On the Consequences of Demographic Change for International Capital Flows, Rates of Returns to Capital, and the Distribution of Wealth and Welfare*

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Abstract

In all major industrialized countries the population is aging over time, reducing the fraction of the population in working age. Consequently labor is expected to be scarce, relative to capital, with an ensuing decline in real returns on capital and increases in real wages. This paper employs a large scale OLG model with intra-cohort heterogeneity to ask what are the distributional consequences of these changes in factor prices induced by changes in the demographic structure. Since these demographic changes occur at different speed in industrialized economies we develop a multi-region (the US, the European Union, the rest of the OECD and the rest of the world) open-economy model that allows for international capital flows. This allows us to evaluate to what extent the distributional consequences of changing factor prices

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for the US and Europe are mitigated or accentuated by the fact that the population is aging at different rates elsewhere in the world.

As a result of the aging of the population worldwide, the return to capital is declining by about half a percentage point from year 2005 until 2080 and gross wages are increasing substantially. However, taking the increasing burden of PAYG financed social security systems into account, *net* wages are predicted to decline quite dramatically - by roughly 8 percent in the US and 11 percent in the European Union. These two regions also are predicted to run substantial current account deficits in the later part of the 21st century, as capital is flowing back to these regions from the rest of the world.

In order to document the distributional consequences of the demographic transition within and across generations, we analyze the evolution of Gini coefficients for earnings, income, consumption and wealth. We find that earnings, income and consumption are slightly more equally distributed in 2080 than in 2005, but that wealth is slightly more unequally distributed. In order to evaluate the welfare consequences of the demographic transition across generations, we ask the following hypothetical question: suppose a household born in 1950, the initial steady state of our model, would live through the economic transition with changing factor prices induced by the demographic change (but keeping her own survival probabilities constant at their 1950 values), how would its welfare have changed, relative to a situation without a demographic transition? We find that households experience significant welfare losses due to the demographic transition, in the order of 1% of consumption for cohorts born in 1950 and increasing to roughly 6% for cohorts born in 2000. These losses are mainly due to the fact that lower future returns to capital make it harder for households to save for retirement and due to the decline of net wages in their working lives. The welfare losses we document have to be traded off against the potential welfare gains from a longer (and healthier) life that is part of the source of the aging of the population (lower birth rates are the other source), and whose welfare benefits we are agnostic about in this paper.

1 Introduction

In all major industrialized countries the population is aging, over time reducing the fraction of the population in working age. Consequently the capital labor ratio is expected to increase, with ensuing declines in real returns on capital and increases in real wages. This paper employs a large scale OLG model with intra-cohort heterogeneity to ask what are the distributional consequences of these changes in factor prices induced by changes in the demographic structure. Since these demographic changes occur at different speed in industrialized economies we develop a multi-region (the US, the European Union and the rest of the OECD) open-economy model that allows for international capital flows. This allows us to evaluate to what extent the distributional consequences of changing factor prices for the US and Europe are mitigated or accentuated by the fact that the population is aging at different rates elsewhere in the world.

In order to answer the questions posited we develop a three region open economy version of the standard large scale-overlapping generation model pioneered by Auerbach and Kotlikoff (1987) and enriched by intra-cohort heterogeneity, as in Imrohoroglu et al. (1995), Conesa and Krueger (1999) and many others.

Both extensions of the basic Auerbach-Kotlikoff model are necessary for the question we want to address. First, intra-cohort heterogeneity will endogenously give rise to some agents deriving most of their income from returns to capital, while the income of others is mainly composed of labor income. Abstracting from this heterogeneity does not allow a meaningful analysis of the distributional consequences of changes in factor prices. Second, in light of substantial differences in the evolution of the age distribution of households across regions, it is important to allow for capital to flow from regions with a larger old-age dependency ratio to those with a larger share of workers in the population. In this way the effects on returns to capital of demographic changes is potentially mitigated for regions with rapidly aging population, and accentuated for those regions with a slower population aging process.

[TO BE COMPLETED]

[SOME OF THE LITERATURE TO BE CITED: Domeij-Floden, Feroli, Attanasio-Kitao-Violante, Brooks, Hendriksen, Fehr-Jokisch-Kotlikoff, Brsch-Supan-Ludwig-Winter; Abel]

The paper is organized as follows. Section 2 introduces a simple analytical model that allows us to illustrate the major effects at work in our quantitative exercise. Our quantitative model is presented in Section 3. We describe our thought experiment in Section 4 and discuss some technical details on calibration and the solution method in Section 5. Our main results are presented in Section 6. Sections 7 contains various forms of sensitivity analyses asking some substantive questions with respect to the roles of pension reforms and openness as well as less substantive questions with respect to some modeling issues. Section 8 concludes.

2 A Simple Model

In this secition we construct a simple two period OLG model that is a special case of our quantitative model in the next section. We can characterize equililibria in this model analytically, and aim at providing some intuition for the quantitative results derived below. We are especially interested in the influence of demographic variables and the size and structure of the social security system on the direction and dynamics of international capital flows.

In every country i there are $N_{t,i}$ young households that live for two periods and have preferences representable by the utility function

$$\log(c_t^y) + \beta \log(c_{t+1}^o)$$

In the first period of their lives households work for a wage w_t, and in the second period they retire and receive social security benefits $b_{t+1,i}$ that are financed via payroll taxes on labor income. Thus the budget constraints read as

$$\begin{aligned} c_t^y + s_t &= (1-\tau_{t,i})w_{t,i} \\ c_{t+1}^o &= (1+r_{t+1})s_t + b_{t+1,i} \end{aligned}$$

where r_{t+1} is the real interest rate between period t and $t+1$ and $\tau_{t,i}$ is the social security tax rate in country i. We assume that capital flows freely across countries, and thus the real interest rate is equalized across the world.

