

The Marginal Worker and The Aggregate Elasticity of Labor Supply

Preliminary and Incomplete

François Gourio and Pierre-Alexandre Noual*

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Abstract

This paper attempts to reconcile the high apparent aggregate elasticity of labor supply with small micro estimates. We elaborate on Rogerson's seminal work (1988) and show that his results rely neither on complete markets nor on lotteries, but rather on the indivisibility and the fact that the workforce is homogeneous at the margin. We derive two robust implications of a setup with indivisible labor but without lotteries, using either a complete markets model or an incomplete markets model (solved numerically). (1) Agents with reservation wages far above or below the market wage are less responsive (in labor supply) to the business cycle than agents whose reservation wage is around the market wage. (2) The aggregate elasticity is given by the marginal homogeneity of the workforce. We test implication (1) using the PSID and find support for it. We build an incomplete markets model and calibrate it to cross-sectional moments of hours worked. We show that it can reproduce the feature (1). This allows us to use the model to evaluate the importance of feature (2), i.e. to estimate the aggregate elasticity of labor supply implied by the marginal homogeneity.

Keywords: indivisible labor, reservation wage distribution, labor supply, business cycles.

“We’d been eating Harry’s food on credit for the last couple of weeks while my father figured his next move. I gathered that things worked this way pretty much every year. My father was a seasonal kind of guy. May through November he was flush, but along about Thanksgiving when the road construction business dwindled, he’d get himself laid off and collect unemployment until late spring. The unemployment was

*Boston University and University of Chicago. Email addresses: fgourio@bu.edu and noual@uchicago.edu. The paper and an extended appendix can be found on <http://people.bu.edu/fgourio> and on <http://home.uchicago.edu/~noual>. We thank Fernando Alvarez, Lars Hansen and Ronni Pavan, and participants at the Economic Dynamics working group in Chicago for their comments. First draft: May 2005. Any error is ours.

meager though, compared to his normal summer earnings.” Richard Russo, *The Risk Pool*, p. 131.

1 Introduction

Consider the labor supply choice of agents differing in productivity, or in labor disutility. If any positive labor supply choice is permitted, all agents will be at interior solutions (provided Inada conditions are verified), and a temporary increase in the aggregate wage will typically induce all agents to work more, generating an increase in the aggregate labor supply. The strength of this increase will be a weighted average of the labor supply elasticities of all the agents: all agents are “marginal”.

However, if the labor supply choice is indivisible, most agents will be irresponsive to changes in the wage, since they are either not working or working, and will not shift from one state to the other. But a few agents, which are nearly indifferent between working and not working, will adjust their labor supply discretely. These few agents are the “marginal workers”; their labor supply elasticity, not the others’, matters for the aggregate behavior of the economy. Indeed, one could imagine that a small minority of agents with highly elastic labor supply does all the cyclical adjustment in the workforce, generating what looks like a highly elastic aggregate labor supply, while the majority of the workforce is irresponsive to macroeconomic conditions.

In this paper, we study whether this combination of indivisibility and heterogeneity can rationalize the high apparent macroeconomic elasticity of labor supply, i.e. the stylized fact that large changes in employment are associated with small variations in wages or productivity. To do so, we study theoretically a complete market model and numerically an incomplete market model. Both models make two important predictions:

- (1) the agents who are “marginal” in terms of the reservation wage will be more responsive to aggregate fluctuations than agents who are at either end of the reservation wage distribution. Hence, our model predicts a hump-shaped pattern of sensitivities to aggregate shocks.

- (2) the aggregate elasticity of labor supply is given by the amount of heterogeneity at the margin.

Point (1) is probably clear. Point (2) deserves more explanation. In our model, the Frisch elasticity of labor supply is the hazard rate of the distribution of reservation wages, measured at the current macroeconomic wage. This means that labor supply is more elastic if there are more agents whose reservation wage is near the market wage, i.e. if the mass of people who are

indifferent between work and leisure at the current market wage is high. Thus, labor supply is more elastic if reservation wages are more homogeneous at the margin.

Empirically we document that (1) is a feature of the data. For instance, when average annual hours worked rise by 100 hours, the average hours of people working “full-time” (i.e. around 2,000 hours a year) rise by about 70 hours, while the hours of the people working around 700 hours rise by 200 hours. We next use our incomplete markets model to examine these questions quantitatively. We choose parameters to match cross-sectional moment of hours worked and ask if it can reproduce pattern (1). Our model is able to do that, and we can then use it to measure the impact of heterogeneity on the aggregate elasticity of labor supply.

We find it useful to relate our contribution to Rogerson’s seminal paper on lotteries (1988). He showed in a static, deterministic model that in the presence of indivisibilities, allowing agents to randomize their labor supply is generally Pareto-improving, if agents can insulate consumption from these labor supply fluctuations through complete markets. Moreover, the introduction of lotteries generates a very elastic labor supply curve in the aggregate. This much-admired contribution has failed to convince some economists. We believe three reasons stand out for this. First, the interpretation of the lotteries is difficult (“we don’t see people flipping a coin to decide whether to work or not”). Second, this result holds under complete markets, but the complete markets model is empirically unappealing (this point is made for instance by Ljungqvist and Sargent (2003)). Third, even under complete markets, the elasticity of labor supply crucially depends on the shape of the distribution of reservation wages. The infinite elasticity of labor supply in Rogerson (1988) rests on the homogeneity of workers.

Our contribution is to note that one can take away the lotteries and the complete markets assumptions, and that the high elasticity of labor supply of Rogerson is really driven by indivisibility and marginal homogeneity. This explanation of aggregate labor supply by heterogeneity and indivisibility instead of lotteries is compelling - at least to us. But the quantitative question remains, how big is the marginal homogeneity required to make sense of macroeconomic facts? Is this consistent with microeconomic data? This motivates our quantitative exercise using the incomplete market model. Future work will elaborate on this.

Related Literature.

