Search, Market Power, and Inflation Dynamics

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Abstract

The short-run non-neutrality of money and its implications for inflation dynamics are examined in a monetary search economy with heterogenous agents. Lump-sum money injections affect the distribution of money holdings in equilibrium and thus generate short-run non-neutrality. The response of prices and inflation to shocks of this type depends on the changes in households' search intensity that they induce. Monetary shocks change the distribution of prices in equilibrium and thus alter the returns to search. The resulting changes in optimal search intensity affect sellers' profit maximizing markups and thus may result in sluggish price adjustment and persistent inflation despite the absence of restrictions on sellers' ability to set prices in every period.

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1. Introduction

In this paper, we examine short-run monetary non-neutrality and the dynamics of both prices and inflation using a monetary search economy with heterogeneous agents. In our economy, short-run non-neutrality results as lump-sum monetary injections, whether anticipated or not, change the distribution of money holdings across households and affect real allocations. The degree of price adjustment in response to such shocks and the resulting dynamics of inflation are driven by the changes that they induce in housholds' search intensity. Monetary shocks affect the distribution of wealth and thus have differential effects on agents' returns to search. Changes in search intensity in turn affect sellers' market power and thus induce them to change their markups, affecting the magnitude of the contemporaneous response of prices to the shock. Moreover, as even transitory shocks result in persistent movements of the distribution of money holdings across households, the responses of search intensity and the price movements they generate are persistent as well.

Our research contributes to the large literature on monetary non-neutrality and the dynamics of inflation, much of which is aimed at developing practical models of monetary policy (see *e.g.* Woodford (2003) and the references therein). In most of this literature monetary non-neutrality is effectively assumed as restrictions are imposed on firms ability to change prices in response to shocks. This is typically justified as being in accordance with empirical evidence that the contemporaneous response of prices to such shocks (and to real shocks as well) is muted (*e.g.* Gali and Gertler (1999)). This is often interpreted as "price stickiness" and modeled accordingly with timing restrictions and/or costs associated with price changes.

Our economy, in contrast, has no restrictions on sellers ability to change their prices. Short-run monetary non-neutrality stems instead from the wealth effects of unequal monetary transfers to heterogenous households. Non-neutrality of this sort is studied in search models also by Berentsen, Camera, and Waller (2004), Molico (2005), and Williamson (2004). Sluggish price adjustment arises from the effect that both contemporaneous changes in the distribution of money across households and changes in households' expected future values of money affect the distribution of prices and thus the return to search. In equilibrium our economy thus may exhibit a form of price stickiness emanating not from exogenous restrictions on the *ability* of sellers to change prices, but rather from the effects of changes in search intensity on their market power and *desire* to do so.

We incorporate the posted-price selling mechanism of Burdett and Judd (1983) and Head and Kumar (2005) into a dynamic model with heterogenous households similar to those studied by Head and Shi (2003) and Williamson (2005). In general, in our economy a higher than anticipated increase (decrease) in the growth rate of money raises (lowers) the degree of price dispersion. Changes in dispersion affect the returns to search and thus increase (decrease) search intensity. This lowers (raises) firms desired markups and so may result in a reduction (increase) of real prices. If real prices fall in response to a monetary shock, then nominal prices must rise by less than the money stock. We refer to incomplete contemporaneous adjustment of prices to changes in the quantity of money as price stickiness, and we focus on the adjustment of both the price level and inflation rate over time.

Because dynamics result from the wealth effects of differential money transfers, some aspects of our results are similar to those obtained in studies using "limited participation" models (*e.g.* Alvarez, Atkeson, and Kehoe (2002) and Chiu (2005). Unlike these models, however, in ours there is no portfolio allocation problem associated with the demand for money. Rather, our model is more closely related to those of Williamson (2004, 2005) and Head and Shi (2003) in that money shocks directly affect the distribution of money holdings across households. Movements in this distribution affect both prices and the rate at which money flows from households of one type to another.

In an earlier paper, Head, Kumar, and Lapham (2005) consider a stochastic model similar in many respects but different in that households are always identical. In that model the distribution of money holdings across households is always degenerate and monetary shocks have no effect on the *distribution* of wealth. Thus, money is neutral in both the short and long run. That paper focus on the non-superneutrality of money and thus does not speak to the issue of dynamics in response to transitory shocks. Here we allow households to be heterogeneous to a limited degree. As in both Head and Shi (2003) and Williamson (2005), however, the distribution of money holdings across households in a stationary equilibrium may be described by a low dimension vector. This is crucial in enabling us to analyze dynamics in response to aggregate shocks.

This version of the paper is very preliminary and is circulated only for discussion purposes. In the next section we describe the environment. In section 3 we define a particular class of stationary monetary equilibrium and present some results characterizing its basic features. Section 4 contains some examples illustrating the dynamics that can arise from both transitory and persistent monetary shocks. Section 5 outlines the remaining work to be done. Proofs of propositions are in the appendix. Also, some propositions that we believe to be true but have not yet fully proven are presented as "conjectures".

2. The Economy

2.1. The Environment

Time is discrete with an infinite horizon. There are $H \geq 3$ non-storable goods which may be produced and consumed. In a manner similar to that employed by Head and Shi (2003), the world is comprised of two equally sized groups of households who we will refer to as type 1 and type 2 households.[1] All households in each of these two groups are symmetric in all respects *except* with regard to the goods they produce and consume. Within each type there unit measures of $H \geq 3$ different sub-types of household. Regardless of type (1 or 2), a sub-type h household produces good h and derives utility only from consuming good $h+1$, modulo H. All households, regardless of type or sub-type, are comprised of unit measures each of two different kinds of members: buyers and sellers. Individual household members do not have independent preferences and do not undertake independent actions. Rather, they act only on instructions from the household.

Let types be indexed by $i = 1, 2$, Consider a household of any sub-type, $h \in \{1, \ldots, H\}$, belonging to type i. This is without loss of generality, since sub-types are entirely symmetric. Members of this household who are sellers may produce good h at constant disutility ϕ_{it} utils per unit at time t. Production costs are stochastic, and in each time period all sellers from households of type i have the same cost. Let $\phi_{it} \in \{\phi_L, \phi_M, \phi_H\}$ for all i and t. Also suppose that ϕ_{it} evolves via a discrete Markov Chain with

$$\text{Prob}\left[\phi_{it+1} = \phi' | \phi_{it} = \phi\right] \equiv \pi^{\phi}(\phi', \phi) \qquad i = 1, 2, \forall t. \tag{2.1}$$

Let y_{it} denote the total quantity produced by the sellers of a type i household (of any sub-type) at time t. Then this household's total disutility from production in period t is equal to $\phi_{it} y_{it}$.

Members of this household who are *buyers* observe individually random numbers of price quotes and may purchase good $h+1$ at the lowest price that they observe. Each period, all buyers observe a single price quote without cost. Additional price quotes arrive at each buyer at Poisson rate n_{it}, where this arrival rate is common across buyers and chosen by the household. At the end of a unit interval of time, each buyer is able to recall and purchase at the lowest price observed individually. Increases in the arrival rate of additional price quotes is subject to a proportional utility cost, μ. In period t, if a household's buyers observe additional prices at rate n_{it}, then its search costs for that period are equal to μn_{it}.

[1] The extension to L types is logically straightforward but both notationally and numerically complex.

The price posted by an individual seller is observable only by buyers of a particular type. We will think of buyers of a particular type and sub-type congregating in a particular "home" market or location. Sellers of the same type may post prices in the home market/location of the appropriate sub-type without cost. If they want their prices to be available to buyers of the other type, however, they must "travel" to these buyers' location at a cost. Households choose measures of sellers which target different types of buyers. We assume that there is fraction of sellers that were originally chosen to target each type of buyer. Let $\bar{z}_{i,-i}$ denote the fraction of type i sellers originally designated to sell in the *other* market $(-i)$. Note that $\bar{z}_i^i = 0$ for $i = 1, 2$ is a possibility. Deviations from having measure $\bar{z}_{i,-i}$ of sellers targeting "foreign" buyers result in adjustment costs of the following form:

$$\mathcal{Z}(z_{i,-it}) = \xi(z_{i,-it} - \bar{z}_{i,-i})^{\nu}, \qquad \nu \geq 1, \quad \xi \geq 0, \tag{2.2}$$

where the case of zero adjustment cost may be attained by setting $\xi = 0$.