The production function in each country is given by

$$Y_{t,i} = K_{t,i}^{\alpha} \left(Z_i A_t N_{t,i}\right)^{1-\alpha},$$

where Z_i is the country-specific technology level and $A_t = (1+g)^t$ is exogenously growing productivity. Thus we allow for differences in technology levels across countries, but not its growth rate. We fursther assume that capital depreciates fully after use in production.

The production technology in each country is operated by a representative firm that behaves competitively in product and factor markets. Profit maximization of firms therefore implies that

$$\begin{aligned} 1+r_t &= \alpha k_t^{\alpha-1} \\ w_{t,i} &= (1-\alpha)Z_i A_t k_t^{\alpha}, \end{aligned} \tag{1}$$

where

$$k_t = k_{t,i} = \frac{K_{t,i}}{Z_i A_t N_{t,i}} \forall i$$

is the capital stock per efficiency unit of labor.

We assume that the social security system is a pure pay-as-you-go (PAYGO) system that balances the budget in every period. Therefore

$$\tau_{t,i} w_{t,i} N_{t,i} = b_{t,i} N_{t-1,i}$$

Finally, market clearing in the world capital market requires that

$$K_{t+1} = \sum_i K_{t+1,i} = \sum_i N_{t,i} s_{t,i}$$

2.1 Analysis

Equilibria in this model can be characterized analytically. For that purpose we first solve the household problem and then aggregate across households (countries).

2.1.1 Optimal Household Savings Behavior

From the household problem we can solve for saving of the young in country i as

$$s_{t,i} = \frac{\beta}{1+\beta} w_{t,i}(1-\tau_{t,i}) - \frac{b_{t+1,i}}{(1+\beta)(1+r_{t+1})} \tag{2}$$

The budget constraint of the social security system implies that

$$b_{t,i} = \frac{N_{t,i}}{N_{t-1,i}} w_t \tau_{t,i} = \gamma_{t,i}^N w_{t,i} \tau_{t,i}$$

where $\gamma_{t,i}^N$ is the gross growth rate of the young cohort in country i between period $t-1$ and t.[1] Using this expression for benefits and substituting out for wages and interest rates from (1) in (2) yields

$$s_{t,i} \quad = \quad \frac{\beta(1-\tau_{t,i})(1-\alpha)}{1+\beta} Z_i A_t k_t^\alpha - \frac{\gamma_{t+1,i}^N \tau_{t+1,i}(1-\alpha)}{(1+\beta)\alpha} Z_i A_{t+1} k_{t+1} \tag{3}$$

[1]The population of a country i at time t is given by

$$Pop_{t,i} = N_{t,i} + N_{t-1,i}$$

and the share of old people in the population by

$$sh_{t,i} = \frac{N_{t-1,i}}{Pop_{t,i}}.$$

2.1.2 Aggregation

For further reference, define by $\tilde{N}_t = \sum_i Z_i N_{t,i}$ the efficiency weighted world population, by $\tilde{\theta}_{t,i} = \frac{Z_i N_{t,i}}{\tilde{N}_t} = \frac{\tilde{N}_{t,i}}{\tilde{N}_t}$ the relative share of the efficiency-weighted population in country i and by $\tilde{\gamma}_t^N = \frac{\tilde{N}_t}{\tilde{N}_{t-1}}$ the growth rate of aggregate (world) efficiency weighted population.

The capital market clearing condition reads as

$$\sum_i s_{t,i} N_{t,i} = \sum_i K_{t+1,i} = k_{t+1} \sum_i Z_i A_{t+1} N_{t+1,i} = k_{t+1} A_{t+1} \tilde{N}_{t+1} \tag{4}$$

Aggregating household savings decisions across countries yields, from (3):

$$\sum_i s_{t,i} N_{t,i} = \frac{(1-\alpha)\beta A_t k_t^\alpha}{1+\beta} \sum_i (1-\tau_{t,i}) Z_i N_{t,i} - \frac{(1-\alpha)A_{t+1}k_{t+1}}{(1+\beta)\alpha} \sum_i Z_i N_{t+1,i} \tau_{t+1,i}$$

Using this in (4) and simplifying yields

$$k_{t+1} = \sigma(\tilde{\gamma}_{t+1}^N, \gamma^A, \overrightarrow{\tau}_t, \overrightarrow{\tau}_{t+1}, \overrightarrow{\tilde{\theta}}_t, \overrightarrow{\tilde{\theta}}_{t+1}) k_t^\alpha \tag{5}$$

where

$$\begin{aligned} & \sigma(\tilde{\gamma}_{t+1}^N, \gamma_{t+1}^A, \overrightarrow{\tau}_t, \overrightarrow{\tau}_{t+1}, \overrightarrow{\tilde{\theta}}_t, \overrightarrow{\tilde{\theta}}_{t+1}) \\ = \quad & :\sigma_t = \frac{\alpha(1-\alpha)\beta(1-\tau_t^a)}{\tilde{\gamma}_{t+1}^N \gamma^A \left(\alpha(1+\beta) + (1-\alpha)\tau_{t+1}^a\right)} \end{aligned}$$

is the aggregate saving rate of the economy in period t, with $\tau_t^a = \sum_i \tau_{t,i} \tilde{\theta}_{t,i}$ denoting the average social security contribution rate in the world and $\gamma^A = 1 + g$ is the growth rate of the technology.

Equation (5), as a function of the policy and demographic parameters of the model, describes the dynamics of the aggregate capital stock, given the

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The we can easily compute the growth rate of the population as

$$\gamma_{t,i}^{Pop} = \frac{Pop_{t+1,i}}{Pop_{t,i}} = \frac{1+\gamma_{t,i}^N}{1+\frac{1}{\gamma_{t-1,i}^N}}$$

In a steady state

$$\gamma_i^{Pop} = \gamma_i^N$$

Also $sh_{t,i} = \frac{1}{1+\gamma_{t,i}^N}$. Thus $\gamma_{t,i}^N$ measures both the population growth rate as well as the age distribution in the economy.

initial condition $k_0 = \frac{\sum_i s_{-1,i} N_{-1,i}}{A_0 \sum_i Z_i N_{0,i}}$. Here $s_{-1,i}N_{-1,i}$ denotes total assets held by the initial old generation in country i.

