# Mismatch and Mobility*

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The goal of this paper is to extend Shimer (2005) to an environment with endogenous mobility. Rather than work directly in the original model directly, however, I work in a related framework, the stock-flow matching model (Taylor, 1995; Coles and Muthoo, 1998). One of the contributions of this paper is therefore to compare how the solution to the stock-flow matching model compares with the solution to Shimer (2005); their predictions are quantitatively similar.

I start by discussing that model briefly and then summarize how I will introduce endogenous mobility into the model. Note that this paper is preliminary and (obviously) incomplete.

# 1 Stock-Flow Matching Model

## 1.1 Model

There is a fixed measure $M = 1$ of workers and a fixed measure $N$ of jobs. Take any particular worker. Within any measure $v$ of jobs, the probability that the worker is a suitable match for at least one of those jobs is $1 - e^{-\alpha v}$, where the parameter $\alpha$ measures the difficulty of matching. Symmetrically, the probability that a job is not suitable for any worker in a measure $u$ is $1 - e^{-\alpha u}$.

Large $\alpha$ is associated with matching being easier. Although this exact functional form is not necessary for what follows, it is a sensible restriction. In particular, the probability of not finding a suitable match in neither a measure $v_1$ of jobs nor a disjoint measure $v_2$ of jobs is just equal to the probability of not finding a job in a measure $v_1 + v_2$ of jobs.

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## 1.2 Beveridge Curve

At any point in time, $u$ is the measure of unemployed workers and $v$ is the measure of vacant jobs. Existing matches end exogenously at rate $s$. When this happens, the worker looks across the vacant jobs to see if one is available. If there is an available job, she takes it. Otherwise she becomes unemployed. Thus the transition rate into unemployment is

$$s(1-u)e^{-\alpha v},$$

the product of the separation rate, the measure of employed workers, and the probability that a newly unemployed worker is not a good match for any of the vacancies. An unemployed worker finds a job if she is a good match for a newly-vacant job. This occurs to some unemployed worker at rate

$$s(N-v)(1-e^{-\alpha u}).$$

Note that the number of filled jobs is equal to the number of unemployed workers, $N-v=1-u$, and so in steady state

$$v=-\frac{1}{\alpha}\log\left(1-e^{-\alpha u}\right). \tag{1}$$

This is the Beveridge curve. Like in Shimer (2005), it is driven purely by aggregation. For fixed $\alpha$, it gives a decreasing relationship between the number of unemployed workers and the number of vacancies, with movements along the Beveridge curve induced by movements in the number of jobs $N$. It is almost indistinguishable from the Beveridge curve in Shimer (2005), and hence indistinguishable from U.S. data. For example, evaluating at $u=v$, the slope of the Beveridge curve is $-1$.

## 1.3 Reduced-Form Matching Function

Building on the previous analysis, unemployed workers find a job at rate

$$\frac{s(1-u)(1-e^{-\alpha u})}{u},$$

while the vacancy-unemployment ratio is

$$\frac{-\log(1-e^{-\alpha u})}{\alpha u}.$$

A change in the number of jobs changes unemployment and hence both of these quantities, giving rise to a reduced-form matching function. One can implicitly differentiate this to compute the elasticity of the job finding rate w.r.t. to the v-u ratio, the elasticity of the reduced-form matching function:

$$\frac{\log(1-e^{-\alpha u})\big(1-e^{-\alpha u}\big(1+\alpha u(1-u)\big)\big)}{(1-u)\big(\alpha u e^{-\alpha u}-(1-e^{-\alpha u})\log(1-e^{-\alpha u})\big)}.$$

If $u = v$, $\alpha u = \log 2$, and this reduces to

$$\frac{\alpha(1 - \log 2) + (\log 2)^2}{2\alpha - \log 4}.$$

In particular, when $\alpha$ is large, we get that the elasticity of the reduced-form matching function is $\frac{1-\log 2}{2} \approx 0.153$ at $u = v$. For reasonable parameter values, I compute an elasticity that is slightly larger and not quite constant, say about 0.18. This is slightly smaller than the comparable number is Shimer (2005), about 0.21.