All households have identical preferences over their preferred good. A representative type i household (of any sub-type) acts to maximize the expected discounted sum of its period utility over an infinite horizon:

$$U = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[u(c_{it}) - \phi_{it} y_{it} - \xi(z_{i,-it} - \bar{z}_{i,-i})^{\nu} - \mu n_{it} \right] \right]. \tag{2.3}$$

The household's period utility equals that which it receives from consumption of goods purchased by its buyers minus the production disutility incurred by its sellers and its search costs. Consumption utility is given by $u(c_{it})$ where c_{it} is the total purchase of good $h+1$ by the household's buyers. For all types and in all time periods, $u(\cdot)$ is strictly increasing and concave with $\lim_{c \to 0} u'_c(c)c = \infty$. In examples later we will specialize utility to have the constant relative risk aversion form:

$$u(c_{it}) = \frac{{c_{it}}^{1-\alpha} - 1}{1 - \alpha}, \qquad \alpha > 1, \qquad i = 1, 2. \tag{2.4}$$

Since goods are non-storable and a sub-type h household produces good h and consumes good $h+1$, a double coincidence of wants between memebers of any two households is impossible. Moreover, it is assumed that households of a give sub-type (regardless of type) are anonymous. As a result, direct exchanges of goods cannot be mutually beneficial. Thus, we focus on exchange facilitated by the existence of perfectly durable and intrinsically worthless *fiat money*. A household may acquire money by producing and selling goods to buyers of other households. The money may then be used by the household's buyers to acquire the appropriate consumption good in future periods.

In the initial period ($t = 0$) *all* households are endowed with M_0 units of fiat money. The *per household* stock of this money is denoted M_t for each t. At the beginning of each period $t \geq 1$, households receive a common lump-sum transfer, $(\gamma_t - 1)M_{t-1}$, of new units of fiat money from a central "monetary authority". Thus, the *per household* money stock evolves as follows:

$$M_{t+1} = \gamma_t M_t. \tag{2.5}$$

The money growth rate is stochastic, and like the cost parameters ϕ_1 and ϕ_2, follows a Markov chain with $\gamma \in \{\gamma_L, \gamma_M, \gamma_H\}$ and

$$\text{Prob}\left[\gamma_{t+1} = \gamma' | \gamma_t = \gamma\right] \equiv \pi^\gamma(\gamma', \gamma) \qquad i = 1, 2, \forall t. \tag{2.6}$$

It is useful for notational purposes to define the vector $\sigma_t = (\phi_{1t}, \phi_{2t}, \gamma_t)$, of exogenous stochastic parameters. Since both ϕ and γ are independently and identically distributed across both types and time, using (2.1) and (2.6) we may define a time invariant probability distribution over the twenty-seven possible values of σ.

The final component of our environment which we describe here is the timing of events within a period. At the beginning of period t, all households observe the state of the economy and have individual (post-transfer) money holdings, m_t. The economy wide state will be written

$$s_t = \left(\frac{M_{1t}}{M_t}, \frac{M_{2t}}{M_t}, \sigma_t\right) = (M_1, M_2, \sigma_t). \tag{2.7}$$

In (2.7) we introduce the notation, M_i, $i = 1, 2$ which without a time subscript will be used to denote type i households' *share* of the aggregate money stock in the current period.

The household chooses the rate at which its buyers observe additional price quotes and issues instructions to both its buyers and sellers. The buyers and sellers then split up for a trading session, the details of which will be described below. After trading, the individual members return to their respective households; buyers with their goods purchases and unspent money balances (if any), and sellers with their sales receipts in fiat money. These balances are then augmented with next period's transfer to become the household's individual holdings at the beginning of the next period, m_{t+1}.

2.2. The current period trading session

Before analyzing the household's dynamic optimization problem, it is useful to describe the trading session of the current period. In so doing we will take as given households' money holdings and valuations of money carried into the *next* period, as both of these are determined before the start of the trading session. For expositional purposes we will suppress time subscripts wherever possible in this section.[2]

In each period, trade for each consumption good takes place in two distinct markets, each associated with buyers from households of a particular type. All buyers at each location are identical. Sellers of the appropriate sub-type may be directed by their household to sell at either location. Thus, it is possible that a location, while it contains only buyers of one particular type, may contain sellers from more than one type. It is, however, costly to travel to any location *other* than that containing buyers from the same type and therefore it is also possible that certain locations will contain only sellers of one type, or no sellers at all.

In each market, sellers post fiat money prices at which they are willing to sell their good for fiat money. Buyers (again of type i households) observe a single price quote with certainty, and additional quotes which arrive at Poisson rate n_i for a unit interval of time. At the end of this period of search, buyers can recall all of their received price quotes and may purchase at the lowest price they have observed. Alternatively, they may not purchase at all and return to the household with their money holdings unspent.

We treat the household's choices regarding the current trading session in two stages. To begin with, we fix both the Poisson arrival rate of additional price quotes to buyers and the measures of sellers visiting the different market/locations. Holding these fixed, we consider households' choices of trading instructions to its individual buyers and sellers. We then return to the problems of search intensity and the allocation of sellers across locations. This two-stage separation is appropriate because trade takes place only at the end of the search interval. Note that because it is not until this time that the household knows either the number of prices observed by individual buyers or the number of times a given seller's posted price has been observed, it has no incentive to treat either buyers or sellers asymmetrically. For this reason we assume that households distribute money holdings equally to all buyers and issue to the same instructions to all buyers and to all sellers.

Denote the probability with which a buyer of a type i household observes $k \geq 1$ additional price quotes (that is, $1 + k$ quotes total) during the current trading session q_{ik}, where

$$q_{ik} = \frac{e^{-n_i} n_i^k}{k!} \qquad k \geq 1, \tag{2.8}$$

[2] Specifically, *real* variables, by which we mean probabilities and quantities, will be written without time subscripts whereas we will keep the time subscript on *nominal* quantities including distributions (*i.e. cdf*'s) of nominal prices.

Since the household contains a unit measure of buyers, all of whom receive price quotes at the same rate, for any k, q_{ik} may also be interpreted as the measure of the household's *ex ante* identical buyers who observe $k+1$ prices (total) in the current period.

With the measures of buyers observing different numbers of prices fixed, the mechanism by which buyers and sellers are matched is similar to the "noisy sequential search" process of Burdett and Judd (1983) as extended to a monetary economy by Head and Kumar (2005) and Mortensen (2005). The household knows the distribution of prices offered by sellers, but individual buyers may only purchase at a price they are quoted by a specific seller in a particular period. Let $F_{it}(p_t)$ denote the cumulative distribution function (*cdf*), of the prices posted by all sellers in location i at time t, and let $\mathcal{F}_{it}$ denote its support. Time subscripts are included here to indicate that this notation refers to a distribution of *nominal* prices. Note that the sellers posting these prices may be from households not of type i.