Since, from the firms' first order condition, interest rates are given by

$$1 + r_t = \alpha k_t^{\alpha-1}$$

the dynamics of the real interest rate is given by

$$1 + r_{t+1} = \left(\frac{\alpha}{\sigma_t}\right)^{1-\alpha} (1 + r_t)^{\alpha}$$

with initial condition $1 + r_0 = \alpha k_0^{\alpha-1}$.

Finally, we can characterize international capital flows. Net foreign assets of country i at the beginning of period $t+1$ (or the end of period t) are given by

$$\begin{aligned} F_{t+1,i} &= N_{t,i}s_{t,i} - K_{t+1,i} = N_{t,i}s_{t,i} - Z_i A_{t+1} N_{t+1,i} k_{t+1} \\ &= \left(\frac{(1-\alpha)\beta(1-\tau_{t,i})}{1+\beta} - \gamma^N_{t+1,i}\gamma^A \sigma_t \left(1 + \frac{\tau_{t+1,i}(1-\alpha)}{(1+\beta)\alpha}\right)\right) Y_{t,i} \end{aligned}$$

and thus

$$\begin{aligned} \frac{F_{t+1,i}}{Y_{t,i}} &= \left(\frac{(1-\alpha)\beta(1-\tau_{t,i})}{1+\beta} - \gamma^N_{t+1,i}\gamma^A \sigma_t \left(1 + \frac{\tau_{t+1,i}(1-\alpha)}{(1+\beta)\alpha}\right)\right) \\ &= f_{t+1,i}(\gamma^N_{t+1,i}, \gamma^A, \tau_{t,i}, \tau_{t+1,i}, \tilde{\gamma}_{N,t+1}, \overrightarrow{\tau}_t, \overrightarrow{\tau}_{t+1}, \overrightarrow{\tilde{\theta}}_t, \overrightarrow{\tilde{\theta}}_{t+1}) \quad (6) \end{aligned}$$

determines the size and sign of net foreign assets to GDP of country i. Furthermore, the current account, relative to output, is defined as

$$ca_{t,i} = \frac{CA_{t,i}}{Y_{t,i}} = \frac{F_{t+1,i} - F_{t,i}}{Y_{t,i}} = f_{t+1,i} - f_{t,i}/\gamma^Y_{t,i} \quad (7)$$

where $\gamma^Y_{t,i} = \frac{Y_{t,i}}{Y_{t-1,i}} = \gamma^A_t \gamma^N_{t,i} \left(\sigma_{t-1} k^{\alpha-1}_{t-1}\right)^{\alpha}$ is the growth rate of output in country i.

2.2 Qualitative Results

2.2.1 Balanced Growth Path

Let the growth rates of populations and social security tax rates be constant over time. Then in a balanced growth path the economy is growing at rate

$\gamma^N \gamma^A$ and the the capital stock per efficieny unit of labor is given as

$$\begin{aligned} k^* &= (\sigma)^{\frac{1}{1-\alpha}} \\ &= \left(\frac{\alpha(1-\alpha)\beta(1-\tau^a)}{\tilde{\gamma}^N \gamma^A \left(\alpha(1+\beta) + (1-\alpha)\tau^a\right)} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

Evidently the steady state capital stock per labor efficiency units is strictly decreasing in the effective population growth rate of the world, $\tilde{\gamma}^N$ as well as the average social security contribution rate of the world economy, τ^a. The reverse is true for the world interest rate.

In the balanced growth path, net foreign asset positions and the current account of country i are given by[2]

$$\begin{aligned} f_i &= \frac{\beta(1-\alpha)(1-\tau_i)}{1+\beta} \left[1 - \frac{\tilde{\gamma}_i^N (1-\tau^a)(\alpha(1+\beta) + (1-\alpha)\tau_i)}{\tilde{\gamma}_N (1-\tau_i)(\alpha(1+\beta) + (1-\alpha)\tau^a)} \right] \\ ca_i &= f_i \left(1 - \left(\tilde{\gamma}_i^N \gamma^A \right)^{-1} \right) \end{aligned}$$

Thus our simple model has the following qualitative predicitions. First we observe that in the empirically relevant case that $\tilde{\gamma}_i^N \gamma^A > 1$, the sign of the current account coincides with that of the net foreign asset position. Thus we focus our discussion on the later.

1. If all countries have identical population growth rates and social security contribution rates ($\tilde{\gamma}_i^N = \tilde{\gamma}^N$ and $\tau_i = \tau^a$), then net asset positions and current accounts are zero in the long run.

2. If all countries have the same size of the social security system ($\tau_i = \tau^a$), then
$$f_i = \frac{\beta(1-\alpha)(1-\tau_i)}{1+\beta} \left[1 - \frac{\tilde{\gamma}_i^N}{\tilde{\gamma}^N} \right]$$
Thus countries with higher than world average population growth have a negative net asset position and current accounts, countries with lower than average population growth rates have positive net asset positions and current accounts. Capital flows from old to young regions.

3. If all countries have identical population growth rates ($\tilde{\gamma}_i^N = \tilde{\gamma}^N$) then
$$f_i = \frac{\beta(1-\alpha)(1-\tau_i)}{1+\beta} \left[1 - \frac{(1-\tau^a)(\alpha(1+\beta) + (1-\alpha)\tau_i)}{(1-\tau_i)(\alpha(1+\beta) + (1-\alpha)\tau^a)} \right]$$

[2] We made use of the fact that $\tilde{N}_{t,i} = Z_i N_{t,i}$ and thus $\tilde{\gamma}_i^N = \gamma_i^N$

and countries with higher than average social security contribution rates, $\tau_i > \tau^a$ have negative net asset positions and current accounts, those with lower contribution rates have positive net asset positions and current accounts.

