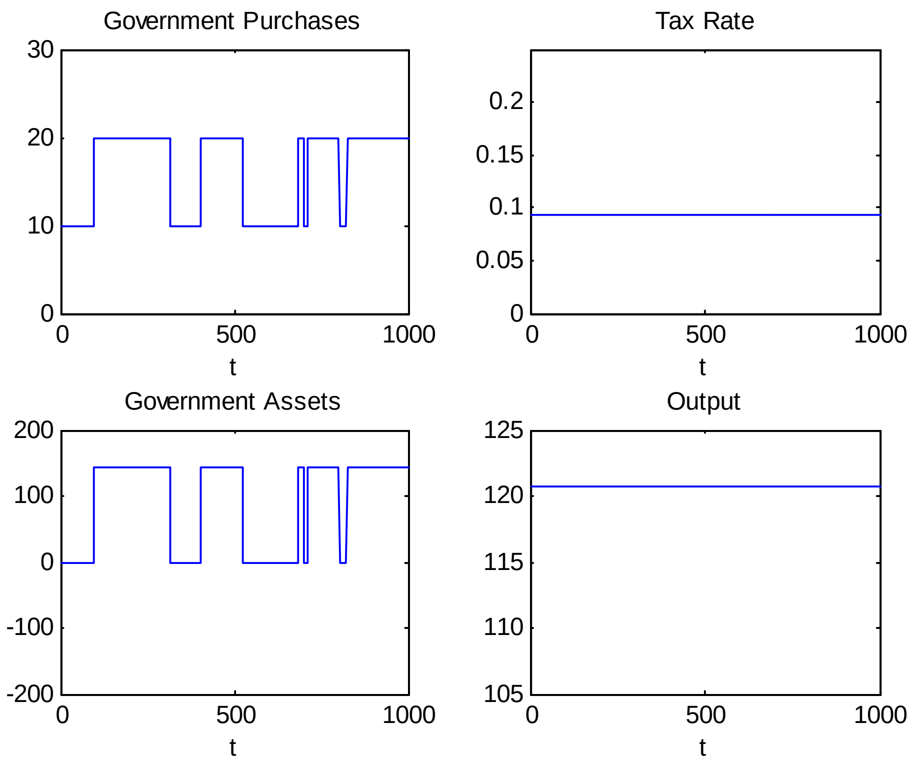
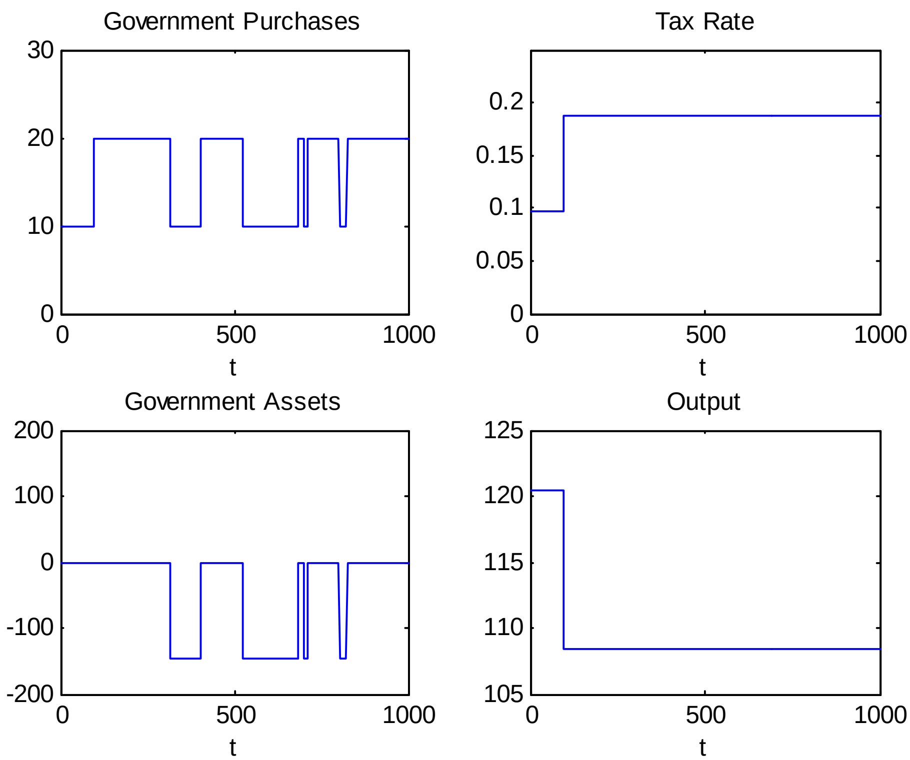


Figure 2 presents an economy managed by a dictator. In this economy, taxes are increasing along the equilibrium path and output is decreasing along the equilibrium path. While the government would like to hold assets to hedge against the high government purchase shock, it cannot do so as a consequence of the incentive compatibility constraint on the dictator. This means that after the first occurence of the high government purchase shock, the tax rate must permanently increase to a new level to accomodate the increase in government liabilities. After this initial increase, the tax rate is at a steady state. In the long run, the government holds debt whenever the low shock occurs since it can run a surplus, and the government holds zero assets under the high shock. This simple example shows how the political economy constraint can bias the government towards using debt as opposed to assets as a hedging instrument.