Following Mortensen (2005) (with the modification that here we have assumed all buyers observe at least one price quote) it is straightforward to show that the *cdf* of the lowest price observed by an individual buyer from a type i household is given by

$$J_{it}(p_t) = \sum_{k=1}^{\infty} \left[1 - (1 - F_{it}(p_t))^k\right] q_{ik} = \frac{1 - ne^{-n_i F_{it}(p_t)}}{1 - e^{-n_i}} \qquad p_t \in \mathcal{F}_{it}. \tag{2.9}$$

An individual buyer who purchases does so at the lowest price observed, spending $x_{it}(p_t)$ conditional on the price paid as instructed by the household. Buyers are constrained to spend no more in a trading session than the money allocated to them at the beginning of the period by the household:

$$x_{it}(p_t) \leq m_{it} \qquad i = 1, 2,\ p_t. \tag{2.10}$$

Having allocated the same quantity of money to all buyers, the household will optimally instruct them all to behave symmetrically. Moreover, because households contain a continuum of symmetric buyers, they face no uncertainty with regard to their overall trading opportunities and total consumption in the current period. Realized household consumption purchases in this period are then

$$c_{it} = \int_{\mathcal{F}_{it}} \frac{x_{it}(p_t)}{p_t} dJ_{it}(p_t). \tag{2.11}$$

Individual sellers produce to meet the demand of the buyers who observe their price and wish to purchase. The expected quantity of goods sold in the current period for a type i seller (of any sub-type h) who posts p_t in market/location ℓ is given by:

$$y_{i\ell}(p_t) = \frac{X_{\ell t}(p_t)}{p_t} \left[\frac{1}{W_\ell} \sum_{k=1}^{\infty} Q_{\ell k} k \left[1 - F_{\ell t}(p_t)\right]^{k-1}\right] \tag{2.12}$$

$$= \frac{X_{\ell t}(p_t)}{p_t}\left[\frac{1}{W_\ell}\frac{N_\ell e^{-N_\ell F_{\ell t}(p_t)}}{1-e^{-N_\ell}}\right], \tag{2.13}$$

Note that sales depend on the conditions in the market in which the seller operates, ℓ in (2.12) and (2.13). Here $X_{\ell t}(p_t)$ is the spending rule of a type ℓ buyer, $Q_{\ell k}$ is the aggregate fraction of type ℓ buyers observing k additional prices, and N_ℓ is the average search intensity of buyers in market/location ℓ. Also, W_ℓ is the measure of sellers in market/location ℓ in the current period and is given by

$$W_\ell = Z_{\ell\ell} + Z_{-\ell\ell}, \tag{2.14}$$

where $Z_{-\ell,\ell}$ is the measure of sellers chosen to visit market/location ℓ by households of type $-\ell \neq \ell$ in the current period.

In both (2.12) and (2.13), $X_{\ell t}(p_t)/p_t$ represents the quantity of goods exchanged per sale and the bracketed term is the expected number of sales. The expected number of sales conditional on the price posted is the expected number of times that the seller's price will be observed *and* be the lowest of the prices observed by that buyer. This is the product of the ratio of buyers to sellers in the particular location ($1/W_\ell$ because the measure of buyers in each location is always one), the total number of price observations, and the probability that of the k prices observed by any buyer, the posted price, p_t is the lowest. The algebraic steps performed in going from (2.12) to (2.13) are similar to those taken by Mortensen (2005).

Let $\hat{F}_{i\ell t}(p_t)$ be the distribution of prices posted by sellers of a type i household in market/location ℓ in the current period , and let $\hat{\mathcal{F}}_{i\ell t}$ denote its support. Since the household contains a continuum of sellers, it faces no uncertainty with regard to its total sales in the current trading session. These sales are the sum of those by its sellers in all the markets in which they participate:

$$y_i = \sum_{\ell=1}^{2} z_{i\ell}\int_{\hat{\mathcal{F}}_{i\ell t}} y_{i\ell}(p_t)\,d\hat{F}_{i\ell t}(p_t). \tag{2.15}$$

Note that expected sales in any market/location depend only on the price posted, not on the seller's type. A type i household's sales revenue in units of fiat money is given by

$$rev_{it} = \sum_{\ell=1}^{2} z_{i\ell}\int_{\hat{\mathcal{F}}_{i\ell t}} p_t y_{i\ell}(p_t)\,d\hat{F}_{i\ell t}(p_t). \tag{2.16}$$

We then have an expression describing the law of motion for a representative type i household's money holdings:

$$m_{it+1} = m_{it} - \int_{\mathcal{F}_{it}} x_{it}(p_t)\,dJ_{it}(p_t) + rev_{it} + \frac{(\gamma_{t+1}-1)}{2}M_t. \tag{2.17}$$

A representative households money holdings going into next period's trading session are its money holdings at the beginning of *this* period minus the amount spent by its buyers plus its sellers' sales revenue plus the transfer received at the beginning of the next period. Note that all households receive an equal transfer.

We now characterize households' choices of instructions $x_{it}(p_t)$ and $\hat{F}_{i\ell t}(p_t)$, $\ell = 1, 2$, to its buyers and sellers respectively. Consider first the expenditure rule given to the households' buyers, $x_{it}(p_t)$. The household's gain to having a buyer exchange $x_{it}(p_t)$ units of money for consumption at price p_t is given by the household's marginal utility of current consumption, $u_c(c_{it})$ times the quantity of consumption good purchased, $x_{it}(p_t)/p_t$. The household's cost of this exchange is the number of currency units given up, $x_{it}(p_t)$ times the marginal value of a unit of money in the trading session of the next period, which we denote ω_{it}. Note that ω_{it} is the value of relaxing constraint (2.17) marginally.

Individual buyers are small and the household may not reallocate money balances across buyers once the trading session has started and they have begun to observe prices. We then have the following proposition:

Proposition 1: Households issues the common expenditure rule to all buyers:

$$x_{it}(p_t) = \begin{cases} m_{it} & \text{for } p_t < \frac{u_c(c_{it})}{\omega_{it}} \\ m_{it} & \text{with probability } 1 - \epsilon \text{ for } p_t = \frac{u_c(c_{it})}{\omega_{it}} \\ 0 & \text{otherwise.} \end{cases} \tag{2.18}$$

That is, households instruct buyers to spend their entire money holdings if the lowest price they observe is below a *reservation price* and to return with money holdings unspent otherwise. If the lowest price is exactly equal to the reservation price, with some (possibly very small) probability, ϵ, the buyer refuses to buy.[3]

Next, consider the household's price posting strategies (*i.e.* the instructions it gives to its sellers). The expected return to a type i household from having a seller post price p_t in market/location ℓ in the current trading session is given by

$$r_{i\ell t} = \left[\omega_{it} X_{\ell t}(p_t) - \phi_i \frac{X_{\ell t}(p_t)}{p_t}\right] \left[\frac{1}{W_\ell} \frac{N_\ell e^{-N_\ell F_{\ell t}(p_t)}}{1 - e^{-N_\ell}}\right], \tag{2.19}$$

[3] Choosing a small positive probability with which buyers do not trade at the reservation price (when the household is indifferent to trade) enables us to rule out a number of uninteresting equilibria. We return to this issue in the next section.

where again $F_{\ell t}(p_t)$ is the *cdf* of prices posted by *all* sellers in market/location ℓ and $X_{\ell t}(p_t)$ and N_ℓ are the expenditure rule and search intensity of type ℓ buyers. Note that in (2.19) the components which are specific to type i sellers are the valuation of money, ω_{it}, and the production cost, ϕ_i, both of which are components of the return to each sale. The probability of making a sale depends only on the price posted and the search intensity of buyers.

It is clear from (2.19) that no type i seller would be instructed to post any price lower than $p^*_{it} = \phi_i/\omega_{it}$ in *any* market, as the return to doing so is negative. Also, the return to *any* seller of posting a price greater than the common reservation price of the buyers present in any market is zero. Households maximize returns by instructing sellers to post in the various markets only prices such that

$$p_t \in \underset{p_t}{\operatorname{argmax}}\, r_{i\ell t}(p_t) \equiv \hat{\mathcal{F}}_{i\ell t}. \tag{2.20}$$

Because households receive the same expected return from a seller who posts any price in $\hat{\mathcal{F}}_{i\ell t}$, we express its instructions as a *cdf*, $\hat{F}_{i\ell t}$, on support $\hat{\mathcal{F}}_{i\ell t}$ and think of sellers as drawing their prices randomly from this distribution. At this stage, however, we make no claims about the characteristics of this distribution.

In addition to instructions regarding the actions of its individual members, the household must also choose search intensity and the measure of its sellers that visit the other market. Consider first the former. As all buyers transact in their home market/location and the household takes as given the distribution of posted prices, $F_{it}(p_t)$, optimal search intensity in the current period satisfies

$$n_i = \underset{n}{\operatorname{argmax}} \left\{ u(c_{it})|_{F_{it}(p_t)} - \mu n_i \right\}. \tag{2.21}$$

The household takes as given the distribution of prices posted in the market, but considers how the distribution of the lowest price observed by its own buyers varies with search intensity, n_i.