4. Higher population growth rates, ceteris paribus[3], reduce the net asset position of a country in the BGP. The same is true for higher social security contribution rates.

2.2.2 Dynamics

As long as $\tilde{\gamma}_{t+1}^N \gamma^A \geq 1$, the economy converges monotonically from its initial condition to the balanced growth path characterized in the last section. We can explicitly characterize this dynamics. Define as percentage deviation from the balanced growth path

$$\hat{k}_t = \log(k_t) - \log(k^*)$$

We then can then write

$$\hat{k}_{t+1} = \hat{\sigma}_t + \alpha \hat{k}_t \tag{8}$$

where

$$\begin{aligned}
\hat{\sigma}_t &= \log(\sigma_t/\sigma_t^*) \\
&= \log(1-\tau_t^a) - \log(1-\tau^{a*}) \\
&\quad - \left(\log \tilde{\gamma}_{t+1}^N - \log \gamma_{t+1}^A\right) - \left(\log \tilde{\gamma}^N - \log \gamma^A\right) \\
&\quad - \left(\log(\alpha(1+\beta) + (1-\alpha)\tau_{t+1}^a) - \log((\alpha(1+\beta) + (1-\alpha)\tau^{a*}))\right)
\end{aligned} \tag{9}$$

Equation (8), in conjunction with equation (9) can be used to deduce the impulse response of the capital stock per worker (and thus the total capital stock etc.) with respect to shocks in population growth rates and social security contribution rates. Also note that

$$\hat{r}_t = r_t - r^* \approx \log(1+r_t) - \log(1+r^*) = -(1-\alpha)\hat{k}_t$$

so one can easily deduce the dynamics of returns to capital from the dynamics of capital itself. Also, using the results in (6) and (7) in conjunction with (9) allows to deduce the dynamics of the net asset position and the current account following a shock in population growth rates or social security contribution rates.

[TO BE COMPLETED]

[3] Strictly speaking, an increase in $\tilde{\gamma}_i^N$ or τ_i change τ^a as well. This is meant by ceteris paribus. Also note that a meaningful balanced growth path does not exist with country heterogeneity in $\tilde{\gamma}_i^N$.

3 The Quantitative Model

In this section we describe the quantitative model that we use to evaluate the consequences of demographic changes around the world for international capital flows and their consequences for the returns to capital and wages, as well as the distributional consequences emanating form these changes.

In our quantitative work we consider (at most) four countries/regions: the US, the European Union, the rest of the OECD and the rest of the world.

3.1 Demographics

The demographic evolution in the countries of interest is taken as exogenous (that is, we do not model fertility, mortality or migration endogenously) and as the main driving force of our model.

Households start their economic life at age 20, retire at age 65 and life at most until age 95. Since we do not model the first 19 years of a household explicitly, we denote its twentieth year of life by $j = 1$, its retirement age by $jr = 45$ and the terminal age of life by $J = 85$. Households face an idiosyncratic, time- and country-dependent probability of surviving from age j to age $j+1$, which we denote by $s_{t,j,i}$.

For each country $i \in \{1, \ldots, I\}$ we have data or forecasts for populations of model age $j \in \{1, \ldots, 85\}$ at time $t = \{1950, \ldots, 2400\}$, denoted by $N_{t,j,i}$. These are explained in detail in appendix A. The survival probabilities are then computed as[4]

$$s_{t,j,i} = \frac{N_{t+1,j+1,i}}{N_{t,j,i}}$$

3.2 Endowments and Preferences

Households value consumption and, if the labor-leisure choice is endogenous, labor over the life cycle $\{c_j, l_j\}$ according to a standard time-separable utility function

$$E\left\{\sum_{j=1}^{J} \beta^j u(c_j, l_j)\right\}$$

[4]For simplicity we assume that all migration takes place at or before age $j = 1$ in the model, so that we can treat migrants and agents born inside the country of interest symmetrically.

where β is the time discount factor and expectations are taken over idiosyncratic survival probabilities and stochastic labor productivity, described now.

Households are heterogenous with respect to age, their deterministic earnings potential and their stochastic labor productivity. All these sources of heterogeneity affect a household's labor productivity and thus wages. First, households labor productivity differs according to their age; let ε_j denote average age-specific productivity of cohort j.

Second, each household belongs to a particular group $k \in \{1, \ldots, K\}$ that shares the same average productivity θ_k. Differences in groups stand in for differences in education or ability, characteristics that are fixed at entry into the labor market and affect a group's relative wage. We introduce these differences in order to generate part of cross-sectional income and thus wealth dispersion that does not come from our last source of heterogeneity, idiosyncratic productivity shocks. That is, lastly, a household's labor productivity is affected by an idiosyncratic shock $\eta_t \in \{1, \ldots, E\}$ that follows a time-invariant Markov chain with transition probabilities

$$\pi(\eta_{t+1}|\eta_t) > 0.$$

Let Π denote the unique invariant distribution associated with π. Therefore, labor productivity of a household of age j, in group k and with idiosyncratic shock η_t is given by

$$\theta_k \varepsilon_j \eta_t.$$

3.3 Technology

In each country the single consumption good is being produced according to a standard neoclassical production function

$$Y_{t,i} = A_{t,i} K_{t,i}^{\alpha} L_{t,i}^{1-\alpha}$$

where $Y_{t,i}$ is output in country i at date t, $K_{t,i}$ and $L_{t,i}$ are labor and capital inputs and $A_{t,i}$ is total factor productivity at time t in country i. The parameter α measures the capital share and is assumed to be constant over time and across countries. Furthermore, in each country capital used in production depreciates at a rate δ, again assumed to be time- and country-independent. Since in each country production takes place with a constant-returns to scale production function and since we assume perfect competition, the number of firms is indeterminate in equilibrium and without loss of generality we assume that in each country a single representative firm operates.