Figure 2: Dictator



In terms of welfare, the period zero continuation utility of households under a benevolent government is 1094, and under a dictatorship it is 1055. This means that in order to make households under a dictatorship as well off as they would be under a benevolent government, consumption needs to increase by 1.97 on average in every period, which represents a 1.8% increase in overall consumption starting from period 0, which is substantial. Given that the extracted rents are 1.67 in every period, this means that the welfare effect of the political economy constraint works primarily through the resource constraint, as opposed to the implied bounds on assets.[21]

However, these welfare effects are conditional on choosing the optimal sustainable policy described in Theorem 1 under a dictatorship. If, alternatively, one were to solve (20)

[21]This is not surprising, given that previous analyses suggest that the welfare implications of even more severe forms of market incompletion are not very large. See Aiyagari, Marcet, Sargent, and Seppala (2002).

subject to $\tau_t = \underline{\tau}$, meaning according to the prescribed policy under a benevolent government, the solution would yield $\underline{\tau} = \underline{\tau}(s_{\max})$, which would effectively force the dictator to extract higher rents in order that (9) hold. This would yield a period zero continuation utility to households equal to 881. This means that the gains from transitioning from a fixed tax regime to the optimal backloaded tax regime in a dictatorship is equivalent to increasing consumption in a fixed tax regime by 8.68 in every period, where this represents a 10% increase in consumption in such an economy, which is large.

5 Taxation Under Democratization Threats

In the above analysis, we have assumed that households replace a dictator with an equally selfish dictator, so that by prescribing symmetric strategies to the two dictators, we can ignore (4), the incentive compatibility constraint on the households which is essential for maintaining the same dictator in power. As a result, it is possible to backload the tax rate so that households become worse off along the equilibrium path, where households accept this transition since they do not expect any replacement dictator to choose different policies.

In this section, we more seriously consider the possibility that households can replace the dictator with a better alternative through some costly political process, and we do this to capture the commitment problem by households to maintaining the dictator in power. Specifically, imagine if

$$i_t = \{1, 2, \delta\}, \tag{28}$$

where, as before, 1 and 2 represent two selfish dictators, though now δ represents a benevolent government with full commitment power in a democracy where any transition to δ is permanent. To make the problem interesting, we impose that a choice of δ is costly to capture the fact that revolutions are destructive. Specifically, if $i_{t-1} \neq \delta$ and $i_t = \delta$ households pay an additive cost $\Gamma(s_t)$. Moreover, as a simplification, if $i_{t-1} \neq \delta$ and $i_t = \delta$, then $b_{t-1}(s_t) = 0$ (as perceived at t), so that the financial market shuts down during a democratization episode.[22]

Note that the payoff of choosing democracy in state s is the payoff associated with the

[22] This assumption can be microfounded in a more complete model where the benevolent government can default and can also choose to not collect all household claims.

solution to

$$\max_{c,n} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t | s_0\right) \left(c\left(s^t\right) - \eta \frac{n\left(s^t\right)^{\gamma}}{\gamma} \right) \tag{29}$$

$$\text{s.t. } (9), \ c\left(s^t\right) + g\left(s^t\right) + \underline{\mathbf{x}} = n\left(s^t\right) \ \forall s^t, \text{ and } s_0 = s,$$

where we have assumed that the benevolent social planner will choose reservation rents $\underline{\mathbf{x}}$. Define $n^{\delta}(s_t)$ as the fixed allocation of labor associated with the solution to (29), and let

$$Q^{\delta}(s_t) = \frac{n^{\delta}(s_t) - \underline{\mathbf{x}} - \eta \frac{n^{\delta}(s_t)^{\gamma}}{\gamma}}{1-\beta} - \Gamma(s_t)$$

represent the payoff gross of government purchases from transitioning to a democracy at s_t.

Given that we are interested in an economy under weak institutions, we consider an equilibrium where it is most efficient for democratization to never occur along the equilibrium path. This is always the case, for instance if $\Gamma(s_t)$ is high enough, meaning democratization is excessively costly.[23] In such an equilibrium, households can always transition to a democracy which means that

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k | s^t\right) \left(n\left(s^k\right) - \eta \frac{n\left(s^k\right)^{\gamma}}{\gamma} - x\left(s^k\right) \right) \geq Q^{\delta}(s_t) \ \forall s^t, \tag{30}$$

where we have used the fact that $G(s_t)$, the price of total government liabilities, is paid both on and off the equilibrium path. We can show that analogous results to Proposition 3 hold, though we add constraint (30) to capture the incentive compatibility constraint on households.

Proposition 5 *An allocation* $\{c, n, g, x\}$ *constitutes a sustainable competitive equilibrium without binding debt limits under* $i\left(s^t\right) \neq \delta \ \forall s^t$ *if and only if it satisfies* $(9), (10), (12)$ *and* (30)*, and if* $\underline{\mathbf{b}}$ *is sufficiently low and* $\overline{\mathbf{b}}$ *is sufficiently large.*

Proof. *See Appendix.* ■

[23] If the process for g is deterministic, democratization will either occur in the first period or never, since a dictator expecting to be replaced with certainty would always not implement government projects which provides a large disutility to households.

As a consequence we can write the program as

$$\max_{\{c,n,g,x\}} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) \left[c\left(s^t\right) - \eta \frac{n\left(s^t\right)^\gamma}{\gamma} + z\left(g\left(s^t\right), s_t\right)\right] \tag{31}$$

$$\text{s.t. } (9), (10), (13), (14), \text{ and } (30). \tag{32}$$

The results of Lemmas 1 and 2 continue to hold in this economy.