The household chooses the measures of its sellers that visit different locations to equate the marginal returns across markets net of participation costs. That is, to solve

$$\max_{z_{i\ell} \in [0,1]} z_{ii} r_{iit} + z_{i,-i} r_{i,-it} - \xi(z_{i,-i} - \bar{z}_{i,-i})^\nu \tag{2.22}$$

subject to:

$$z_{ii} + z_{i,-i} = 1. \tag{2.23}$$

This problem gives rise to the following system of first-order and complementary slackness conditions:

$$r_{i,-i} - \nu\xi(z_{i,-i} - \bar{z}_{i,-i})^{\nu-1} - \upsilon \le 0, \quad z_{i,-i} \ge 0, \quad z_{i,-i}[r_{i,-i} - \nu\xi(z_{i,-i} - \bar{z}_{i,-i})^{\nu-1} - \upsilon] = 0, \tag{2.24}$$

where υ is a Lagrange multiplier associated (2.23).

2.3: Dynamic optimization

At time t, the relevant state variables for a representative type i household are its beginning of period individual money holdings, m_t; the distribution of money holdings across households of different types, M_{1t}, M_{2t} and σ_t. Since we consider situations in which the aggregate money stock grows, as in (2.9) it is useful to divide all nominal variables by the economy-wide money-stock, M_t. From this point on all nominal variables (*i.e.* quantities of money and prices) will have been normalized by M_t to obtain "real" counterparts. Also, time subscripts will be suppressed in the usual fashion when using dynamic programming.

We represent the dynamic optimization problem of a type i household by the following Bellman equation:

$$v^i(m, M_1, M_2, \sigma) =$$

$$\max_{m', n_i, x_i(p), z_{i,-i}, \hat{F}_{i\ell}(p)} \left\{ u(c_i) - \phi_i y_i - \xi(z_{i,-i} - \bar{z}_{i,-i})^{\nu} - \mu n_i + \beta \mathrm{E}\left[v^i(m', M_1', M_2', \sigma')\right] \right\} \tag{2.25}$$

subject to:

$$(2.1), (2.5), (2.6), (2.8) - (2.11), \quad \text{and} \quad (2.13) - (2.17).$$

The household takes as given the actions of other households and the aggregate distributions of posted prices at both relevant market/locations, all of which it treats as functions of the aggregate state, (M_1, M_2, σ). From the Bellman equation, we have first-order conditions associated with choice of m':

$$\omega_i(m, M_1, M_2, \sigma) = \beta \mathrm{E}\left[v_m^i(m', M_1', M_2', \sigma')\right], \tag{2.26}$$

and with choice of $x_i(p)$:

$$u_c(c_i)\frac{1}{p} - \lambda_i(p; m, M_1, M_2, \sigma) - \omega_i(m, M_1, M_2, \sigma) = 0 \qquad \forall p \in \mathcal{F}_i, \tag{2.27}$$

where $\lambda_i(p; \cdot)$ is a Lagrange multiplier on the buyers' expenditure constraint, (2.10). The first-order and complimentary slackness conditions associated with the choice of $z_{i,-i}$ are given by (2.24), and the first-order condition associate with choice of n_i is

$$\mu = u_c(c_i)\left[\frac{\partial c_i}{\partial n_i}\right]_{F_i(p)}. \tag{2.28}$$

Finally, we have the envelope condition,

$$v_m^i = \int_{\mathcal{F}_i} \lambda_i(p; \cdot) dJ_i(p) + \omega_i(\cdot). \tag{2.29}$$

Equations (2.26)—(2.29), (2.21), and (2.24); together with the buyers' expenditure rules, (2.18), and the optimal price-posting conditions, (2.20), characterize a (type i) household's optimal behaviour condition on its money holdings, m, the aggregate state, (M_1, M_2, σ), and its beliefs regarding the actions of other households.

3. Equilibrium

We consider only equilibria that are symmetric and Markov. By symmetric, we mean that in equilibrium all households of the same type make the same choices and thus have the same money holdings and the same marginal valuation of money, Ω_{it} in all periods. By Markov, we mean that the quantitites, C_{it} and Y_{it}, search intensity, N_{it}, the allocation of sellers across market/locations, Z^1_{2t} and Z^2_{1t}, and the distributions of *real prices* (*i.e.* nominal prices divided by the per household money stock, M_t), are all time invariant functions of the aggregate state, (M_1, M_2, σ), which itself is Markov, evolving according to (2.7) and (2.19)

In such an equilibrium, if one exists, all buyers of a given type have common reservation prices and equal money holdings so that we have

$$C_i(M_1, M_2, \sigma) = M_i \int_{\mathcal{F}_i} \frac{1}{p} dJ_i(p), \tag{3.1}$$

where all nominal variables have been normalized by the aggregate money stock, M_t. In order for (3.1) to be satisfied, must depend only on the aggregate state. If conditional on (M_1, M_2, σ), *all* nominal prices are proportional to the aggregate money stock M, then there exist state-contingent (but time invariant) distributions of *real* posted prices characterized by supports $\mathcal{F}_i(M_1, M_2, \sigma) \equiv \{p_t/M_t, p_t \in \mathcal{F}_{it}$; for all t such that the state is given by (M_1, M_2, σ), and conditional *cdf*'s:

$$F_i(p|M_1, M_2\sigma) = F_{it}(p_t) \qquad \forall p \in \mathcal{F}_i(M_1, M_2\sigma). \tag{3.2}$$

If conditional distributions satisfying (3.2) exist, then we may think of buyer as observing real price quotes, and define conditional distributions of lowest real prices observed:

$$J_i(p \,|\, M_1, M_2, \sigma) = \frac{1 - N_i e^{-N_i F_i(p|M_1, M_2, \sigma)}}{1 - e^{-N_i}}. \tag{3.3}$$

We then have the following definition:

Definition: A *symmetric monetary equilibrium* (SME) is a collection of time-invariant, type-specific household choices, $n^i(m.M_1, M_2\sigma)$, $z_{i,-i}(\cdot)$, $x_i(p;\cdot)$. $\hat{F}_{i\ell}(p;\cdot)$ m; common expenditure rules, $X_i(p; M_1, M_2, \sigma)$, search intensities, $N_i(\cdot)$, seller allocations, $Z_{i,-i}(\cdot)$, and distributions of posted prices, $F_i(p;\cdot)$, for all $i, \ell = 1, 2$ such that:

1. Taking as given the distributions of posted prices, $F_i(p;\cdot)$, common expenditure rules, $X_i(p;\cdot)$, allocation of sellers, $Z_{i,-i}(\cdot)$, and search intensities, $N_i(\cdot)$. representative households of each type choose n_i, m', $x_i(p)$, $z_{i,-i}$, and the price distributions $\hat{F}_{i\ell}(p)$ to satisfy the household Bellman equation, (2.25).

2. For both types, individual choices equal their per household counterparts: $n_i = N_i(M_1, M_2, \sigma)$, $z_{i,-i} = Z_{i,-i}(\cdot)$, $x_i(p) = X_i(p\,|\,\cdot)$, individual household money holdings equal average holdings for each type, $m_i = M_i(\cdot)$, and the distributions of prices posted at each market/location are compositions of the distributions posted by sellers from the various types. That is, they are given by

$$F_i(p\,|\,\cdot) = F_{1i}(\,|\,\cdot) + F_{2i}(\,|\,\cdot) \qquad \forall p \in \mathcal{F}_i(\cdot). \tag{3.4}$$

3. Money has value in all states:

For all (M_1, M_2, σ), $F_i(p\,|\,\cdot) > 0$ for some $i = 1, 2$ and some $p < \infty$.

In characterizing an SME for this economy, we begin by operating under the assumptions that such an equilibrium exists and that effort to obtain additional price quotes (N_i for $i = 1, 2$) is strictly positive in both markets for all states. We then derive restrictions that such an equilibrium must satisfy under these assumptions. We then demonstrate both that such an equilibrium exists and that any SME by the definition above must exhibit $N_i(\cdot) > 0$ for all states.