One can also think about issues like job-to-job transitions in this model. One thing that appears in Shimer (2005) but not in this model is duration dependence. Here all unemployed are in the same situation, as are all employed workers, and so there is no duration dependence.

## 1.4 Free Entry

To close the model, one has to determine the number of vacancies. It is not obvious how to do this because labor markets are not competitive in the stock-flow matching model.[1] Here I focus on the efficient allocation. I assume workers and firms are risk-neutral and infinitely-lived, discounting future payoffs at rate $r$. Thus the planner's objective is to maximize the expected present value of output net of job creation costs.

Let $\gamma$ be the cost of creating a new job, $x$ the productivity of an employed worker, $z$ the productivity of an unemployed worker, and $s$ the separation rate, i.e. the rate at which jobs are destroyed. Then the social planner choose a gross job creation rate $c$ to solve the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} rW(u,v) = \max_c \Bigg( & \frac{x-z}{r+s}\big(s(1-u)(1-e^{-\alpha v}) + c(1-e^{-\alpha u})\big) - \gamma c \\ & + W_u(u,v)\big(s(1-u)e^{-\alpha v} - c(1-e^{-\alpha u})\big) + W_v(u,v)\big(ce^{-\alpha u} - s(1-u)(1-e^{-\alpha v})\big)\Bigg). \end{aligned}$$

The first line gives the expected present value of output in newly created matches, $(x - z)/(r + s)$, times the rate at which new matches are created when either a worker loses her job and finds a vacancy or a new job is created and finds a worker, minus the cost of creating new jobs. The second line has the partial derivative of the value functions multiplied by the net rate of increase in the state variables.

The steady state solution to this equation and the Beveridge curve in (1) is

$$\gamma = \frac{x-z}{r+s}\big(1 - e^{-\alpha u}\big) + \frac{s\gamma}{r+s}e^{-\alpha u}.$$

Intuitively, the cost of creating a job $\gamma$ must balance the sum of two terms: the probability that a new job leads to a match immediately, $1 - e^{-\alpha u}$, times the expected output from the

[1] This is an important difference between the stock-flow model and the model in Shimer (2005).

match, $(x - z)/(r + s)$; plus the probability it does not immediately lead to a new match, $e^{-\alpha u}$, times the saving from being able to postpone creation of a job until this job ends, with hazard rate $se^{-st}$. This pins down the unemployment rate $u$, with vacancies given by the Beveridge curve in (1) and the number of jobs by $N = 1 - u + v$.

# 2 Mobility

The chief advantage of the stock-flow matching model is that one can more easily think about mobility. The simplest way to do this is to assume an unemployed worker can pay a fixed cost $c$ and change her 'type'. This gives her a new draw from each of the vacant jobs, and so she gets a match with probability $1 - e^{-\alpha v}$. An important insight is that the return to mobility is highest when $v$ is high. Indeed, in steady state this should simply put a cap on $v$. If there are too many vacancies, unemployed workers have an incentive to pay the mobility cost, reducing both $u$ and $v$ until the incentive disappears. Effectively this introduces a flat section in the Beveridge curve in (1):

$$v = \min\left\{-\frac{1}{\alpha}\log\left(1 - e^{-\alpha u}\right), \bar{v}\right\}.$$

This approach is stark and hence delivers stark conclusions.

An alternative assumption is that unemployed workers periodically have a chance to move. They draw a mobility opportunity at rate $\lambda$; the mobility cost is drawn from some distribution $F(c)$. Now mobility will be high either when there is a large return to mobility (for the reason explained in the previous paragraph) or when there are many potential movers (since there are more potential low-cost movers). It is unclear how this will change the slope of the Beveridge curve.

But one thing that will happen, is that unemployment benefits and other labor market policies will affect the location of the Beveridge curve. Generous unemployment benefits make workers more reluctant to pay the mobility cost and hence raise both unemployment and vacancies. Conversely, subsidies to retraining or moving will shift the Beveridge curve in. Thus one can start to think about the implications of labor market policies for the location of the Beveridge curve, a possibility that is absent from most versions of the matching model (Pissarides, 2000).

# References

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