Lemma 4 *Lemmas 1 and 2 continue to hold under* $i\left(s^t\right) \neq \delta \; \forall s^t$.
Proof. *See Appendix.* ■

By analogous reasoning to that used in Section 4.1, we can write the recursive program with an additional constraint corresponding to the recursive version of (30):

$$Q(s, J) = \max_{n \in \left[n^{\max}, n^{fb}\right], \{J_k\}_{k \in S}} \left\{ n - \eta \frac{n^\gamma}{\gamma} + \beta \sum_{k \in S} \pi_{ks} Q(k, J_k) \right\} \tag{33}$$

s.t.

$$J = R(n) + \beta \sum_{k \in S} \pi_{ks} J_k \tag{34}$$

$$J_k \geq G(k) + R^{\max} + \beta \frac{\mathbf{x}}{1-\beta} \;\; \forall k \in S \tag{35}$$

$$Q(k, J_k) \geq Q^\delta(k) + R^{\max} + \beta \frac{\mathbf{x}}{1-\beta} \;\; \forall k \in S \tag{36}$$

Equation (36) represents the additional constraint that households prefer not to democratize along the equilibrium path, and it takes into account equation (19), since x^* is excluded from (33). Let λ, $\beta \pi_{ks} \phi_k$, and $\beta \pi_{ks} \psi_k$ represent the Lagrange multiplier on (34), (35), and (36), respectively. Taking first order conditions, we achieve (24), (26) and

$$-Q_J(k, J_k) = \frac{\lambda + \phi_k}{1 + \psi_k}, \tag{37}$$

which replaces (25) and which implies that $\lambda_k = -Q_J(k, J_k) \gtreqless \lambda = -Q_J(s, J)$, since the dictator must be induced to not steal and households must be induced to not democratize. Using analogous arguments to those of Theorem 1 we can establish a law of motion for the tax rate.

Theorem 2 *The solution to* (31) − (32) *admits an optimal tax policy characterized by*

$$\begin{aligned}
\tau\left(s^t\right) &= \begin{cases} \overline{\tau}\left(s_t\right) & \text{if } \tau\left(s^{t-1}\right) > \overline{\tau}\left(s_t\right) \\ \tau\left(s^{t-1}\right) & \text{if } \tau\left(s^{t-1}\right) \in \left[\underline{\tau}\left(s_t\right), \overline{\tau}\left(s_t\right)\right] \\ \underline{\tau}\left(s_t\right) & \text{if } \tau\left(s^{t-1}\right) < \underline{\tau}\left(s_t\right) \end{cases} \\
\text{and } \tau_0 &= \underline{\tau}\left(s_0\right)
\end{aligned} \tag{38}$$

Proof. See Appendix. ■

Embedded in Theorem 2 is the fact that the tax rate will increase whenever dictator is tempted to steal assets ((35) binds), and the tax rate will decrease whenever households are tempted to democratize ((36) binds). The reason for which the tax rate increases when (35) binds follows from our discussion in Section 4.1. Why must the tax rate decrease when (36) binds? The reason is that the dictator can generate support from households by reducing the tax rate which increases output and household consumption and increases household utility. The government cannot hold too much debt in such a state, since associated with high levels of debt are high future tax rates associated with higher primary surpluses to pay for this debt. There is in fact a parallel between our problem and one with ad-hoc debt and asset bounds. Formally, constraints (35) and (36) can be replaced by $J_k \in \left[\underline{J}_k, \overline{J}_k\right]$, corresponding to ad-hoc debt and asset bounds, and this would yield the same results in our analysis for properly chosen values of $\underline{J}_k$ and $\overline{J}_k$. Therefore, while providing incentives to the dictator to not steal imposes bounds on assets, providing incentives to households to support the dictator imposes bounds on debt.

A natural question is whether the tax rate is constant in the long run. The following proposition provides a sufficient condition for which this is not a possibility.

Proposition 6 *If there is a unique invariant long run distribution with full support over* S*, then conditional on the existence of an equilibrium under* $i\left(s^t\right) \neq \delta\ \forall s^t$*, the tax rate is not constant in the long run if there does not exist an* n^* *which satisfies*

$$\frac{n^* - \eta n^{*\gamma}}{1-\beta} \geq \max_{s \in S} G\left(s\right) + R^{\max} + \beta \frac{\underline{x}}{1-\beta} \text{ and} \tag{39}$$

$$\frac{n^* - \eta \frac{n^{*\gamma}}{\gamma}}{1-\beta} \geq \max_{s \in S} Q^{\delta}\left(s\right) + R^{\max} + \beta \frac{\underline{x}}{1-\beta}. \tag{40}$$