3.4 Government Policies

In the benchmark model the government simply collects assets of households that die before age J and redistributes them in a lump-sum fashion among the citizens of the country as accidental bequests $Tr_{t,i}$. As sensitivity analysis we explore also how our results are affected by the presence of a pure pay-as-you-go public pension system, whose taxes and benefits have to be adjusted to the demographic changes in each country.

This social security system is modelled as follows. On the revenue side, households pay a flat payroll tax rate $\tau_{t,i}$ on their labor earnings. Retired households receive benefits ...[To be completed]

3.5 Market Structure

In each period there are spot markets for the consumption good, for labor and for capital services. Whereas the labor market is a national market where labor demand and labor supply are equalized country by country, the markets for the consumption good and capital services are international in that goods and capital can flow freely, and without any transaction costs, between countries. The supply of capital stems from households in all countries who purchase capital as assets in order to save for retirement and to smooth out idiosyncratic productivity shocks. The supply of consumption goods stem from the representative firms in each country.

Again, as sensitivity analysis we explore how the US would be affected by its demographic changes if it were a closed economy. In that exercise the capital used by US firms equals the assets that US citizens accumulate for life cycle and precautionary reasons.

3.6 Equilibrium

Individual households, at the beginning of period t, are indexed by their group k, their country of origin i, their age j, their idiosyncratic productivity chock η and their asset holdings a. Thus their maximization problem reads as

$$\begin{aligned} W(t,i,j,k,\eta,a) &= \max_{c,a',l}\{u(c,l)+\beta s_{t,j,i}\sum_{\eta'}\pi(\eta'|\eta)W(t+1,i,j+1,k,\eta',a')\} \quad (10) \\ s.t.\ c+a' &= w_{i,t}\theta_k\varepsilon_j\eta l+(1+r_t)a+Tr_{i,t} \\ a',c &\geq 0 \text{ and } l\in[0,1] \end{aligned}$$

where $w_{i,t}$ is the wage rate per efficiency unit of labor and r_t is the real interest rate. We denote the cross-sectional measure of households in country i at time t by $\Phi_{t,i}$ We then can define a competitive equilibrium as follows.

Definition 1 *Given initial capital stocks and distributions* $\{K_{0,i}, \Phi_{0,i}\}_{i\in I}$ *a competitive equilibrium are sequences of individual functions for the household,* $\{W(t,\cdot), c(t,\cdot), l(t,\cdot), a'(t,\cdot)\}_{t=0}^{\infty}$*, sequences of production plans for firms* $\{L_{t,i}, K_{t,i}\}_{t=0,i\in I}^{\infty}$*, prices* $\{w_{t,i}, r_t\}_{t=0,i\in I}^{\infty}$ *and transfers* $\{Tr_{t,i}\}_{t=0,i\in I}^{\infty}$ *and measures* $\{\Phi_{t,i}\}_{t=0,i\in I}^{\infty}$ *such that*

1. *Given prices, transfers and initial condition,* $W(t,\cdot)$ *solves equation* (10)*, and* $c(t,\cdot), l(t,\cdot), a'(t,\cdot)$ *are the associated policy functions.*

2. *Interest rates and wages satisfy*

$$\begin{aligned} r_t &= \alpha A_{t,i}\left(\frac{L_{t,i}}{K_{t,i}}\right)^{1-\alpha} - \delta \\ w_{t,i} &= (1-\alpha)A_{t,i}\left(\frac{L_{t,i}}{K_{t,i}}\right)^{-\alpha} \end{aligned}$$

3. *Transfers are given by*

$$Tr_{i,t+1} = \frac{\int (1-s_{t,j,i})a'(t,i,j,k,\eta,a)\Phi_{t,i}(dj\times dk\times d\eta\times da)}{\int \Phi_{t+1,i}(dj\times dk\times d\eta\times da)}$$

4. *Market clearing*

$$\begin{aligned} L_{t,i} &= \int \theta_k\varepsilon_j\eta l(t,i,j,k,\eta,a)\Phi_{t,i}(dj\times dk\times d\eta\times da) \text{ for all } i \\ \sum_{i=1}^{I} K_{i,t+1} &= \sum_{i=1}^{I}\int a'(t,i,j,k,\eta,a)\Phi_{t,i}(dj\times dk\times d\eta\times da) \\ & \sum_{i=1}^{I}\int c(t,i,j,k,\eta,a)\Phi_{t,i}(dj\times dk\times d\eta\times da) + \sum_{i=1}^{I} K_{i,t+1} \\ &= \sum_{i=1}^{I} A_{t,i}K_{t,i}^{\alpha}L_{t,i}^{1-\alpha} + (1-\delta)\sum_{i=1}^{I} K_{i,t} \end{aligned}$$

5. *Law of Motion for cross-sectional measures* Φ*: The cross-sectional measures evolve as*

$$\Phi_{t+1,i}(\mathcal{J}\times\mathcal{K}\times\mathcal{E}\times\mathcal{A}) = \int P_{t,i}((j,k,\eta,a),\mathcal{J}\times\mathcal{K}\times\mathcal{E}\times\mathcal{A})\Phi_{t,i}(dj\times dk\times d\eta\times da)$$

where the Markov transition functions $P_{t,i}$ are given by

$$P_{t,i}((j,k,\eta,a),\mathcal{J}\times\mathcal{K}\times\mathcal{E}\times\mathcal{A}) = \begin{cases} \pi(\eta,\mathcal{E})s_{t,i,j} & \begin{array}{c} \textit{if } a'(t,i,j,k,\eta,a)\in\mathcal{A} \\ k\in\mathcal{K}, j+1\in\mathcal{J}\end{array} \\ 0 & \textit{else} \end{cases}$$

and for newborns

$$\Phi_{t+1,i}(\{1\}\times\mathcal{K}\times\mathcal{E}\times\mathcal{A}) = N_{t+1,i,1}\begin{cases} \Pi(\mathcal{E}) & \textit{if } 0\in\mathcal{A} \\ 0 & \textit{else}\end{cases}$$