Our paper is most closely related to Chang and Kim (2006). They consider an economy with indivisible labor, heterogeneous productivity, and incomplete markets. They calibrate the skills distribution to match features of income and wealth heterogeneity. Our main difference with their work is our attempt to bring the model to the data perhaps more directly, and to emphasize the

role of the reservation wage distribution at the margin, i.e. around the market wage.

Mulligan (2001) argues that an indivisible labor model is equivalent to a representative agent model with any Frisch elasticity of labor supply, for some process for idiosyncratic shocks. We find a similar result in Section 2. But while Mulligan viewed this result as a compelling reason to use a macro model, we believe the opposite is true: the micro model can be brought to the data to estimate the aggregate elasticity of labor supply that empirically reasonable heterogeneity will generate.

Organization.

Section 2 analyzes a model with heterogeneity in disutility of labor, indivisible labor supply, no lotteries, and complete markets. We first use a simple partial equilibrium model to illustrate our main point. We then move to a standard RBC model. We show analytically that under some conditions there is a one-to-one mapping between the marginal heterogeneity of workers and the Frisch elasticity of labor supply. In section 3 we extend the analysis to incomplete markets. Our model is calibrated to match features of the distribution of hours worked. It also displays hump-shaped sensitivities to the business cycle: we present some regressions on simulated data which mimic those that we run in the data in section 4. The model can be used to infer the aggregate elasticity of labor supply. Finally in Section 4 we present results from PSID data over 1967-1996. We provide evidence of heterogeneous sensitivities to macroeconomic conditions. Section 5 concludes with a discussion of possible lines of continuation for this project.

2 Complete Markets, Indivisible Labor, and Heterogeneity

This section analyzes two complete markets models with labor supply indivisibility and ex-ante identical agents who become heterogeneous due to stochastic tastes. The first model is a partial equilibrium model (it takes prices as exogenous), and the second one is a variant of the standard RBC model. We first describe the household side of our model, which is common to both the partial and general equilibrium (RBC) versions. Then in each subsection, we add the assumptions about technology and market structure which close the model.

Preferences

Flow utility is defined over consumption and hours worked and is additively separable: $u(c) - \theta v(n)$. Agents differ in stochastic labor disutility, or cost of working, or taste for leisure, θ . The function u is increasing, concave, and it satisfies the Inada condition. Total utility is the expected

discounted sum of these flow utilities:

$$U = \mathbb{E} \sum_{t \geq 0} \beta^t (u(c_t) - \theta_t v(n_t)).$$

Heterogeneity

We make few assumptions about the process for θ at this point: we only require that it is stationary and has an invariant distribution H . The initial draw of θ is from this invariant distribution H , so in any future period the cross-sectional distribution of θ is H . The conditional probabilities of histories are denoted $\pi_t^\theta(\theta^t) = \Pr(\theta^t)$, where as usual θ^t denotes the history $(\theta_0, \dots, \theta_t)$. We assume that θ is continuous, so $\pi_t^\theta(\theta^t)$ is a joint density. (Note that the probabilities here are not conditioned on θ_0 .)

This framework can accommodate both fixed effects (“laziness”) and transitory shocks (e.g., a woman bearing a child) which influence the cost of working. Two particular cases are interesting: (i) θ is constant, (ii) θ follows a Markov process with full mixing and a unique invariant distribution.¹

Indivisibility

We assume that the labor input is indivisible: $n \in \{0, \bar{n}\}$. We do not allow agents to use lotteries in labor supply as in Rogerson (1988), i.e. to randomize their labor supply choice and make their income contingent on the outcome of this randomization.

Complete Markets

We assume that agents can trade in complete markets based on the histories z^t and θ^t .² In the two models of this section, insurance against idiosyncratic shocks will trade at the risk-neutral prices, since by definition these shocks have no aggregate effect. Letting $p_t^\theta(\theta^t)$ be the price at time 0 of a claim to one good in time t , history θ^t , we have $p_t^\theta(\theta^t) = \beta^t \pi_t^\theta(\theta^t)$.

A. The Partial Equilibrium Model

In this section, we make two additional assumptions which make prices exogenous:

- (1) The technology is linear in labor: $Y_t = z_t N_t$, where the average -and marginal- product of

¹We also considered models with heterogeneity in market productivity, which yield nearly identical results. When we go to the data, we want to think of θ as standing for the relative disadvantage at market production, relative to “home production” or leisure.

²In principle, the state of the economy includes all combinations of histories of all the agents, but as usual agents would not want to trade assets based on someone else’s idiosyncratic shock, so we simplify the notation by not considering these contingencies: we consider simultaneously the histories θ^t and z^t which are “orthogonal”.

labor z_t is an exogenous stationary process, with continuous support; the joint density of history z^t is $\pi_t(z^t)$, and it has an invariant distribution G .

(2) There is a “foreign bank” with which the households can trade at risk-neutral prices. Hence the state-contingent price for the delivery of goods at time t and after history z^t is $p_t(z^t) = \beta^t \pi_t(z^t)$.

Under these assumptions, and assuming that all wealth is labor income, we can write the *individual problem* as:

$$\max_{\{c_t(\theta^t, z^t), n_t(\theta^t, z^t)\}} \sum_{t \geq 0} \beta^t \int_0^\infty \int_0^\infty \pi_t(z^t) \pi_t^\theta(\theta^t) [u(c_t(\theta^t, z^t)) - \theta_t v(n_t(\theta^t, z^t))] dz^t d\theta^t,$$

$$s.t. : \forall t \geq 0, \forall z^t \in Z^t, \forall \theta^t \in \Theta^t : n_t(\theta^t, z^t) \in \{0, \bar{n}\},$$

$$\sum_{t \geq 0} \int_0^\infty \int_0^\infty \beta^t \pi_t(z^t) \pi_t^\theta(\theta^t) c_t(\theta^t, z^t) dz^t d\theta^t \leq \sum_{t \geq 0} \int_0^\infty \int_0^\infty \beta^t \pi_t(z^t) \pi_t^\theta(\theta^t) z_t n_t(\theta^t, z^t) dz^t d\theta^t.$$

The first-order condition with respect to consumption yields the usual perfect risk-sharing rule, given that insurance across both idiosyncratic and aggregate

shocks is sold at the risk-neutral prices, and that utility is separable:

$$u'(c_t(\theta^t, z^t)) = \lambda,$$

and thus $c_t(\theta^t, z^t) = c$, a constant.³ It is easy to prove then that the labor supply choice will involve a threshold rule, with the agent working if the ratio of the wage to his taste shock θ is above some given threshold x .