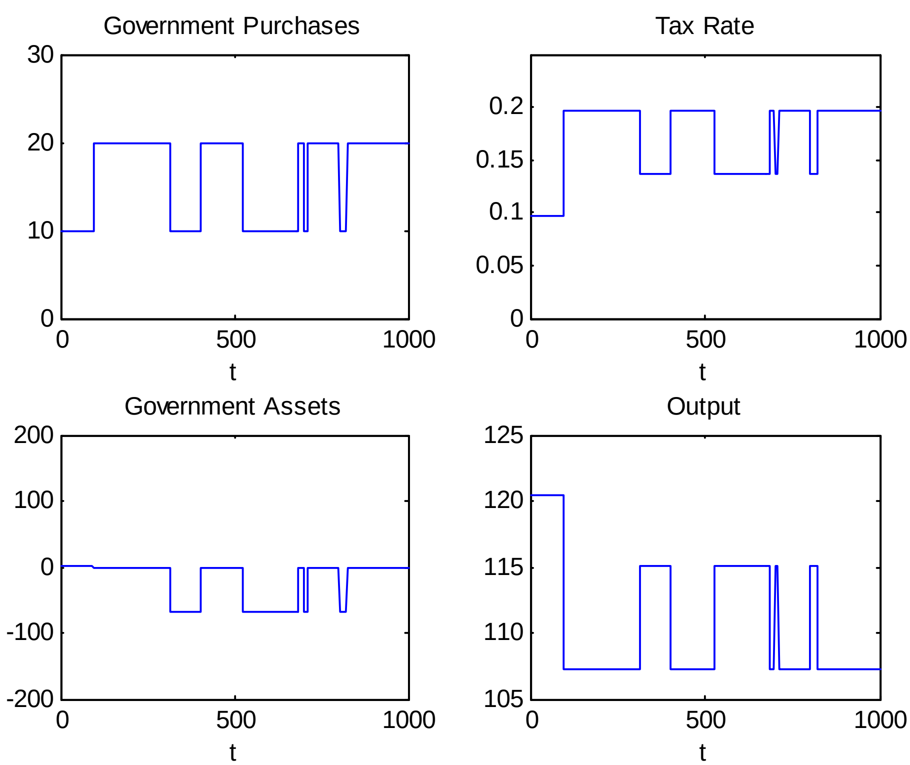
Proof. *If* τ^* *is fixed, then this requires a fixed* n^* *which satisfies* (14) *and* (30)*, and thus requires* (39) *and* (40)*.* ■

To illustrate the logic of our argument, consider the numerical simulation in Section 4.3 and let

$$\Gamma_t = \begin{cases} 50 & \text{if } g_t = 10 \\ \infty & \text{if } g_t = 20 \end{cases}.$$

This means that democratization is only possible when government purchases are low. Figure 3 displays the time path of government purchases, the tax rate, government assets, and output. We can see that associated with constraint (30) is an upper bound on government debt equal to 68.13, which is why the government cannot smooth long run taxes as in Figure 2. Underlying this bound on debt is the dictator's desire to reduce taxes in order to receive support from households that are tempted to replace him with a democracy. Therefore, in the long run taxes increase whenever government spending goes up and taxes decrease whenever government spending goes down. In contrast to Figure 2, the tax rate and output is much more volatile.

Figure 3: Dictator Under Democratization Threat



6 Conclusion

We have shown that weakly institutionalized economies experience endogenous limitations on the size of assets and debt which their governments can hold, where these limitations emerge through the strategic interaction between households and the dictator in power. These constraints hinder the government's ability to use financial markets to hedge against aggregate risk, which means that taxes must be more volatile and more persistent than in an economy managed by a benevolent social planner. Moreover, our numerical simulations suggest that it is important to allow these policies to be more volatile and more persistent, since less volatile and less persistent policies associated with a fixed tax rate would generate excess rents for the dictator.

In our analysis, we have made some assumptions which have caused us to neglect certain features of fiscal policy under weak institutions. First, our assumption of quasi-linearity of household utility which is crucial for the tractability of the model has shut down the government's use of the interest rate as a tool to smooth the distortions due to taxation. Second, by assuming the perfect observability of the dictator's actions, we have generated a model which requires the same dictator stay in power forever. Relaxing this assumption would create political turnover along the equilibrium path, since households would exercise replacement as a disciplining device to induce the optimal fiscal policy. Third, our model neglects the important interaction between fiscal policy and monetary policy in emerging economies by ignoring money and nominal bonds. Allowing for these policy tools could enrich the model and the possible deviations by the dictator off the equilibrium path. We plan to explore many of these extensions in future research.

7 Appendix

Proof of Proposition 1. The proof of necessity for an economy without default is in the text. Now consider an economy under some default. Equation (6) implies that if $D(s^{t+1}) = 1$, then $((1 - D(s^{t+1})) \max\{0, b(s^{t+1}|s^t)\} + \min\{0, b(s^{t+1}|s^t)\}) = b(s^{t+1}|s^t) \leq 0$, so that the same exercise as in the text can be used to achieve (8) and (9). For sufficiency, choose $\tau(s^t)$ so as to satisfy (5), choose $D(s^t) = 0$, choose $q(s^{t+1}|s^t)$ so as to satisfy (6), and let $b(s^t|s^{t-1})$ be determined by (8), where this is always possible if $\underline{\mathbf{b}}$ is sufficiently low and $\overline{\mathbf{b}}$ is sufficiently large. Satisfaction of (2) is implied by the satisfaction of (3) and (10). **Q.E.D.**