Definition 2 *A stationary equilibrium is a competitive equilibrium in which all individual functions are constant over time and all aggregate variables grow at a constant rate.*

4 The Thought Experiment

The exogenous driving process of our model is a time-varying demographic structure. We allow country specific survival, fertility and migration rates to change over time, inducing a demographic transition from an initial distribution towards an new steady state population distribution that arises once all time changes in these rate have been completed and the population structure has settled down to its new steady state. Induced by this transition of the population structure is a transition path of the economies of the model, both in terms of aggregate variables as well as cross-sectional distributions of wealth and welfare. Summary measures of these changes will provide us with answers as to how the changes in the demographic structure of the economy, by changing returns to capital and wages, impacts the distribution of welfare over time and across people in the economy.

[TO BE COMPLETED]

5 Calibration

In this section we discuss how we specify the parameters for our benchmark model. This entails choosing parameters governing the demographic transition, the preferences and endowment specification of households and the production technology by firms. We take as length of the period one year.

5.1 Demographics

Throughout the world, demographic processes are determined by the demographic transition that is characterized by falling mortality rates followed by a decline in birth rates, resulting in population aging and reducing the population growth rate (in some countries, even turning it negative). While demographic change occurs in almost all countries across the world, extent and timing differ substantially. Europe and some Asian countries have almost passed the closing stages of the demographic transition process while Latin America and Africa are only at the beginning stages (Bloom and Williamson, 1998; United Nations, 2002).

We focus on three different groups of industrialized countries, the US, the European Union and all other OECD countries. All demographic parameters used in the model can be deduced from our data for populations for these three regions, broken down by age groups. These population numbers determine both the idiosyncratic survival probabilities as well as the relative sizes of total populations in the three countries/regions in all time periods under consideration.

Figure 1 illustrates the differential impact of demographic change on total population numbers and population growth rates for the period 1950-2100. As the right panel shows, the US, while also experiencing a decrease of population growth rates, will have positive population growth rates also in the far distant future, whereas population growth rates drop below zero for the other two regions. Europe starts from low levels of population growth and population growth numbers are negative after 2005.

Figure 2 shows the impact of demographic change on working-age population ratios - the ratios of the working-age population (of age 15-65) to the total population - and old-age dependency ratios - the ratio of the working age population to the old-age population (of age 65+). As the figure illustrates, the three regions are differentially affected by the impact of demographic change: While working-age population ratios decrease in all regions (and old-age correspondingly increase), these effects are much stronger in the EU and the other OECD countries (ROECD). In terms of levels, the EU is the oldest region, but the figure also shows that the speed of demographic change is slightly higher in ROECD.

[FIGURES TO BE ADDED]

5.2 Endowments and Preferences

Households start their life with no assets and are endowed with one unit of time per period. Labor productivity is given by the product of three components, a deterministic age component ε_j, a deterministic group component θ_k and a stochastic idiosyncratic component η.

The age-productivity profile $\{\varepsilon_j\}_{j=1}^{J}$ is taken from Hansen (1993) and generates an average life-cycle wage profile consistent with the data. Conditional on age, the natural logarithm of wages is given by

$$\log(\theta_k) + \log(\eta).$$

We choose the number of groups to be $K = 2$ and lets groups be of equal size. We choose $\{\theta_1, \theta_2\}$ such that average-group productivity is equal to 1 and the variance of implied labor incomes of entrants to the labor market coincides with that reported by Storesletten et al. (2004). This requires $\exp(\theta_1) = 0.73$ and $\exp(\theta_1) = 1.27$.

For the idiosyncratic part of labor productivity we use a 2 state Markov chain with persistence parameter $\rho = 0.98$ and implied conditional variance of 8%.[5]

We assume that their period utility function is given by

$$u(c, l) = \frac{\left(c^{\kappa}(1-l)^{1-\kappa}\right)^{1-\sigma}}{1-\sigma}$$

where σ governs the relative risk aversion of the household and κ measures the relative importance of consumption, relative to leisure. In the benchmark economy we set $\sigma = 2$ and $\kappa = 1$, that is, assume that labor supply is exogenously given at $l = 1$.

In addition we have to specify the time discount factor of households. We choose a β such that the resulting world return on capital equals to 2.5%. This requires a $\beta = 0.9677$.

5.3 Technology

The capital share is assumed to time- and country-invariant and to equal $\alpha = 0.36$. As depreciation rate we choose $\delta = 8\%$ on an annual level. For the sequence of country-specific productivity levels $A_{t,i}$ we choose a structure

$$A_{t,i} = A_i(1+g)^t$$

[5] We are in the process of generating results with a Markov chain with more than two states, which is not conceptually difficult but time-consuming.

where g is the common productivity growth rate and A_i is a country-specific productivity constant. In our benchmark calibration we set $g = 0$. We choose the A_i such that relative outputs per capita in our model for the three regions coincide with that in the data for an average for the period 1950 to 2005. This requires an $A_{US} = 1.2$, $A_{EU} = 0.95$, $A_{RE} = 0.855$

[TO BE COMPLETED]

6 Results for the Benchmark Model

[TO BE COMPLETED]

6.1 Measuring International Capital Flows

In order to document our results about the direction and size of international capital flows we will document the evolution of the current account and the net asset position of the countries/regions under consideration. Define the net foreign asset position of country i at time t at the beginning of period t as

$$F_{t,i} = A_{t,i} - K_{t,i}$$

The current account in period t is then defined as the change in the net asset position of a country,

$$CA_{t,i} = F_{t+1,i} - F_{t,i}.$$

When reporting these statistics we will always divide them by output $Y_{t,i}$. Note that in a closed economy $F_{t,i} = C_{t,i} = 0$, and that in a balanced growth path of an open economy $CA_{t,i} = g\,(A_{t,i} - K_{t,i})$. Furthermore

$$\sum_i F_{t,i} = \sum_i CA_{t,i} = 0 \text{ for all } t.$$

6.2 Aggregate Variables

[TO BE COMPLETED]