Given these decisions rules, the problem can be rewritten as a static problem, as of time 0 :

$$\max_{\{c, x\}} \left(u(c) - v(\bar{n}) \int_0^\infty \int_0^\infty \theta \mathbf{1}_{\{\frac{z}{\theta} \geq x\}} dH(\theta) dG(z) \right),$$

$$s.t. : c \leq \int_0^\infty \int_0^\infty \mathbf{1}_{\{\frac{z}{\theta} \geq x\}} z dH(\theta) dG(z),$$

where G is the invariant distribution of z . This finally yields the consumption c and a threshold x , implying a decision rule to work in all periods where $z_t \geq \theta_t x$. This can be expressed as a minimum z required to work given θ , or $z^*(\theta)$. Note that this problem implicitly assumes that

³If utility $u(c, n)$ were not separable in consumption and leisure, we would have to solve for two consumption levels c^w and c^l , such that $u'(c^w, \bar{n}) = u'(c^l, 0) = \lambda$. Nonseparability modifies the consumption allocation but has no impact on our results.

each agent is insured against his initial θ_0 as well as against future shocks. Agents are ex-ante identical, so they all make the same choices; this conveniently removes wealth effects.

The similarity with Rogerson’s analysis is interesting. By choosing a threshold x , the agent chooses – from an ex-ante point of view – a probability of working in future periods. He conditions his labor supply on the future aggregate state z_t and on the future idiosyncratic state θ_t . By borrowing and lending in complete markets, the agent can insulate his consumption from the variation in his labor supply. Since labor supply is random and independent of his consumption, this allocation is equivalent to a lottery, but the interpretation - not as an exogenous randomization but as a series of choices in a dynamic, stochastic environment - is much more reasonable.

Aggregation

In any future period, the distribution of θ is the invariant distribution H , and given that all agents have identical thresholds x , this yields the labor supply of the economy:

$$L_t = \bar{n} \int_{z_t \geq x\theta} dH(\theta) = L(z_t) = \bar{n}H(\theta_t^*),$$

where $\theta_t^* = z_t/x$ is the θ of the “marginal household”, who is just at the margin of working. The elasticity of the labor supply is then given by the marginal heterogeneity of θ :

$$\begin{aligned} \frac{d \log L_t}{d \log z_t} &= \frac{\frac{z_t}{x} h\left(\frac{z_t}{x}\right)}{H\left(\frac{z_t}{x}\right)}, \\ &= \frac{\theta_t^* h(\theta_t^*)}{H(\theta_t^*)}. \end{aligned} \tag{2.1}$$

The elasticity of aggregate labor supply is given by the marginal heterogeneity (the hazard rate). The smaller the marginal heterogeneity, i.e. the greater $\frac{\theta^* h(\theta^*)}{H(\theta^*)}$, the greater the aggregate elasticity. Intuitively, if the hazard rate is large, a small increase in wage will be enough to draw a large number of people in the labor force. The shape of the distribution of disutility of labor *around the marginal agent* determines the aggregate elasticity. This is the relevant object for macroeconomics, and the next step to our research is to find a way to estimate this distribution.

Heterogeneous responsiveness

In this economy, all the adjustment takes place at the extensive margin: in any given period, the agents with high θ will never work, the agents with low θ will always work, and some marginal agent will be indifferent. If θ is persistent, an aggregate shock will not affect the labor supply of the very high or very low θ , since they will remain well above (below) the threshold. Only the “marginal” agents, those for which θx is close to z , might shift in and out of the labor force. This is an interesting cross-sectional prediction: the responsiveness of labor supply should be

largest for the intermediate θ agents. In Section 3, we show that this prediction still holds with incomplete markets, and in Section 4, we test this prediction using the PSID.

Formally, assuming that θ and z are both Markov with conditional probabilities $F(\cdot | \theta)$ and $\Pi(\cdot | z)$, then the probability of working today given that the wage was $z_t = z$ and the type was $\theta_t = \theta$ yesterday is:

$$\begin{aligned}\phi(z, \theta) &= \Pr(n_{t+1} = 1 | z_t = z, \theta_t = \theta) \\ &= \Pr\left(\theta_{t+1} < \frac{z_{t+1}}{x} | z_t = z, \theta_t = \theta\right), \\ &= \int_0^\infty F\left(\frac{z'}{x} | \theta\right) d\Pi(z' | z),\end{aligned}$$

where F is the cumulative distribution of θ' given θ . Because of the persistence in z and θ ,⁴ ϕ satisfies $\partial^2 \phi / \partial z \partial \theta > 0$. This shows that this probability depends less on z for the low θ , who work in period t . A similar argument shows that the probability of not working tomorrow depends less on z for the high θ agents, who do not work in period t .

Robustness to wealth effects

So far, we assumed that θ_0 is unknown to the agent. As a result, the household is insured against θ_0 . In the data, there are probably fixed effects in θ , against which the agent cannot insure. Since we are interested in a competitive equilibrium where each agent has to finance his consumption, it is important to consider what happens if we relax this assumption.⁵

In that case the optimal consumption and labor supply rule would depend on θ_0 . One can show that (1) each agent has a consumption which is constant over time and across states, but will depend on his θ_0 which affects his willingness to work and thus his wealth; (2) each agent has a threshold rule which now depends on his initial θ_0 as well, i.e. he works when $z_t / \theta_t \geq x(\theta_0)$. As a result, the aggregate labor supply is

$$L_t(z_t) = \bar{n} \int_0^\infty \int_0^\infty \pi_t^\theta(\theta_t | \theta_0) 1_{z_t / \theta_t \geq x(\theta_0)} d\theta_t dH(\theta_0),$$

where the integrals sum over θ_0 first, then over the histories θ^t given θ_0 . To explain this expression more generally, recall that the decision rule is to work whenever $z_t \geq \theta_t x$, where x is a threshold which is the same for everyone (i.e. independent of θ_0) in the section above. Now x will not be the same for everyone: its distribution will depend on the distribution of θ_0 , and so the aggregate

⁴i.e. $F(\cdot | \theta_1)$ first-order stochastically dominates $F(\cdot | \theta_2)$ for $\theta_1 \geq \theta_2$, and similarly $\Pi(\cdot | z_1)$ first-order stochastically dominates $\Pi(\cdot | z_2)$ for $z_1 \geq z_2$.