Proof of Proposition 2. We first check that the continuation policy induces a competitive equilibrium. Using (5) and (6), let

$$
\begin{aligned}
n\left(s^t\right) &= \left(\frac{1}{\eta}(1-\tau_t)\right)^{1/(\gamma-1)} && (41)\\
b\left(s^{t+1}|s^t\right) &= \begin{cases} \underline{\mathbf{b}} & \text{if } q_t(s_{t+1}) > \beta\pi(s_{t+1}|s_t) \\ 0 & \text{otherwise} \end{cases} && (42)\\
c\left(s^t\right) &= (1-\tau_t)\, n\left(s^t\right) + ((1-D_t)\max\{0, b_{t-1}(s_t)\} + \min\{0, b_{t-1}(s_t)\}) && (43)\\
&\quad - \sum_{s_{t+1}\in S} q_t(s_{t+1})\, b\left(s^{t+1}|s^t\right),
\end{aligned}
$$

and where Assumption 2 ensures feasibility and (2) is satisfied at t. For $k > t$, let

$$
\begin{aligned}
n\left(s^k\right) &= n^{\max} \\
b\left(s^{k+1}|s^k\right) &= 0 \\
c\left(s^k\right) &= \begin{cases} n^{\max}(1-\tau^{\max}) + \underline{\mathbf{b}} & \text{if } k = t+1 \text{ and } q_t(s_{t+1}) > \beta\pi(s_{t+1}|s_t) \\ n^{\max}(1-\tau^{\max}) & \text{otherwise} \end{cases}
\end{aligned}
$$

where Assumption 2 ensures feasibility, (2) is satisfied at k, and the value of $\widehat{x}$ ensures that (3) is satisfied at k.

To prove sustainability, let any dictator in power choose $\widehat{\rho}$ for $k > t$ for all possible histories, where a dictator is permanently thrown out of power if he deviates from $\widehat{\rho}$ and is kept in power otherwise. Households are indifferent between the dictators, so that (4) is satisfied. To consider the best deviation from $\widehat{\rho}$ at k given that $\widehat{\rho}$ is implemented for all

$l > k$, the dictator at k must maximize x_k subject to (3) and (41) − (43):

$$\max_{\rho_k} x_k \tag{44}$$
$$\text{s.t.}$$
$$g_k + x_k - \sum_{s_{t+1} \in S} q_k(s_{k+1}) \, \mathbf{\Psi}(q_k(s_{k+1})) \, \underline{\mathbf{b}} =$$
$$\tau_k \left(\frac{1}{\eta}(1-\tau_k) \right)^{1/(\gamma-1)} - ((1-D_k) \max\{0, b_{k-1}(s_k)\} + \min\{0, b_{k-1}(s_k)\}),$$
$$\text{where } \mathbf{\Psi}(q_k(s_{k+1})) = \begin{cases} 1 & \text{if } q_k(s_{k+1}) > \beta\pi(s_{k+1}|s_k) \\ 0 & \text{otherwise} \end{cases}$$

which yields a solution equal to $R^{\max} + \max\{0, -b(s_{k-1}(s_k))\}$, which is equal to $\widehat{x}$, so that the strategy is sustainable. **Q.E.D.**

Proof of Corollary 2. The proof of Proposition 2 establishes that $\widehat{\rho}$ solves (44). **Q.E.D.**

Proof of Lemma 1. Maximize (15) s.t. $(10), (13), (14)$, and (18), and imagine if the solution $\{\widetilde{c}, \widetilde{n}, \widetilde{g}, \widetilde{x}\}$ is such that (18) does not bind. By Assumption 1, the solution to the program satisfies (17), and we can show that $\widetilde{\tau}(s^t) \geq 0$ for all s^t. Imagine if $\widetilde{\tau}(s^t) < 0$, meaning $\widetilde{n}(s^t) > n^{fb}$ for some s^t. Any such equilibrium can be strictly improved by choosing $\widetilde{\widetilde{n}}(s^t) = n^{fb}$, $\widetilde{\widetilde{c}}(s^t) = \widetilde{c}(s^t) + n^{fb} - \widetilde{n}(s^t)$, and $\widetilde{\widetilde{x}}(s^t) = \widetilde{x}(s^t)$ for all such s^t where $\widetilde{\tau}(s^t) < 0$, and this satisfies $(10), (13), (14)$, and (18) by the concavity of $R(n)$. Now consider an equilibrium where $\widetilde{\tau}(s^t) \geq 0$ for all s^t, and where (18) does not bind. We will prove using induction that this constraint must bind. Given the satisfaction of (13), if (18) does not bind, then (14) does not bind at $t = 0$. It must therefore be that $\widetilde{n}(s^0) = n^{fb}$. Imagine instead if $\widetilde{n}(s^0) < n^{fb}$. Then this allocation can be strictly improved by choosing $\widetilde{\widetilde{n}}(s^0) = \widetilde{n}(s^0) + \varepsilon(s^0)$, $\widetilde{\widetilde{c}}(s^0) = \widetilde{c}(s^0) + \varepsilon(s^0)$, and $\widetilde{\widetilde{x}}(s^0) = \widetilde{x}(s^0)$ for $0 < \varepsilon(s^0) < n^{fb} - \widetilde{n}(s^0)$ which is sufficiently small while continuing to satisfy $(18), (10), (13),$ (14), and (18). Since $\widetilde{n}(s^0) = n^{fb}$ and $R(n^{fb}) = 0$, the satisfaction of (14) at $t = 0$ implies that

$$\sum_{s^1 \in S^1} \beta\pi(s^1|s^0) \left[\sum_{k=1}^{\infty} \sum_{s^k \in S^k} \beta^{k-1} \pi(s^k|s^1) \begin{pmatrix} R(n(s^k)) - \\ (g(s_k) + R^{\max}(1-\beta) + \beta\underline{\mathbf{x}}) \end{pmatrix} \right] \tag{45}$$
$$> g(s_0) + R^{\max}(1-\beta) + \beta\underline{\mathbf{x}},$$

and since the right hand side of (45) is positive by the fact that $g(1) + \underline{\mathbf{x}} > 0$ and $g(1) > 0$, it follows that (14) cannot bind for some s^1. We will refer to all such s^1 as $\widehat{s}^1$. Now consider some $\widehat{s}^k$ for $k \geq 1$ where (14) does not bind for all subhistories $\widehat{s}^t \subset \widehat{s}^k$ for $t < k$ (histories along the path which lead to $\widehat{s}^k$). If (14) binds at $\widehat{s}^k$, then

$$\sum_{l=k}^{\infty} \sum_{s^l \in S^l} \beta^{l-k} \pi\left(s^l | \widehat{s}^k\right)\left(R\left(n\left(s^l\right)\right) - \left(g\left(s_l\right) + R^{\max}\left(1-\beta\right) + \beta \underline{\mathbf{x}}\right)\right) = 0. \tag{46}$$