6.3 Distributional and Welfare Consequences of the Demographic Transition

In this section we document who benefits and who loses from the demographic transition. In performing this exercise we have to take into account that household's lifetime utility is bound to change simply because in expectation they live longer over time. We therefore first describe the thought experiment we carry out in order to quantify the welfare conseqeunces from the demographic transition

A household's welfare is affected by two consequences of the demographic change. First, her lifetime utility changes because her own survival probabilities increase; this is in part what triggers the aging of the population (the other source are declines in birth rates). Second, due to the demographic transition she faces different factor prices and government transfers and taxes (from the social security system and from accidental bequests) than without changes in the demographic structure.

We want to isolate the welfare impact of the second effect. For this we compare lifetime utility of agents born and already alive in 2006 under two different scenarios. For both scenarios we fix a household's individual survival probabilities at their 2005 values; of course they fully retain their age-dependence. Then we solve each household's problem under two different assumptions about factor prices and taxes/transfers. Let $\overline{W}(t,i,j,k,\eta,a)$ denote the lifetime utility of an agent at time $t \geq 2006$ in country i with individual characteristics (j,k,η,a) that faces the sequence of equilibrium prices as documented in the previous section, but constant 2005 survival probabilities, and let $\overline{W}_{2006}(t,i,j,k,\eta,a)$ denote the lifetime utility of the same agent that faces prices and taxes/transfers that are held constant at their 2006 value. Finally, denote by $g(t,i,j,k,\eta,a)$ the percentage increase in consumption that needs to be given to an agent (t,i,j,k,η,a) at each date and contingency in her remaining lifetime (keeping labor supply allocations fixed) at fixed prices to make her as well off as under the situation with changing prices.[6] Negative numbers for $g(t,i,j,k,\eta,a)$ thus indicate that households suffer welfare losses from the general equilibrium effects of the demographic changes.[7] Of particular interest are the numbers $g(t=2006,i,j=1,k,\eta,a=0)$, that is, the welfare consequences for newborn agents in 2006 (remember that newborns start their life with zero assets).

[6] For the Cobb-Douglas utility specification for $\sigma \neq 1$ the number $g(t,i,j,k,\eta,a)$ can easily be computed as

$$g(t,i,j,k,\eta,a) = \left[\frac{\overline{W}(t,i,j,k,\eta,a)}{\overline{W}_{2006}(t,i,j,k,\eta,a)}\right]^{\frac{1}{\kappa_i(1-\sigma)}}.$$

[7] We alos computed these numbers taking 1950 as the base year of comparison. The results are availbale upen request.

[TO BE COMPLETED]

7 Sensitivity Analysis

[TO BE WRITTEN]

7.1 The US as Closed Economy

7.1.1 A Two Country World

7.2 Social Security Reform

7.2.1 Holding the Replacement Rate Fixed

7.2.2 Abstracting from Social Security

7.3 The Role of Endogenous Labor Supply

8 Conclusions

[TO BE WRITTEN]

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A Details of the Demographic Projections

For each country $i \in \{1, \ldots, I\}$ we base our demographic data on the official demographic data and projections by the United Nations (United Nations, 2002). Starting from a given initial age-distribution of population, $N_{1950,j,i}$, in year $t = 1950$ for actual age $j \in \{0, \ldots, 96\}$ demography in each year t is given recursively by

$$\begin{aligned} N_{t+1,i,j+1} &= N_{t,i,j}(s_{t,i,j} + m_{t,j,i}), \quad m_{t,i,j} = 0 \text{ for } j > 19 \\ N_{t+1,i,0} &= \sum_{j=15}^{50} f_{t,i,j} N_{t,i,j} \end{aligned}$$

where $m_{t,i,j}(f_{t,i,j})$ denotes time, age and country specific migration (fertility) rates. Our assumption, that migration rates are zero for ages above 19 allows us to treat newborns and immigrants in the economic model alike, compare footnote 4.

The United Nations provide demographic data on $N_{t,i,j}$, $s_{t,i,j}$ and $f_{t,i,j}$ on an annual basis for the years 1950-2050, but for age-groups of five only. We interpolate the initial distribution of the population, $N_{1950,i,j}$, and the data on $s_{t,i,j}$ and $f_{t,i,j}$ for all $t \in \{1950, \ldots, 2050\}$ between age-groups to get age-specific data. As for migration we use the UN data on aggregate migration, $M_{t,i}$, and assume that migration numbers are equally distributed across ages for $j \in \{1, \ldots, 19\}$. These approximations result in a decent fit of our demographic model to the official UN figures.

We further forecast demography beyond the UN forecasting horizon until 2400. First, while holding fertility rates constant, we assume that life-expectancy continues to increase at constant rates until year 2100. We then hold age-specific survival rates constant and assume that fertility rates adjust such that the number of newborns is constant in each successive year until 2200. This adjustment procedure implies that stationary population numbers are reached in year 2200. To support the steady state in our economic model, we hold demography constant for an additional 200 years until 2400.