⁵While this step is conceptually important, we do not believe that is empirically very important: in the data agents with high productivity work a lot, so the income effect is probably not very strong.

labor supply is the number of agents who have $\theta_t x \leq z_t$, where θ_t and x both vary with the individual, and are correlated since x depends on θ_0 which is correlated with θ_t . Hence our double integral where we first integrate on θ_t given θ_0 , then on θ_0 . Letting $y_t = \theta_t x$, we have that the slope of the aggregate labor supply curve will depend on the shape of the distribution of y_t .

The variable we wish to measure is now akin to a reservation wage. Rather than the distribution of taste shocks (see equation 2.1), it is the distribution of reservation wages that determines the elasticity of labor supply.

B. The RBC Model

In this section we do not assume that there is a “foreign bank”, and we consider a standard technology with capital instead of a linear, labor-only technology. As a result, aggregate risk on z_t , which is now a standard TFP shock, must be shared within the economy. The idiosyncratic risk however is still fully diversifiable.

Technology

The aggregate technology is standard: the production function takes the Cobb-Douglas form:

$$Y_t = K_t^\alpha (z_t N_t)^{1-\alpha},$$

where N_t is the sum of hours worked:

$$N_t(z^t) = \int_0^\infty n_t(\theta^t, z^t) \pi_t^\theta(\theta^t) d\theta^t.$$

The capital accumulation law is:

$$K_{t+1}(z^t) = (1 - \delta)K_t(z^{t-1}) + I_t(z^t),$$

and the aggregate resource constraint is:

$$C_t(z^t) + I_t(z^t) \leq Y_t(z^t).$$

Aggregate Shock

The productivity shock z_t follows a standard AR(1) process:

$$\ln z_t = \rho \ln z_{t-1} + \sigma \varepsilon_t, \tag{2.2}$$

where ε_t is iid $N(0, 1)$. This shock is independent of the idiosyncratic shocks to θ . We denote the unconditional densities $\pi_t(z^t)$ (where as usual, $z^t = (z_0, z_1, \dots, z_t)$ denotes an entire history of realizations).

Social Planner Problem

Since the First Welfare Theorem holds in this economy, the equilibrium is Pareto-efficient, and we can use a social planner problem to find the competitive equilibrium. We again simplify by assuming equal weights, which is equivalent to assuming that agents are insured against their initial θ . This leads to the following program:

$$\begin{aligned} & \max_{\{c_t(\theta^t, z^t), n_t(\theta^t, z^t), K_{t+1}(z^t)\}} \sum_{t \geq 0} \int_0^\infty \int_0^\infty \beta^t \pi_t(z^t) \pi_t^\theta(\theta^t) [u(c_t(\theta^t, z^t)) - \theta_t v(n_t(\theta^t, z^t))] dz^t d\theta^t \\ \text{s.t.} \quad & K_0(z^{-1}) \text{ given,} \\ & \forall t \geq 0, \forall z^t \in Z^t, \forall \theta \in \Theta^t : n_t(\theta^t, z^t) \in \{0, \bar{n}\}, \\ & \forall t \geq 0, \forall z^t \in Z^t : N_t(z^t) = \int_0^\infty \pi_t^\theta(\theta^t) n_t(\theta^t, z^t) d\theta^t \\ & \forall t \geq 0, \forall z^t \in Z^t : K_{t+1}(z^t) + \int_0^\infty \pi_t^\theta(\theta^t) c_t(\theta^t, z^t) d\theta^t \leq F(K_t(z^{t-1}), z_t N_t(z^t)) + (1 - \delta)K_t(z^{t-1}). \end{aligned}$$

In our context, the absence of lotteries means that we do not allow the planner to randomize an agent's allocation. The probabilities of histories θ^t show up in the resource constraints because they also indicate the number of agents who have experienced each history. Note that the agents have equal weights in the planner's objective. Let $\lambda_t(z^t)\beta^t\pi_t(z^t)$ be the multiplier on the resource constraint. The first-order conditions with respect to $c_t(\theta^t, z^t)$ are:

$$u'(c_t(\theta^t, z^t)) = \lambda_t(z^t).$$

This implies that $c_t(\theta^t, z^t)$ is independent of θ^t and equal to $c_t(z^t)$: with separable utility and equal weights, everyone consumes the same amount. The labor supply choice is characterized by a threshold rule: agents work if and only if $\theta \leq \theta_t^*(z^t)$, where $\theta_t^*(z^t)$ is a time- and state- dependent threshold⁶. We thus rewrite the social planner problem as: choose $\{c_t(z^t), \theta_t^*(z^t), K_{t+1}(z^t)\}$ to maximize

$$\sum_{t \geq 0} \beta^t \int_0^\infty \pi_t(z^t) \left[u(c_t(z^t)) - \left(\int_0^{\theta_t^*(z^t)} \theta dH(\theta) \right) v(\bar{n}) \right] dz^t$$

$$\text{s.t.} \quad K_0(z^{-1}) \text{ given}$$

$$\forall t, z^t : K_{t+1}(z^t) + c_t(z^t) \leq F(K_t(z^{t-1}), z_t N_t(z^t)) + (1 - \delta)K_t(z^{t-1}).$$

⁶Since the equilibrium is recursive with only z_t and K_t as state variables, θ_t^* , and all the other variables, depend actually only on z_t and K_t .