Under the assumption that an SME with these properties exists, key quantities describing the equilibrium are the households' marginal valuations of money, $\Omega_i(\cdot)$, as these determine the returns to both buyers and sellers from transacting at a particular price in each state. Returning to the household optimization problem and combining (2.28), (2.29), and (2.31), we have

$$\omega_i(m, M_1, M_2, \sigma) = \beta \mathrm{E}\left[u_c(C_i') \int_{\mathcal{F}_i'} \frac{1}{p}\, dJ_i(p\,|\,m', M_1', M_2', \sigma')\right]. \tag{3.5}$$

Using (3.1), in an SME, (3.5) becomes

$$\Omega_i(M_1, M_2, \sigma) = \mathrm{E}\left[\frac{\beta}{\gamma' M_i'}\left(u_c[C_i(M_1', M_2'\sigma')]C_i(M_1', M_2', \sigma')\right)\right]. \tag{3.6}$$

We thus associate an SME with two functions $\Omega_i(\cdot)$ for $i = 1, 2$. Note to begin with that Ω_i depends on the future distribution of money holdings across households, M_1', M_2', through the presence of M_i' in the denominator. This will be the source of short-run monetary non-neutrality.

Still under the assumption that an SME exists, we may then establish certain properties which it must exhibit. First, consider households' price-posting behaviour $\hat{F}^{i}_{i'}(p\,|\,\cdot)$ conditional on their beliefs regarding buyers' search intensity in the two markets, N_1 and N_2. To begin with we have the following proposition adapted from both Burdett and Judd (1983) and Head and Kumar (2005):

Proposition 2: If an SME exists in which $N_i > 0$ for $i = 1, 2$ in all states, then the distributions of posted prices in each market must be non-degenerate and continuous on connected supports.

This proposition establishes two basic aspects of the equilibria that we consider. First, there is price dispersion in all states. Second there are neither gaps in the support of the price distributions nor any mass points rendering the distributions discontinuous.

A second proposition characterizes the effects of heterogeneity among sellers that may be present in any market location in an SME.

Proposition 3: In an SME, in any state a seller whose costs (measured by ϕ_i/ω_i) are lower than those of another seller ($\phi_{i'}$) will post only prices that are below those of the higher cost seller.

This proposition is adapted from Mortensen (2003) and establishes that in any market, the support of the distribution of posted prices will be divided into two regions, with the lower cost sellers posting below the higher cost ones. Note, however, that from Proposition 2 there can be no gap in the *overall* support of the price distribution in any market/location. Since costs will differ across sellers in all states in which types differ in any respect, situations with high and low cost sellers by this definition are norm (*i.e.* they occur generically).[4] Thus, in any state, *households* may be classified as low cost and high cost, and so it is useful to refer to "low" and "high" cost types of households, which we will distiniguish with subscripts, "L" and "H", respectively.

Using Propositions 1, 2, and 3, we may then construct expressions for the distributions of posted prices that must arise in any SME as functions of the marginal valuations of fiat money, Ω_i, $i = 1, 2$. First, combining households' expenditure rules (2.20) with (2.22) we have that in any SME the support of the distribution of posted prices in market/location i must be contained in the interval $[p^*_i, \bar{p}_i]$ where

$$p^*_i = \frac{\phi_L}{\Omega_L} \qquad \text{and} \qquad \bar{p}_i = \frac{u'(C_i)}{\Omega_i} \tag{3.7}$$

[4] In situations in which all sellers are identical in all respects, the symmetry conditions in the definition of an SME require them to behave identically.

with ϕ_L/Ω_L representing the real marginal cost of the lower cost seller. Note that the sellers' costs do not depend on which market they are selling in.

The upper support of the distribution of posted prices is given by the reservation price $\bar{p}_i$ of the buyers in that market since a positive measure, e^{-N_i}, of buyers observes a single price. If the upper support of the price distribution were below $\bar{p}_i$, any household can profitably deviate by having all of its sellers post $\bar{p}_i$. This clearly is not consistent with equilibrium.

Making use of Propositions 2 and 3, we may then write out the following set of four equations characterizing the "equal profit" conditions governing the supports of the distributions of prices posted by high and low cost sellers in each of the two markets:

$$\begin{aligned} \left(\Omega_L - \frac{\phi_L}{p}\right) e^{-N_i F_{Li}(p)} &= \left(\Omega_L - \frac{\phi_L}{\bar{p}_i^L}\right) e^{-N_i V_i} \\ \left(\Omega_H - \frac{\phi_H}{p}\right) e^{-N_i F_{Hi}(p)} &= \left(\Omega_H - \frac{\phi_H \Omega_i}{u'(C_i)}\right) e^{-N_i} \end{aligned} \qquad i = 1, 2. \tag{3.8}$$

In (3.8) the first (two) equations establish that all prices posted by low cost sellers in each market must generate equal profits. In this equation $\bar{p}_i^L$ represents the maximal price charged by low cost sellers in market i and $V_i = F_i(\bar{p}_i^L)$, the value of the *cdf* of posted prices in market-location i at this price. Given Proposition 3, we have

$$V_i = \frac{Z_{Li}}{Z_{Li} + Z_{Hi}}, \tag{3.9}$$

where Z_{Li} and Z_{Hi} denote the measures of low and high cost sellers present in market/location i. Note that in states in which one households differ by costs it is sometimes also useful to refer to the market in which the buyers come from low-cost households the "low" market, etc. Finally, note that (3.8) reflects the result that the highest price posted by the low cost sellers is equal to the lowest price posted by the high cost sellers.

The system of equations (3.8) may then be straightforwardly solved for the following expressions for the distributions of prices posted by both high and low cost sellers in each market-location in any SME:

$$\begin{aligned} F_{Li}(p) &= V_i - \frac{1}{N_i}\left[\ln\left(1 - \frac{\phi_L}{\Omega_L \bar{p}_i^L}\right) - \ln\left(1 - \frac{\phi_L}{\Omega_L p}\right)\right] \\ F_{Hi}(p) &= 1 - \frac{1}{N_i}\left[\ln\left(1 - \frac{\phi_H \Omega_i}{\Omega_H u'(C_i)}\right) - \ln\left(1 - \frac{\phi_H}{\Omega_H p}\right)\right] \end{aligned} \qquad i = 1, 2. \tag{3.10}$$

The overall distributions of posted prices in each market-location are then compositions of these type-specific distributions:

$$F_i(p) = \begin{cases} F_{Hi}(p) & \text{if } p \in (\bar{p}_{it}^L, \bar{p}_i] \\ F_{Li}(p) & \text{if } p \leq \bar{p}_i^L. \end{cases} \qquad i = 1, 2. \tag{3.11}$$

Given that by Proposition 3 the supports of high and low cost sellers do not overlap, (3.11) is consistent with (3.4). Finally, note that the distribution of transactions prices in each market location is given by (2.11) evaluated at the appropriate distribution in (3.11):

$$J_i(p) = \frac{1 - N_i e^{-N_i F_i(p)}}{1 - e^{-N_i}}. \tag{3.12}$$

An important aspect of the evolution of the distribution of money holdings across household types is apparent by comparing (3.10) and (3.11). Because low cost sellers make more sales than their high cost competitors, they will collect money in greater proportion to their numbers in either market. The magnitude of the flows of money between different types of households is thus determined not only by the relative numbers of sellers present in each market, but also by the search intensities of the buyers present. The latter determines the extent to which the distribution of transactions prices differs from that of posted prices. We now turn to the determination of search intensity and the degree of participation in the "foreign" market, making use of the restrictions (3.8)—(3.12) on the distributions of posted and transactions prices in an SME.

To this point we have assumed that in equilibrium the expected number of additional price quotes observed by all buyers in an SME is always interior. We now have the following proposition establishing that this is indeed true.

Proposition 4: For any strictly positive probability with which buyers do not exchange at the reservation price (i.e. for any $\epsilon > 0$) there exists a search cost parameter, μ low enough that $N_i > 0$ for all i in all states in any SME.

This proposition eliminates the possibility that buyers and sellers will coordinate on strategies such that in some states and market-locations buyers do not search at all ($N_i = 0$) and all buyers charge the reservation ("monopoly") price. Note that even if $\epsilon = 0$, there can be no monetary equilibrium in which $N_i = 0$ in *all* states (Head, Kumar, and Lapham (2005)).