If alternatively (14) does not bind at $\widehat{s}^k$, then it must be that $\widetilde{n}\left(\widehat{s}^k\right) = n^{fb}$. If not, then the allocation can be strictly improved by choosing $\widetilde{\widetilde{n}}\left(\widehat{s}^k\right) = \widetilde{n}\left(\widehat{s}^k\right) + \varepsilon\left(\widehat{s}^k\right)$, $\widetilde{\widetilde{c}}\left(\widehat{s}^k\right) = \widetilde{c}\left(\widehat{s}^k\right) + \varepsilon\left(\widehat{s}^k\right)$, and $\widetilde{\widetilde{x}}\left(\widehat{s}^k\right) = \widetilde{x}\left(\widehat{s}^k\right)$ for $0 < \varepsilon\left(\widehat{s}^k\right) < n^{fb} - \widetilde{n}\left(\widehat{s}^k\right)$ which is sufficiently small while continuing to satisfy $(10), (13), (14)$, and (18). The fact that (14) is relaxed at k implies

$$\sum_{s^{k+1} \in S^{k+1}} \beta \pi\left(s^{k+1} | \widehat{s}^k\right)\left[\sum_{l=k+1}^{\infty} \sum_{s^l \in S^l} \beta^{l-k-1} \pi\left(s^l | s^{k+1}\right)\left(\begin{array}{c} R\left(n\left(s^l\right)\right) - \\ \left(g\left(s_l\right) + R^{\max}\left(1-\beta\right) + \beta \underline{\mathbf{x}}\right) \end{array}\right)\right] \tag{47}$$

$$> g\left(\widehat{s}_k\right) + R^{\max}\left(1-\beta\right) + \beta \underline{\mathbf{x}},$$

where (47) implies that (14) is relaxed for some $\widehat{s}^{k+1}$ where $\widehat{s}^k \subset \widehat{s}^{k+1}$. For all such $\widehat{s}^k$ where (14) does not bind, then

$$R\left(n\left(\widehat{s}^k\right)\right) - \left(g\left(\widehat{s}_k\right) + R^{\max}\left(1-\beta\right) + \beta \underline{\mathbf{x}}\right) < 0, \tag{48}$$

since $R\left(n\left(\widehat{s}^k\right)\right)$. However, (46) and (48) imply that the left hand side of (45) is non-positive which is a contradiction. **Q.E.D.**

Proof of Lemma 2. Using Lemma 1, maximize (15) s.t. $(10), (13), (14)$, and (18). If $\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) x\left(s^t\right) < \tau^{\max} n^{\max} + \beta \frac{x}{1-\beta}$, (13) is violated so that the equilibrium is not sustainable. If $\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) x\left(s^t\right) > \tau^{\max} n^{\max} + \beta \frac{x}{1-\beta}$, one can choose $\widehat{x}\left(s^0\right) = x\left(s^t\right) - \varepsilon$ and $\widehat{c}\left(s^0\right) = c\left(s^t\right) + \varepsilon$ for some $\varepsilon > 0$ which is arbitrarily small, which strictly increases (15) and satisfies all constraints. Therefore,

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) x\left(s^t\right) = \tau^{\max} n^{\max} + \beta \frac{x}{1-\beta}. \tag{49}$$

Given some $\{c, n, g, x\}$ which satisfies (49), there exists a sustainable competitive allocation $\{\widehat{c}, n, g, \widehat{x}\}$ where $\widehat{x} = x^*$ which yields the same value of (15) as $\{c, n, g, x\}$. $\widehat{x}$ clearly satisfies (13), and let $\widehat{c}\left(s^t\right) = n\left(s^t\right) - g\left(s_t\right) - x^*$, so that (10) is satisfied. It follows that $\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) \widehat{c}\left(s^t\right) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) c\left(s^t\right)$ which yields the same value of (15). **Q.E.D.**

Proof of Lemma 3. This proof replicates many of the results in Thomas and Worrall (1988), Lemma 1.