$$N_t(z^t) = \bar{n} \int_0^{\theta_t^*(z^t)} dH(\theta).$$

The first-order conditions of this problem are:

$$\begin{aligned} u'(c_t(z^t)) &= \lambda_t(z^t), \\ \lambda_t(z^t) &= \beta \int_0^\infty \lambda_{t+1}(z^t, z_{t+1}) \pi(z_{t+1} | z^t) (1 - \delta + F_1(K_{t+1}, z_{t+1} N_{t+1})) dz_{t+1}, \\ \frac{v(\bar{n})\theta_t^*}{\lambda_t} &= \bar{n} z_t F_2(K_t(z^{t-1}), z_t N_t(z^t)). \end{aligned} \quad (2.3)$$

The last equation is the only one which differs from the standard RBC model. This equation equates the benefit of shifting the marginal agent from leisure to work, i.e. the MPL $z_t F_2(K_t(z^{t-1}), z_t N_t(z^t)) \stackrel{def}{=} w_t$, with the cost, which is $v(\bar{n})\theta_t^*$ units of utilities, or $\frac{v(\bar{n})\theta_t^*}{\lambda_t}$ units of goods.

It is instructive to contrast this labor supply equation with the one implied by the standard representative agent RBC model. Assume that the utility function is $\log c + v(l)^7$, then the labor supply equation in the representative agent RBC model is

$$\begin{aligned} \frac{u_l(c, l)}{u_c(c, l)} &= w = z F_2(K, zN) \\ c \times v'(l) &= w, \text{ with the log specification,} \end{aligned}$$

and log-linearizing yields the Frisch elasticity of labor supply⁸ as a function of the curvature of v .⁹

⁷Following King, Plosser and Rebelo (1988), real business cycle theorists have used utility functions which are consistent with balanced growth and trendless hours, i.e. $u(c, l) = v(l)c^{1-\gamma}/(1-\gamma)$ for $\gamma \neq 1$, or $u(c, l) = \log c + v(l)$ for $\gamma = 1$. The first type introduces a nonseparability which makes the comparison less straightforward. This is why we stick to the log utility.

⁸Recall that the Frisch elasticity is equal to $\partial \log n / \partial \log w$ where n and c are the solutions of the system $u_c(c, 1-n) = \lambda$, $u_l(c, 1-n) = \lambda w$ for given w and λ .

⁹Denoting with a hat a % deviation from the nonstochastic steady-state, we obtain

$$\frac{v''(l^*)l^*}{v'(l^*)} \hat{l}_t = \hat{\lambda}_t + \hat{w}_t.$$

Let $n = 1 - l$, then

$$-\frac{v''(l^*)l^*}{v'(l^*)} \frac{n^*}{1-n^*} \hat{n}_t = \hat{\lambda}_t + \hat{w}_t,$$

and Frisch elasticity of labor supply is

$$\varepsilon_{nw} = \frac{-v'(l^*)}{v''(l^*)n^*} > 0,$$

since v is concave.

In the indivisible labor model, the Frisch elasticity of labor supply can also be computed exactly, since from equation (2.3):

$$\widehat{\theta}_t^* = \widehat{w}_t + \widehat{\lambda}_t,$$

and there is a one-to-one relation between θ_t^* and N_t :

$$\widehat{N}_t = \frac{\theta^* h(\theta^*)}{\int_0^{\theta^*} dH(\theta)} \widehat{\theta}_t^* = \frac{\theta^* h(\theta^*)}{H(\theta^*)} \widehat{\theta}_t^*,$$

thus

$$\widehat{N}_t = \frac{\theta^* h(\theta^*)}{H(\theta^*)} \widehat{w}_t + \frac{\theta^* h(\theta^*)}{H(\theta^*)} \widehat{\lambda}_t,$$

and the Frisch elasticity of labor supply is

$$\varepsilon_{nw} = \frac{\theta^* h(\theta^*)}{H(\theta^*)}, \quad (2.4)$$

that is, the Frisch elasticity of labor supply is the hazard rate of H , our measure of marginal heterogeneity, as in the partial equilibrium model. Clearly depending on the shape of H , this model can generate any Frisch elasticity, including the very high ones implied by Rogerson’s analysis.

Some Examples

We now present some illustrative simulations of this model. The calibration is standard.¹⁰ The productivity distribution is assumed to be:

$$H(\theta) = \frac{1}{\pi} \left(\arctan(\sigma(\theta - \bar{\theta})) + \frac{\pi}{2} \right),$$

with $\sigma = 5$ and $\bar{\theta} = 1$. $\bar{\theta}$ is a normalization, while σ gives the standard deviation of the disutility of labor. The disutility of working, $v(\bar{n})$, is chosen to generate in our first example an average employment rate $H(\theta^*) = 0.5$ (i.e. in steady-state, 50% of the population is working).

For this first example, displayed in figure 1, the top panels show the responses of θ_t^* and N_t to a persistent ($\rho = .95$) shock to TFP. The threshold disutility of labor θ_t^* rises to draw people in the labor force, and employment increases by a large amount (.6% on impact). The “RA” line on the second graph is the response in the representative agent economy with a Frisch elasticity $\varepsilon_{nw} = 3.08$; this line overlaps with the line representing the response of the indivisible labor

¹⁰Log utility, depreciation rate $\delta = 0.025$, discount rate $\beta = 0.99$, capital share $\alpha = 0.30$.