Given Proposition 4, the optimal choice of search intensity in each state is implicitly characterized by the following first-order condition:

$$\mu = u_c(C_i) \frac{\partial C_i}{\partial n_i}|_{F_{it}(\cdot)} \tag{3.13}$$

where C_i is given by (3.1). Note that in choosing n_i each household takes the distribution of *posted* prices, $F_i(p)$, as given, but considers how its expected transactions prices change with its search intensity. As in Mortensen (2005), system (3.13) (which is algebraically messy) cannot be solved analytically for N_1 and N_2. It can, however, be solved numerically in each state given Ω_i for $i = 1, 2$.

Next we turn to the determination of participation in the "foreign" market. Here we simply note that In an SME, (2.22) may be written:

$$\begin{aligned} r_{Li} &= \frac{M_i}{W_i}\left(\omega_L - \frac{\phi_L}{\bar{p}_i^L}\right)\left[\frac{N_i e^{-N_i V_i}}{1 - e^{-N_i}}\right] \\ r_{Hi} &= \frac{M_i}{W_i}\left(\omega_H - \frac{\phi_H \Omega_i}{u'(C_i)}\right)\left[\frac{N_i e^{-N_i}}{1 - e^{-N_i}}\right] \end{aligned} \qquad i = 1, 2. \tag{3.14}$$

Then, under the assumption of interior solutions, (2.24) may be written:

$$r_{ii} - r_{i,-i} = \nu\xi(z_{i,-i} - \bar{z}_{i,-i})^{\nu-1}, \qquad i = 1, 2. \tag{3.15}$$

For now we maintain the assumption of an interior solution. We have the following (as yet unproven) conjecture:

Conjecture 5: Under certain conditions, there exists an SME in which in all states where households are heterogenous, choices of participation in the foreign market are interior for both types.

This proposition is certainly true, as we can produce examples of such equilibria. What remains to be derived here are the "certain conditions".

At this point we can combine several expressions to write out the laws of motion for money holdings (2.17) in an SME:

$$\begin{aligned} \gamma' M_L' &= \left[\frac{1 - e^{-N_L V_L}}{1 - e^{-N_L}}\right] M_L + \left[\frac{1 - e^{-N_H V_H}}{1 - e^{-N_H}}\right] M_H \\ \gamma' M_H' &= \left[\frac{e^{-N_L V_L} - e^{-N_L}}{1 - e^{-N_L}}\right] M_L + \left[\frac{e^{-N_H V_H} - e^{-N_H}}{1 - e^{-N_H}}\right] M_H. \end{aligned} \tag{3.16}$$

System (3.15) describes the evolution of the distribution of money holdings across household types. Since

$$V_L = \frac{Z_{LL}}{Z_{LL} + Z_{HL}} \qquad \text{and} \qquad V_H = \frac{Z_{LH}}{Z_{LH} + Z_{HH}}, \tag{3.17}$$

It can be seen that both participation in the foreign and the degree of search intensity affect the rate at which money flows between households of different types. Participation affects the rate at which sellers prices are observed by households of the other type. Search intensity affects the share of sales going to the lower cost sellers in both markets.

Using (3.16) it is possible to compare the economy to that of Williamson (2005). In Williamson's economy, the rate at which members of the two types of households meet, π in his notation, plays a role analogous to the participation rates in our economy. The relative sizes of the two types, α in his economy, plays a role in some sense similar to that of search intensity here, which determines the relative numbers of transactions in which the two (equal sized here) types of households engage. A key difference between our economy and Williamson's is that α and π are exogenous parameters in his economy, whereas in ours both search intensity and the degree to which sellers participate in foreign markets are endogenous.

Because participation is endogenous and could equal zero in some states, stability of the distribution of money holdings in a stochastic equilibrium is an issue. At this stage we have not derived explicit conditions for ergodicity. We have, however, generated examples in which the distribution is stable, and we consider one in the next section.

4. An Example

In this section we present a particular example that is illustrative of the type of dynamics that may result from a shock to the money growth rate when types are heterogeneous. In our economy, even if types are entirely symmetric, different realizations of the production disutility parameters, ϕ_1 and ϕ_2 will cause them to have different money holdings at any point in time. Whenever types hold different fractions of the money stock, lump-sum monetary injections of the type considered here will be non-neutral. In order to isolate the effects of particular shocks, however, we restrict the environment in a number of ways.

First, we assume that households are different with regard to their cost parameters and hold these parameters fixed. In particular, we will let type 1 be the low cost household in all periods by setting $\phi_1 = \phi_L < \phi_2 = \phi_H$. In all other respects types are identical. In this way, we can construct an example in which in all periods, type 1 sellers are the low-cost sellers in both markets, and market 1 contains only buyers from low-cost households. The following table lists the parameters for this example:

Parameters for an asymmetric example

$\alpha = 1.5$	Curvature in preferences
$\beta = .99$	Discount factor

$\mu = .0015$	Search cost
$\gamma = 1.0062$	Money creation
$\bar{z}^i_{-i} = .2$	Presence in "foreign" market
$\phi_L = .16$	Low cost disutility
$\phi_H = .2$	High cost disutility

To begin with we consider a steady-state monetary equilibrium with these parameters, by which we mean that both the cost parameters and money growth rate are constant over time. In this economy, the inflation rate is constant and equal to the money growth rate, 2.5%, at "annual rates" ($i.e$ over four periods given $\beta = .99$). With $z_{i,-i} = .2$, the measures of sellers of each type operating in their "home market" is close to eighty percent. This reflects relatively high costs of varying participation in the foreign market away from $\bar{z}_{i,-i}$.

The steady-state equilibrium of the economy described by these parameter values has the following general characteristics:

1. The low cost households consume more than their high cost counterparts and have larger money holdings.

2. They also have higher markups than high cost sellers even though they charge lower prices by Proposition 3.

3. Because they have higher consumption, their value of search is lower and so they search less intensely than their high cost counterparts.

4. Average markups (both within and across markets) are therefore higher (20%) for low cost sellers than for high cost sellers (10%) so that the average markup in the entire economy is roughly 15%.

For our purposes, the most important aspect of the steady-state is the distribution of money across types. In this example, $M_1 = M_L = .55$ in the steady-state, whereas $M_2 = M_H = .45$. Thus, a lump-sum monetary injection will raise the holdings of high cost households by more than those of low cost ones. In the steady-state, however, the fact that low cost sellers make more sales than their high cost counterparts accounts for the fact that the money holdings return to this invariant distribution each period in spite of the fact that there is money creation each period through equal lump-sum transfers.

We now consider the following experiment, intended to match that in conducted in the baseline economy of Altig, Christiano, Eichenbaum, and Linde (2004): The economy is initially in its steady-state, and then experiences an increase of the money growth rate equal to two percent of its average value. The money growth rate then returns to its long-run value with an autocorrelation of .24. Figures 1-4 depict the responses of money holdings, search intensity, inflation, and consumption to this shock.

Considering first Figure 1, note that the shock initially drives money holdings closer together, raising the low money holdings of the high cost sellers and lowering the high money holdings of the low cost sellers. After one period, however, relative money holdings begin to return to their long run level. Money holdings overshoot, however, and eventually return to their long run level from a distribution that is more dispersed than in the steady-state. The particular pattern here appears (based on other experiments) to be typical, but not exclusive. That is, in some cases money holdings are disturbed from their long-run distribution and return monotonically to the steady-state. In all cases, however, the distribution of money holdings evolves over many periods, remaining out of the steady-state long after money growth has returned to its long-run level.

To understand both the adjustment of the distribution of money holdings and movements in prices, it is useful to consider the response of search intensity to the shock. In Figure 2 it can be seen that search intensity rises in the market with low-cost buyers and falls (but by much less) in the other market. These movements in search intensity can be understood by considering the fact that other things equal, an increase in the money growth rate tends to raise real prices only to the extent that it signals that money will be worth less in the future. Since the increase in money growth here is not very persistent, we will treat the direct effect of the shock on real prices as small,

For those households who find their money holdings high as a result of the shock (the high cost household), as long as the increase in real prices due to the inflation tax is small, the return to search is diminished as marginal utility falls due to increased purchasing power. Thus search intensity falls for the high cost agents, but rises for low cost agents for symmetric reasons; their money holdings have fallen.