The methods of Lemma 1 can be used to establish that for $J \geq G(s) + R^{\max} + \beta \frac{x}{1-\beta}$, that $Q(s, J)$ can be redefined such that (22) is relaxed. Therefore, letting I_s be the set of feasible values of J under the relaxed program, if $J' \in I_s$ then $J'' \in I_s$ for all $G(s) + R^{\max} + \beta \frac{x}{1-\beta} \leq J'' < J' \leq \frac{R^{\max}}{1-\beta}$, since the constraint set is no smaller, where the upper bound $\frac{R^{\max}}{1-\beta}$ is due to feasibility. To show that I_s is closed, consider a sequence $J_s'^v \in I_s$ such that $\lim_{v \to \infty} J_s'^v = J_s'$. There will be a corresponding stochastic sequence n^v for each $J_s'^v$. Because each element of n is contained in $\left[n^{\max}, n^{fb}\right]$ and a stochastic sequence n^v specifies a countable number of labor allocations, the space of sequences which includes n^v is sequentially compact in the product topology. This means that there is a sub-sequence of labor allocations converging pointwise to a limiting sequence n^{∞}. Given the continuity of the utility function and $R(n)$ and since $\beta \in (0, 1)$, by the Dominated Convergence Theorem, then the limit of the social welfare function equals the social welfare achieved under the limiting labor sequence, and the limit of the total value of revenues equals the total value of revenues achieved under the limiting labor sequence. This means that n^{∞} is sustainable since n^v is sustainable, and it means n^{∞} achieves J. This establishes the compactness of the feasible values J.

The fact that the frontier $Q(s, J)$ is decreasing follows from the fact that our program is equivalent to a relaxed program. To show that it is strictly concave, consider $J', J'' \in \left[G(s) + R^{\max} + \beta \frac{x}{1-\beta}, \frac{R^{\max}}{1-\beta}\right]$ with associated sequences n', n'' respectively. Let $J''' = \alpha J' + (1-\alpha) J''$ for $\alpha \in (0, 1)$ and define $n''' = \alpha n' + (1-\alpha) n''$. The sequence n''' is sustainable by the convexity of (14) where $G(s^t) = G(s_t)$. Furthermre, n''' provides a welfare greater than $\alpha Q(s, J') + (1-\alpha) Q(s, J'')$ by the concavity of the utility function, which means that $Q(s, \alpha J' + (1-\alpha) J'') > \alpha Q(s, J') + (1-\alpha) Q(s, J'')$. To prove differentiability, consider a sequence n which generates $J \in \left(G(s) + R^{\max} + \beta \frac{x}{1-\beta}, \frac{R^{\max}}{1-\beta}\right)$ starting from s. Consider the sequence n^{ϵ} where the only difference between n and n^{ϵ} is that $n^{\epsilon}(s) = n(s) + \epsilon$, meaning labor at s is different but the continuation equilibrium is identical. Define the function

$$F(\epsilon) = n + \epsilon - \eta \frac{(n+\epsilon)^{\gamma}}{\gamma} + \beta \sum \pi_{ks} Q(k, J_k),$$

where clearly, $F(0) = J$. Optimality implies that $F(\epsilon) \leq Q(s, R(n+\epsilon) + \beta \sum \pi_{ks} J_k)$, where $F(\epsilon)$ is concave and differentiable, which satisfies Lemma 1 of Benveniste and

Scheinkman (1979) so that $Q(\cdot)$ is differentiable. **Q.E.D.**

Proof of Proposition 4. (i) Let $\underline{R}\left(s^{\min}\right)$ and $\underline{R}\left(s^{\max}\right)$ be the revenues generated by $\underline{\tau}\left(s^{\min}\right)$ and $\underline{\tau}\left(s^{\max}\right)$, respectively. Equation (14) means

$$\underline{R}\left(s_{\max}\right)+\beta\left[G\left(s_{\max}\right)+R^{\max}+\beta\frac{\mathbf{x}}{1-\beta}\right]\geq G\left(s_{\max}\right)+R^{\max}+\beta\frac{\mathbf{x}}{1-\beta},$$

since $\sum_{k=1}^{\infty}\sum_{s^k\in S^k}\beta^{k-1}\pi\left(s^k|s_{\max}\right)R\left(n\left(s^k\right)\right)\leq G\left(s_{\max}\right)+R^{\max}+\beta\frac{\mathbf{x}}{1-\beta}$ by equation (27). Analogously, equation (14), implies

$$\underline{R}\left(s_{\min}\right)+\beta\left[G\left(s_{\min}\right)+R^{\max}+\beta\frac{\mathbf{x}}{1-\beta}\right]\leq G\left(s_{\min}\right)+R^{\max}+\beta\frac{\mathbf{x}}{1-\beta},$$

since $\sum_{k=1}^{\infty}\sum_{s^k\in S^k}\beta^{k-1}\pi\left(s^k|s_{\max}\right)R\left(n\left(s^k\right)\right)\geq G\left(s_{\min}\right)+R^{\max}+\beta\frac{\mathbf{x}}{1-\beta}$ by equation (27). Therefore,

$$\underline{R}\left(s_{\max}\right)\geq G\left(s_{\max}\right)>G\left(s_{\min}\right)\geq\underline{R}\left(s_{\min}\right),$$

which means that $\underline{\tau}\left(s_{\min}\right)<\underline{\tau}\left(s_{\max}\right)$.