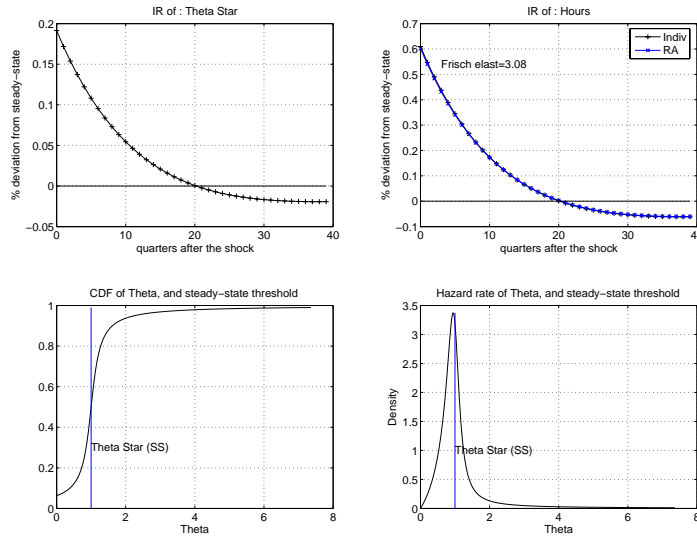


Figure 1: The response of θ^* and N to a persistent TFP shock (calibrated for an employment rate of 50%).

economy, according to (2.4), since 3.08 is the number that makes our indivisible labor economy equivalent to a representative agent economy. The last two figures show the c.d.f. and hazard rate of the invariant distribution of θ , with the threshold θ^* represented by a blue vertical line. Clearly, in this case the marginal heterogeneity is small, since θ^* is near the maximum of the hazard rate. This explains why our economy is equivalent to one with a rather large Frisch elasticity (certainly much higher than any microeconomic estimate).

In our second example, we keep the same distribution but choose $v(\bar{n})$ to obtain an employment rate of 90%. In this case, we obtain that the marginal heterogeneity is large, i.e. at the margin workers are very different in disutility of labor, and consequently our economy is equivalent to one which has an inelastic Frisch labor supply: $\varepsilon_{nw} = 0.27$. This can be seen in the hours response which is much smaller, about .11% on impact.

These examples show how large differences in the aggregate elasticity of labor supply stem from differences in the shape of the distribution H at the margin. (Of course, one can cook up even more extreme examples by playing with the shape of the distribution.) This makes clear how important it is to measure empirically this “marginal heterogeneity”. To our knowledge, this has never been done.

Robustness to wealth effects for the RBC model

The analysis that we have presented assumes equal weights. This can be justified, as in the

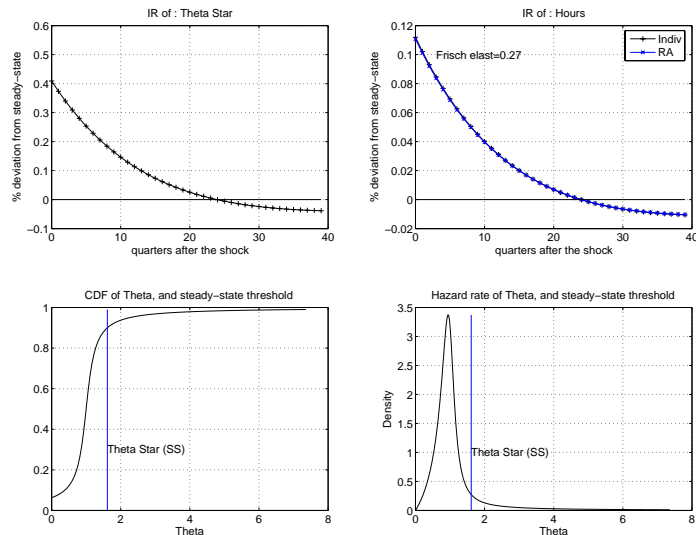


Figure 2: The response of θ^* and N to a persistent TFP shock (calibrated for an employment rate of 90%).

partial equilibrium section, by assuming that agents can insure against θ_0 ex-ante and so are ex-ante identical. But again one may want to relax this assumption and go to a framework with wealth differences between the agents. Here we discuss the results that hold when weights are not equal.

In this case, one can prove (1) that consumption takes the form $c_t(\theta^t, z^t) = k(\theta_0)C_t(z^t)$ where $C_t(z^t)$ is aggregate consumption; (2) each agent has a threshold rule, but the threshold is now state-contingent: the agent works if and only if $z_t/\theta_t \geq x(\theta_0, z^t)$. In this case, it is not the distribution of θ but the distribution of reservation wages that will matter.

Before turning to our empirical investigation, we check whether our results hold when we abandon the assumption of complete markets. It may not be surprising, in light of the quantitative literature on incomplete markets (e.g. Krusell and Smith (1998), Levine and Zame (2001)) that for reasonable parameters, deviations from complete markets do not matter much.

3 A Quantitative Model with Incomplete Markets, Indivisible Labor and Heterogeneity

Full insurance enables households to insulate their consumption from variations in hours worked, and this may be thus affect their labor supply choice.¹¹ For this reason, we wish to

¹¹To put it another way, the menu of available assets determines to what extent households are able to shift labor supply across time: households would like to transfer some of the labor income earned today toward future

examine the quantitative impact of a simple type of market incompleteness: we assume that households can only save using a risk-free bond. To avoid a curse of dimensionality, we make the model partial equilibrium, i.e. we assume an exogenous, constant interest rate, and a linear labor-only technology, so that wages are exogenous as well.¹²

We calibrate our model to reproduce simple cross-sectional statistics and then study its aggregate implications. An important element of our calibration is that we assume that the model (and the indivisibility) applies to monthly data, while we observe annual data. We are interested in two main results. First, given a reasonably calibrated distribution of tastes, what is the aggregate elasticity of labor supply implied by this incomplete market model? Second, we noted in Section 2 that a natural cross-sectional implication is that agents with either high or low taste for leisure θ are less responsive to aggregate shocks than intermediate θ . In Section 4, we will show that this prediction is true in the data. Here we test if our calibration reproduces this pattern of the data. We find that it can. This shows that this model is able to reproduce the sensitivity of the labor supply of the different groups to the business cycle. This gives us some confidence in the aggregate implications of the model. (Future work will extend the model and add more empirical moments to match.)