Changes in search intensity affect the flows of money across household types by altering the fraction of sales that low-type sellers make to high cost buyers and that high-type sellers make to low cost buyers. The movements in search intensity depicted here *reduce* the number of transactions across types in both markets. Since low-cost buyers have high money holdings relative to high cost buyers (even though the gap is smaller than in the steady-state) the increase in transactions of this type can serve to push the money holdings apart, that is back toward their steady-state levels. Or, it could not. The path of money holdings depends on the relative magnitudes of the shift in the fraction of transactions that occur and of the differences in the size of money holdings across household types. For this reason, different patterns for the adjustment of money holdings can occur in different circumstances. It does not appear, however, that the particular pattern is of much importance for aggregate dynamics. What is important, is that the distribution of money holdings is disturbed from the steady-state for a long time.

With regard to the dynamics of inflation, a key relationship is the magnitude of the responses of search intensity in the two markets. An increase in search intensity in the low cost market lowers markups, limiting price increases. In contrast, the reduction in search intensity in the high cost market is associated with an increase in markups. Since in this example the former dominates, we might expect that the average markup falls, generating a reduction in real prices on average and inflation that is less than the increase in the money stock. Considering Figure 3 we can see that this is exactly what happens in the initial period. In subsequent periods, however, it can be the case that the deviations of search intensity and markups from their steady-state levels are not enough to lower real prices in the presence of a greater money supply. In the example depicted here it can be seen that in subsequent periods inflation exceeds the growth rate of the money stock as the economy returns to the steady-state.

Finally, considering Figure 4 it is clear that the shock is expansionary overall as it raises the relatively high consumption of the low-cost types by more than it reduces that of the high-cost types. Thus, aggregate consumption is increased for many periods following the shock.

From the preceding discussion, it is clear that a money shock of this type generates a number of effects which determine the dynamics of inflation and the degree of short-run non-neutrality. Since these effects to some extent conflict with each other, dynamics are really a quantitative issue and thus we cannot conclude much on the basis of an example. This example is sufficient, however, to demonstrate that the following results are possible in our economy:

1. An increase in the money stock is in general non-neutral in the short-run and could result in movements in aggregate consumption for a significant period of time.

2. The response of search intensity to movements in the distribution of money holdings can generate muted responses of the price level (*i.e.* price stickiness) to monetary shocks.

3. Inflation may display more persistence than the money growth rate.

These qualitative aspects of the response to money shocks appear to be typical in our economy. Therefore, our future work will focus on quantifying the effects of such shocks in our economy.

5. Further Work

Our further work on this project will focus mainly on developing quantitative results. To this end we need to solve the model globally (rather than locally around a particular steady-state) and generate the statistical properties of the economy in response to both money growth and cost shocks. For this we will use the state-space discretization method employed by Molico and Zhang (2005). Solutions, however, are much easier to obtain in our environment than in theirs because our distribution of money holdings can be summarized by a single variable, M_1/M_2.

Having generated a complete dynamic characterization of at least one example, there are two further aspects of the work. First, on the theoretical front, establishing general conditions under which stable dynamic equilibria exist. Second, choosing parameters to calibrate the economy so that its quantitative predictions are useful. To begin with, we will replicate the experiment of a monetary policy shock in the model of Altig, Christiano, Eichenbaum, and Linde (2004).

Finally, there are a number of extensions that might help to make the model quantitative. For example, households could participate in a financial market preceding the opening of the trading session in which they decide how much money to carry into trading as opposed to holding as bonds. We would expect this to have quantitative effects, but not to change the overall character of the dynamics. As long as households are heterogeneous at least in the short-run, money will be non-neutral and inflation will deviate from the money growth rate with some degree of persistence.

In the examples we have considered so far, dynamics are dominated by movements in search intensity rather than in the degree of participation. We have so far only considered small shocks in a neighborhood of a stable steady-state, and these shocks simply do not cause participation to move around much. In extensions we will consider larger and perhaps more persistent shocks to examine the role of endogenous participation in dynamics.

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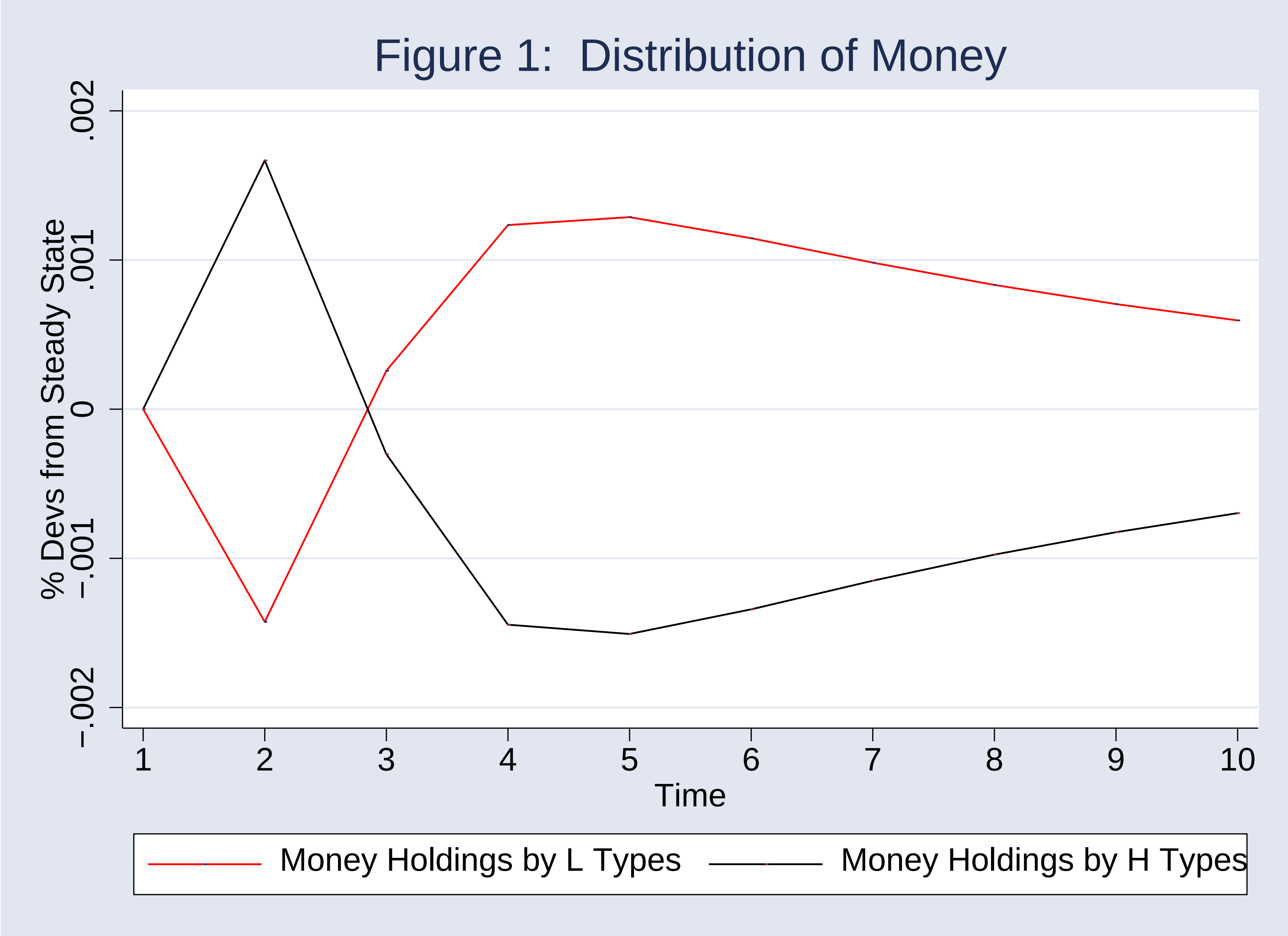

Figure 1: Distribution of Money
% Devs from Steady State
.002
.001
0
-.001
-.002
1
2
3
4
5
6
7
8
9
10
Time
Money Holdings by L Types
Money Holdings by H Types