B Computational Details

B.1 Household Problem

The idea is to iterate on the Euler equation, heavily using ideas developed in Carroll (2005). The dynamic programming problem of the household reads as

$$\begin{aligned} W(t,i,j,k,y,a) &= \max_{c,a'}\{u(c)+\beta s_{t,i,j}\sum_{y'}\pi(y'|y)W(t+1,i,j+1,k,y',a')\} \\ s.t.\ c+a' &= w_t\theta_k\varepsilon_j y+(1+r_t)a+Tr_t \\ a',c &\geq 0 \end{aligned}$$

where t indexes time, i indexes country, k indexes type, j indexes age, η the idiosyncratic income shock and a asset holdings. First define as cash at hand

$$x = w_t\theta_k\varepsilon_j y+(1+r_t)a+Tr_t$$

and rewrite the Bellman equation as

$$\begin{aligned} & V(t,i,j,k,y,x) \\ = & \max_{a'\in[0,x]}\{u(x-a')+\beta s_{t,i,j} * \\ & \sum_{y'}\pi(y'|y)V(t+1,i,j+1,k,y',w_{t+1}\varepsilon_{j+1}y'+(1+r_{t+1})a'+Tr_{t+1})\} \end{aligned}$$

The Euler equation reads as

$$\begin{aligned} u'(c) &\geq \beta s_{t,i,j}(1+r_{t+1})\sum_{y'}\pi(y'|y)V'(t+1,i,j+1,k,y',w_{t+1}\varepsilon_{j+1}y'+(1+r_{t+1})a'+Tr_{t+1}) \\ &= \text{ if } a'>0 \end{aligned} \tag{11}$$

and the envelope condition reads as

$$V'(t,i,j,k,y,x) = u'(c) \tag{12}$$

The algorithm will operate on (11) and (12).

1. Make a grid for savings

$$A = \{0, a_2, \ldots, a_{na}\}$$

2. Make a grid on x for the last generation

$$X_{t,i,nj,k,y} = \{x_1, \ldots, x_{na}\}$$

   One may want to pick $x_{nx} > a_{na}$, e.g. $x_{nx} = \kappa a_{na}$, with $\kappa > 1$. Furthermore choose $x_1 = x_{\min} > 0$, but small. Furthermore let $nx = na + 1$.

3. Economic theory tells us that

$$\begin{aligned} c(t,i,nj,k,y,x) &= x \\ a'(t,i,nj,k,y,x) &= 0 \end{aligned}$$

   for all $x \in X_{t,i,nj,k,y}$. From (12)

$$\begin{aligned} V'(t,i,nj,k,y,x) &= u'(c(t,i,nj,k,y,x)) \\ V(t,i,nj,k,y,x) &= u(c((t,i,nj,k,y,x))) \end{aligned}$$

4. Now iterate on j, $j = nj-1, \ldots, 1$. Given that we know the function $V'(t+1,i,j+1,k,y,x)$ from the previous step, do the following

   (a) For all $a' \in A$, solve

$$c = u'^{-1}\left[\beta s_{t,i,j}(1+r_{t+1})\sum_{y'}\pi(y'|y)V'(t+1,i,j+1,k,y',w_{t+1}\varepsilon_{j+1}y' + (1+r_{t+1})a' + T\right.$$

   for numbers $(c_1, \ldots, c_{na})$. Since

$$w_{t+1}\varepsilon_{j+1}y' + (1+r_{t+1})a' + Tr_{t+1} \notin X_{t+1,i,j+1,k,y'}$$

   in general, this will involve interpolation of the function V, for which it may be useful to do the interpolation on a transformed version of V'. See the remark below.

(b) Equiped with the consumption numbers, define the grid $X_{t,i,j,k,y}$ by

$$\begin{aligned} x_1 &= x_{\min} \\ x_{l+1} &= a_l + c_l \text{ for } l = 1, \ldots, na \end{aligned}$$

and the consumption function

$$\begin{aligned} c(t,i,j,k,y,x_1) &= x_{\min} \\ c(t,i,j,y,k,x_{l+1}) &= c_l \text{ for } l = 1, \ldots, na \\ a'(t,i,j,k,y,x_1) &= 0 \\ a'(t,i,j,k,y,x_{l+1}) &= a_l \text{ for } l = 1, \ldots, na \end{aligned}$$

(c) Update: for all $x \in X_{t,i,j,k,y}$

$$\begin{aligned} & V'(t,i,j,k,y,x) \\ = \; & u'(c(t,i,j,k,y,x)) \\ & V(t,i,j,k,y,x) \\ = \; & u(c(t,i,j,k,y,x)) + \beta s_{t,i,j} * \\ & \sum_{y'} \pi(y'|y) V(t+1,i,j+1,k,y',w_{t+1}\varepsilon_{j+1}y' + (1+r_{t+1})a'(t,i,j,k,y,x) + Tr_{t+1}) \end{aligned}$$

The updating of the value function again involves interpolation, for which one may want to use a transformation of V.

B.2 A Note on Interpolation

We have $V'(t,i,j,k,y,x)$ on $X_{t,i,j,k,y}$, and now want to compute it on $x \in (x_l, x_{l+1})$. One way is simply to have

$$V'(t,i,j,k,y,x) \approx \alpha_1 V'(t,i,j,k,y,x_l) + (1-\alpha_1)V'(t,i,j,k,y,x_{l+1})$$

where α_1 is the appropriate weight. If V' is highly nonlinear, this may yield a bad approximation. But now suppose that a transformation of V', call it W, is truly linear, where $W = g(V')$. If we know V' on $X_{t,i,j,k,y}$, we know W on $X_{t,i,j,k,y}$. Then

$$W(t,i,j,k,y,x) = \alpha_1 W(t,i,j,k,y,x_l) + (1-\alpha_1)W(t,i,j,k,y,x_{l+1})$$

without any approximation error, and thus

$$\begin{aligned} V'(t,i,j,k,y,x) &= g^{-1}\left[W(t,i,j,k,y,x)\right] \\ &= g^{-1}\left[\alpha_1 W(t,i,j,k,y,x_l) + (1-\alpha_1)W(t,i,j,k,y,x_{l+1})\right] \\ &= g^{-1}\left[\alpha_1 g(V(t,i,j,k,y,x_l)) + (1-\alpha_1)g(V(t,i,j,k,y,x_{l+1}))\right] \end{aligned}$$

without any approximation error. Of course this is true only if the true W is really linear. Carroll proposes to use ??? as transformation

[TO BE COMPLETED]