A. Individual problem

Except for the market structure, the environment is the same as in Section 2. Labor supply choice is either 0 or \bar{n} , and no lotteries are traded. At any point in time, an individual's situation is summarized by his asset level a , and the current states of two Markov chains: the idiosyncratic taste shock θ , and the aggregate wage z . The Bellman equation for this problem reads:

$$V(a, \theta, z) = \max_{n \in \{0, \bar{n}\}, c \geq 0, a' \in [\underline{a}, \bar{a}]} \left\{ U(c, n; \theta) + \beta \sum_{z' \in Z} \sum_{\theta' \in \Theta} Q_z(z' | z) Q_\theta(\theta' | \theta) V(a', \theta', z') \right\} \quad (3.1)$$

$$s.t. \quad : \quad a' = R(a + zn + h(\bar{n} - n) - c),$$

where h can be interpreted as the output of some household production, as an unemployment benefit, or as a combination of both. We set the upper bound \bar{a} for the asset choice such that it will never bind; the lower bound \underline{a} is set to zero, but our results seem insensitive to this assumption.

periods where they will not work, and the extent to which this is possible is determined by the assets market structure.

¹²This is in contrast to Chang and Kim (2006), who solve a very similar model but consider the general equilibrium feedback (They use methods adapted from Krusell and Smith (1998)). We do not think that general equilibrium is key for our results.

The transition matrices Q_θ and Q_z are each discrete approximations of an AR(1) process for $\log \theta$ and $\log z$:

$$\begin{aligned}\log z_{t+1} &= \rho_z \log z_t + \sigma_z \varepsilon_{t+1}^z, \\ \log \theta_{t+1} &= \rho_\theta \log \theta_t + \sigma_\theta \varepsilon_{t+1}^\theta,\end{aligned}$$

where $\varepsilon_{t+1}^z \stackrel{i.i.d.}{\sim} N(0, 1)$, and $\varepsilon_{t+1}^\theta \stackrel{i.i.d.}{\sim} N(0, 1)$ are independent processes. As a result, $\mathbb{E} \log z = \mathbb{E} \log \theta = 0$ and the typical wage is 1 (the Jensen term is small).¹³

This savings-labor supply problem is solved numerically using a simple value function iteration algorithm. We then use the optimal policy rules to simulate the cross-sectional distribution over (a, θ) and compute aggregates with this cross-sectional distribution. More precisely, let μ_t be the distribution at time t over $\Theta \times A$ where Θ and A are the discrete grids for which we solve our problem. Then if $g(\theta, a, z)$ is the optimal asset choice next period given the current state variables, we have the law of motion for the distribution μ_t :

$$\forall i' \in \Theta, \forall j' \in A : \mu_{t+1}(i', j') = \sum_{i \in \Theta} \sum_{j \in A} Q_{i, i'}^\theta \mathbf{1}_{j'=g(i, j, z_t)} \mu_t(i, j).$$

Given this distribution μ_t , we have the aggregate labor supply:

$$N_t = \sum_{i \in \Theta} \sum_{j \in A} \mu_t(i, j) n(i, j, z_t),$$

where n is the labor supply optimal policy function. Finally we construct a simulated panel to compare the empirical implications of this model to the data.

B. Calibration of the model

Assumptions

Preferences are assumed to be log in consumption and separable in leisure:

$$U(c, n; \theta) = \log c - \theta \frac{n}{\bar{n}} v(\bar{n}).$$

¹³Some simple implications of this model are worth noting. First, even without (macroeconomic or idiosyncratic) risk, e.g. if θ were a fixed characteristic, most people would exhibit deterministic cycles in and out of work, because their divisible labor choice would fall within $(0, \bar{n})$. Second, we should observe asset accumulation during work periods and asset decumulation during nonemployment: such an asset cycle, synchronized with activity periods, can be observed from the policy functions.

We take the period to be one month, since the indivisibility is only true for small time intervals. For now we set h equal to 50% of the wage. In an appendix, we also consider a second calibration with h equal to 10% of the wage.

Calibration of the aggregate wage process z

We use data from the BLS on average labor productivity (output per hour worked) to calibrate our process for the wage z . We use quarterly data, and we first take out the trend using an HP filter, then fit an AR(1) to the deviation from the trend of the wage (See Table 3.1). Finally, we convert this quarterly estimate in a monthly estimate. We also give the implied ρ_z and σ_z at annual frequency for comparison (See Table 3.2).

	ρ_z	σ_z
Y/L	0.769	0.006
w	0.654	0.008

Table 3.1: $AR(1)$ estimates fitted to the deviations from trend of quarterly labor productivity and wage (BLS, 1947-2005) Deviations from trend computed with an HP filter ($\lambda = 1600$)
 w : nonfarm business sector real compensation per hour,
 Y/L : output per hour for all persons.

	ρ_z monthly	σ_z monthly	ρ_z annual	σ_z annual
Y/L	0.916	0.0035	0.350	0.007
w	0.868	0.005	0.183	0.009

Table 3.2: Left: parameters implied by table 3.1 for our monthly calibration of $\{z_t\}$, Right: parameters implied by table 3.1 at annual frequency (for info; not used in calibration).

Calibration of the Process for θ and the Disutility of Labor using Cross-Sectional Data

We calibrate the three remaining parameters (the disutility of labor $v(\bar{n})$ and the process for idiosyncratic taste shocks i.e. $\rho_\theta, \sigma_\theta$) to match three cross-sectional moments. Our moments are (1) the cross-sectional average of annual hours worked per year; (2) the cross-sectional standard deviation of annual hours worked, and (3) the correlation of annual hours worked between two adjacent years.

We compute these moments in our PSID sample of family heads ¹⁴). The first moment, the mean, is estimated to be 1630 hours; the second, the standard deviation, is 720 hours; and the third one, the correlation, is 0.70.

By using only cross-sectional moments to calibrate our model, we leave completely open the possibility that it will fail to generate interesting aggregate behavior. We find it important to calibrate parameters so as to reproduce features of the cross-section of hours worked, since we showed that the shape of heterogeneity is key to assess the behavior of aggregate labor. Table 3.3 presents the outcome of our calibration. Our model can nearly match all three moments¹⁵, with three parameters (this of course, is a success!). Hence our simple model appears to have the potential to reproduce realistic features of labor supply.