(ii) Consider two states s and k where $k>s$ and imagine if $\underline{\tau}(k)\leq\underline{\tau}(s)$. If shocks are i.i.d., then $Q(s,J)=Q(k,J)$, since $\pi_{ks}=\pi_k$ in (21) – (23). If $\underline{\tau}(k)\leq\underline{\tau}(s)$, this implies that $\underline{\lambda}(k)\leq\underline{\lambda}(s)$ (see proof of Proposition 1), which means that $-Q_J\left(k,G(k)+\tau^{\max}n^{\max}+\beta\frac{x}{1-\beta}\right)<-Q_K\left(s,G(s)+\tau^{\max}n^{\max}+\beta\frac{x}{1-\beta}\right)$, but this contradicts the fact that $G(s)<G(k)$ under i.i.d. shocks. **Q.E.D.**

Proof of Proposition 5. Proposition 1 establishes the necessity and sufficiency of (9) and (10) under a large enough range $\left[\underline{\mathbf{b}},\overline{\mathbf{b}}\right]$ and it establishes (8) under any competitive equilibrium. For the necessity of (12), a dictator's strategy of choosing $\widehat{\rho}$ whenever in power achieves a payoff weakly larger than the right hand side of (12) by Assumption 3, and a dictator can never achieve a payoff strictly larger than the left hand side of (12). For the necessity of (30), households can always choose to transition to democracy. For sufficiency, consider the following equilibrium where $i\left(s^k\right)=1$ $(=2)$ $\forall s^k$. Given a prescribed sequence of policies ρ, any deviation from ρ at t results in $i\left(s^k\right)=\delta$ for all $k>t$ and $\pi\left(s^k|s^t\right)>0$. The same proof as in Corollary 1 can be used establishes that the best deviation by the dictator in power is to choose $\rho_t=\widehat{\rho}$ which yields a payoff equal to the right hand side of (12), which makes dictator 1 (2) weakly worse off than following ρ. To ensure sustainability of the equilibrium from the perspective of the households, imagine that if $i_k\neq 1$ $(\neq 2)$, then the replacement dictator chooses ρ forever, which satisfies (30). Alternatively, households can choose $i_k=\delta$ which they weakly prefer to not do by

(30). Finally, to ensure the credibility of the punishment by households, imagine that if a dictator deviates from ρ at t, households expect $\widehat{\rho}$ by any dictator for all periods into the future, so that they can threaten a deviating dictator with $i_k = \delta$. **Q.E.D.**

Proof of Lemma 4. The proof of Lemma 1 is identical. The same proof as in the proof of Lemma 2 establishes (49) and the fact that $\{\widehat{c}, n, g, \widehat{x}\}$ satisfies (13) and (10) and yields the same value of the program. We are left to show that $\{\widehat{c}, n, g, \widehat{x}\}$ satisfies (30). Satisfaction by $\{c, n, g, x\}$ of (13) and (30) for all s^t implies

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k | s^t\right) \left(n\left(s^k\right) - \eta \frac{n\left(s^k\right)^{\gamma}}{\gamma} - x^*\right) \geq Q^{\delta}\left(s_t\right) \forall s^t,$$

so that $\{\widehat{c}, n, g, \widehat{x}\}$ satisfies (30). **Q.E.D.**

Proof of Theorem 2. Define $\underline{\lambda}_k = -Q_J\left(k, G\left(k\right) + R^{\max} + \beta \frac{x}{1-\beta}\right)$ and $\overline{\lambda}_k = -Q_J\left(k, \overline{J}_k\right)$ for $\overline{J}_k$ such that $Q\left(k, \overline{J}_k\right) = Q^{\delta}\left(k\right) + R^{\max} + \beta \frac{x}{1-\beta}$. We establish that

$$\lambda\left(s^t\right) = \begin{cases} \overline{\lambda}\left(s_t\right) & \text{if } \lambda\left(s^{t-1}\right) > \overline{\lambda}\left(s_t\right) \\ \lambda\left(s^{t-1}\right) & \text{if } \lambda\left(s^{t-1}\right) \in \left[\underline{\lambda}\left(s_t\right), \overline{\lambda}\left(s_t\right)\right] \\ \underline{\lambda}\left(s_t\right) & \text{if } \lambda\left(s^{t-1}\right) < \underline{\lambda}\left(s_t\right) \end{cases}$$

which by (24) will imply (38). Imagine if $\lambda\left(s^t\right) < \underline{\lambda}\left(s\right)$. Equation (26) and the concavity of $Q\left(\cdot\right)$ imply that $J\left(s^t\right) < G\left(s_t\right) + R^{\max} + \beta \frac{x}{1-\beta}$, which violates (14). Imagine if $\lambda\left(s^t\right) > \overline{\lambda}\left(s\right)$. Equation (26) and the concavity of $Q\left(\cdot\right)$ imply that $Q\left(s_t, J\left(s_t\right)\right) < Q^{\delta}\left(s_t\right) + R^{\max} + \beta \frac{x}{1-\beta}$, which violates (30). Now imagine if $\lambda\left(s^{t-1}\right) \in \left[\underline{\lambda}\left(s_t\right), \overline{\lambda}\left(s_t\right)\right]$ but $\lambda\left(s^t\right) \neq \lambda\left(s^{t-1}\right)$. If $\lambda\left(s^t\right) > \lambda\left(s^{t-1}\right)$, equation (37) implies that $\phi\left(s^t\right) > 0$, so that $J\left(s^t\right) = G\left(s_t\right) + R^{\max} + \beta \frac{x}{1-\beta}$, but this contradicts the definition of $\underline{\lambda}\left(s_t\right)$. If $\lambda\left(s^t\right) < \lambda\left(s^{t-1}\right)$, equation (37) implies that $\psi\left(s^t\right) > 0$, so that $Q\left(s_t, J\left(s_t\right)\right) = Q^{\delta}\left(s_t\right) + R^{\max} + \beta \frac{x}{1-\beta}$, but this contradicts the definition of $\overline{\lambda}\left(s_t\right)$. The fact that $\lambda_0 = \underline{\lambda}\left(s_0\right)$ follows from $J_0 = G\left(s_0\right) + R^{\max} + \beta \frac{x}{1-\beta}$. **Q.E.D.**

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