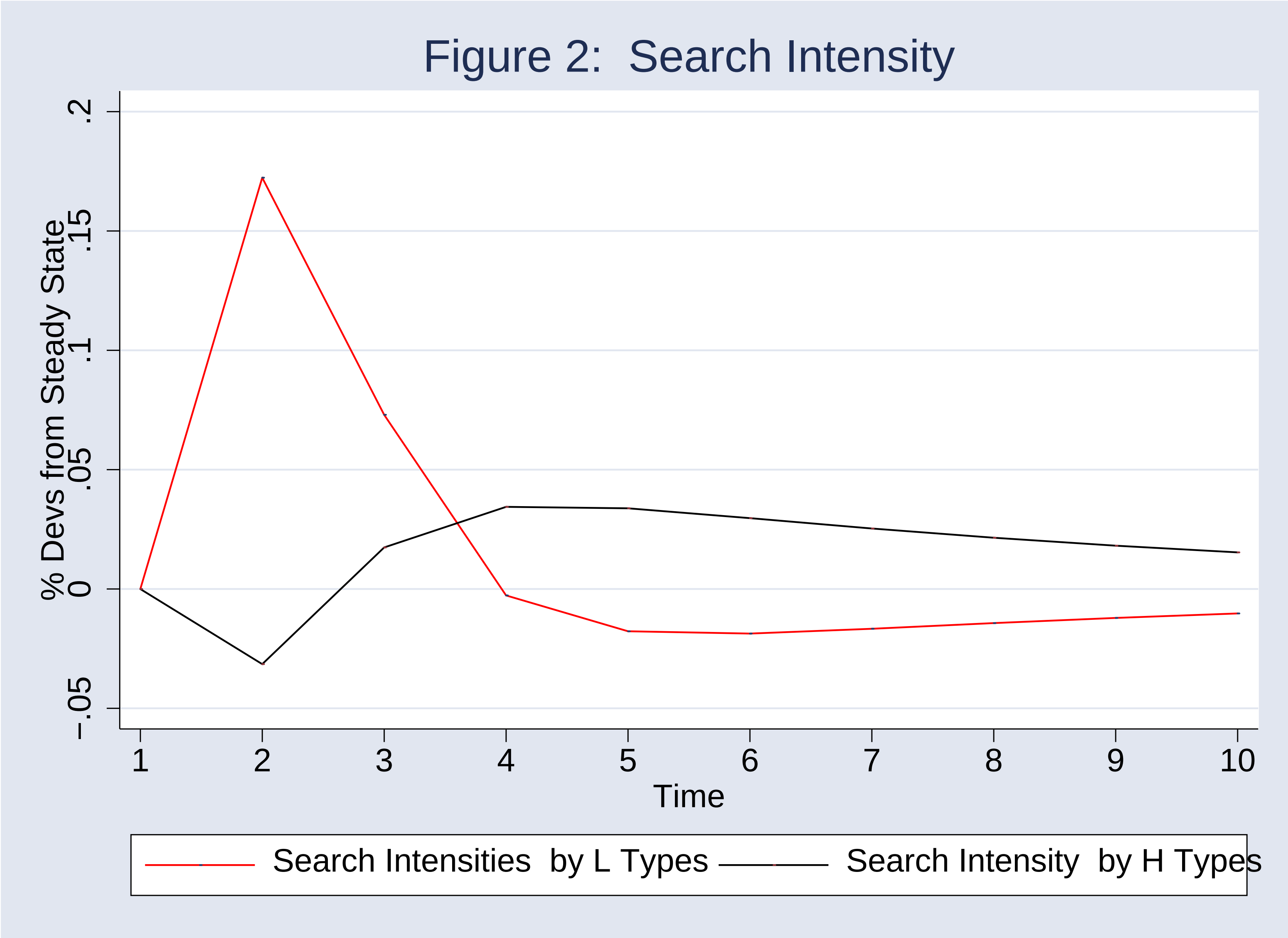
Figure 2: Search Intensity
% Devs from Steady State
-.05
0
.05
.1
.15
.2
1
2
3
4
5
6
7
8
9
10
Time
Search Intensities by L Types
Search Intensity by H Types

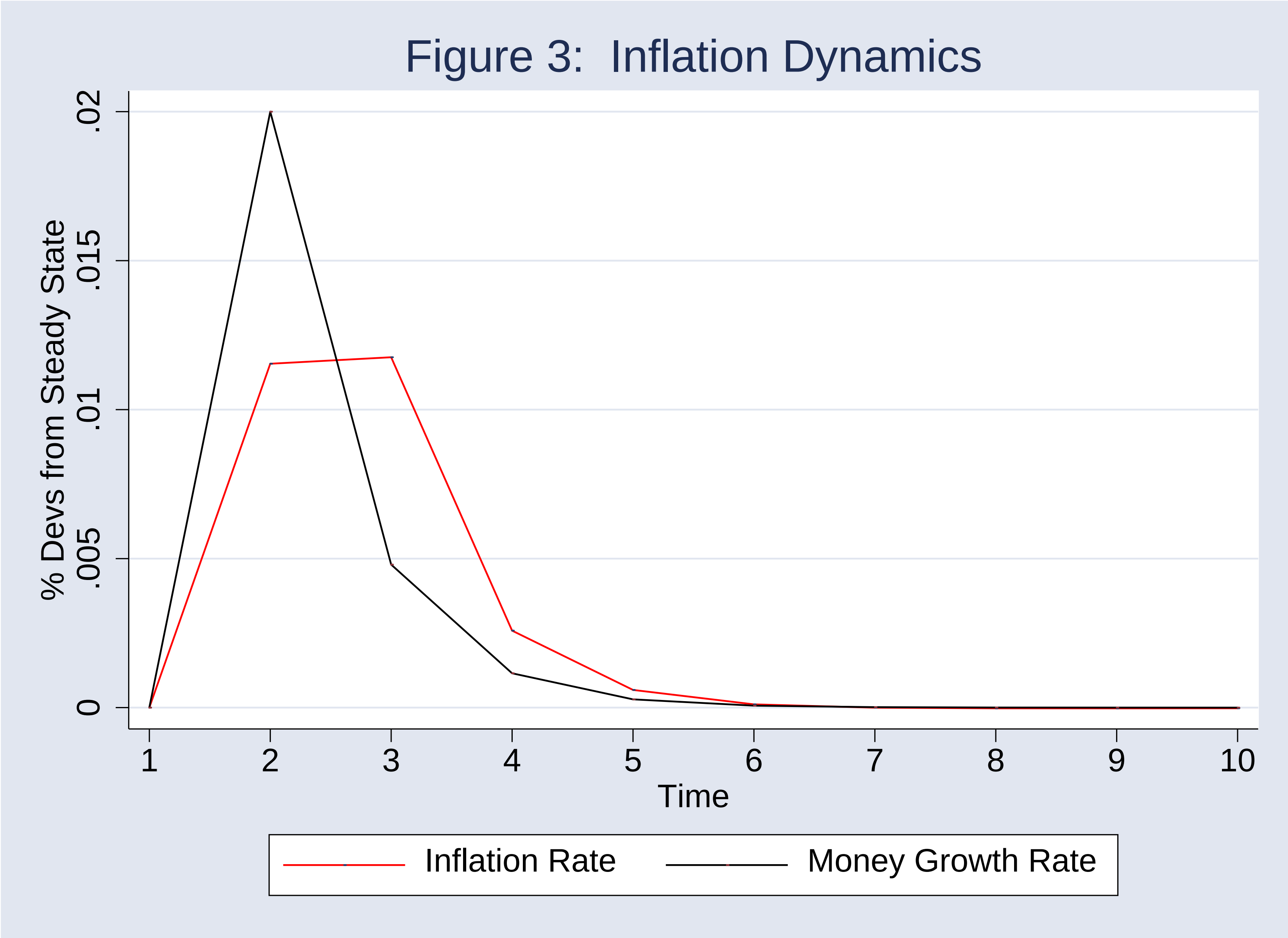

Figure 3: Inflation Dynamics
% Devs from Steady State
0
.005
.01
.015
.02
1
2
3
4
5
6
7
8
9
10
Time
Inflation Rate
Money Growth Rate

Figure 4: Consumption