Moment	Data	Model
Average hours worked per year: $\mathbb{E}[n_{i,t}]$	1630	1636
Standard deviation of hours worked per year: $\sigma[n_{i,t}]$	720	716
Correlation coefficient of hours worked: $Corr(n_{i,t}, n_{i,t+1})$	0.70	0.67

Table 3.3: calibration to cross-sectional moments.¹⁶

We also considered calibrating our model to match the job finding and job separation rates as measured in Shimer (2005). The average job finding rate is 61% per quarter in the data and 39% in the model, and the average job separation rate is 3.5% per quarter in the data and 11% in the model. However, our model makes no distinction between unemployment and inactivity,

¹⁴This footnote provides some details on the exact computation of the moments. Our sample is more fully described in Section 4. First, we restrict the sample to household heads between the ages of 25 and 65. This presumably is a rather inelastic population: for instance, we leave out married women, who are notably more elastic (Heckman (1993)).

Second, we must confront the fact that in the data, some people work more than the indivisible amount of our model. To do so, we put a cap on annual hours worked at 2,100 hours per year (which is an estimate of \bar{n} , around 40 hours per week for 52 weeks). That is, for any worker reporting more than 2,100 hours in a given year, we replace his hours worked by 2,100. This procedure seems justified inasmuch as we consider the participation decision, not an intensive hours decision.

¹⁵Further numerical improvements can probably make it possible to match all three moments exactly. (We are currently limited to a grid search.)

so these figures are not exactly comparable.¹⁷

letter	meaning	value	comments
β	discount factor	0.9913	corresponding annual value 0.9
R	gross interest rate	1.0025	corresponding annual value 1.03
h	benefit if not employed	0.5	parameter chosen a priori
$v(\bar{\pi})$	disutility of labor	0.17	calibrated to cross-sectional moments
ρ_θ	autocorrelation of θ	0.97	calibrated to cross-sectional moments
σ_θ	standard dev. of innovations to θ	0.43	calibrated to cross-sectional moments
ρ_z	autocorrelation of z	0.92	calibrated to aggregate labor productivity
σ_z	standard dev. of innovations to z	0.0035	calibrated to aggregate labor productivity

Table 3.4: Parameter Values for our Numerical Simulations

Table 3.4 summarizes the parameter values that we use. We simulated the economy for 1,000 agents for 3,400 months, and used the last 2,400 months of data to run regressions on the simulated panel.¹⁸

C. Quantitative implications for differences of sensitivities to the business cycle

Our model has predictions for the cross-sectional heterogeneity in sensitivities to the business cycle. As noted in section 2, the model with indivisible labor and heterogeneity implies that people with high or low taste for leisure will be relatively irresponsive to macroeconomic shocks, while people with intermediate values of θ , who are nearly indifferent between work and leisure, may adjust their hours by entering or exiting the workforce. (This is true if there is some persistence in θ , so that a low θ person today is more likely to be a low θ person tomorrow.) In Section 4 we show that this is true in the data. The results we present now are exactly comparable to those constructed from the PSID data in Section 4.

First we note that the prediction carries over from complete to incomplete markets. We proceed in the following (non-standard) way to make this result visible. First, we sort people on annual hours worked at t . This presumably reflects the idiosyncratic shocks θ they experienced

¹⁷This data was constructed by Robert Shimer. For additional details, please see Shimer (2005) and his webpage <http://home.uchicago.edu/~shimer/data/flows/>. When computing average *monthly* transition probabilities between employment and unemployment or inactivity, we find an average job finding rate of around 9%, and an average separation rate of around 5%.

¹⁸The simulation was started from the distribution obtained by running the model for a very long time after shutting the aggregate shocks (i.e. $z = 1$).

through year t (but assets at the beginning of year t may play a role too). Next, we form 10 groups, the deciles of the distribution of hours worked in year t . We use annual data as simulated by our model, to compare them with annual PSID data: so we take time aggregation into account, which obviously dampens the impact of the indivisibility. Finally, we compute a series dn_t^i that records the average variation in hours worked between year t and year $t + 1$ for the agents who were in decile i of the distribution of annual hours worked in t . The last step is to run, for each group i , a time series regression which measures the sensitivity of fluctuations in hours worked to fluctuations in aggregate hours worked:

$$dn_t^i = \alpha^i + \beta^i \cdot dN_t + \varepsilon_t, \quad (3.2)$$

where $dN_t = N_{t+1} - N_t = \frac{1}{10} \sum_{j=1}^{10} dn_t^j$ is the change in average hours over the whole population. Figure 3 plots estimates of β^i for each group i on the left panel, and the associated R^2 of each regression on the right panel. Table 3.5 gives detailed results from these regressions, where each column refers to a decile. Figure 3 shows a clear hump-shaped pattern of sensitivities to the business cycle, which confirms our key cross-sectional implication. This figure is directly comparable with figure 9 (in the appendix of Section 4), where we perform the same exercise on PSID data. Our model economy reproduces our results on PSID data reasonably well: first the distribution of average hours worked by each decile is similar up to decile 6 or 7 (see our lines $\mathbb{E}h_i$ (\cdot , 000) in table 3.5 and for figure 9). We obviously do not capture the fact that some people in the data work more than 2100 hours, our estimate of the indivisibility \bar{n} . The first group has around 10% of individuals, who do not work this year, and this group is not very responsive to macro fluctuations. The next two deciles are the most responsive groups, just as in the PSID. Members of these groups work between 600 and 1,600 hours in year t , and the fluctuations in their hours between t and $t + 1$ is the most correlated with average hours. By contrast, groups that always work (the upper 5 deciles in our model economy) are almost acyclical.

The ability of the model to reproduce this finding is an important success for two reasons. First, the model was calibrated to cross-section data and there is nothing that makes this match an obvious byproduct of our calibration. Second, the fact that we match reasonably well the different sensitivities of different groups to macroeconomic variations suggests that our model captures well the movements of reservation wages over the business cycle. We are thus now ready to draw the aggregate implications from the model.

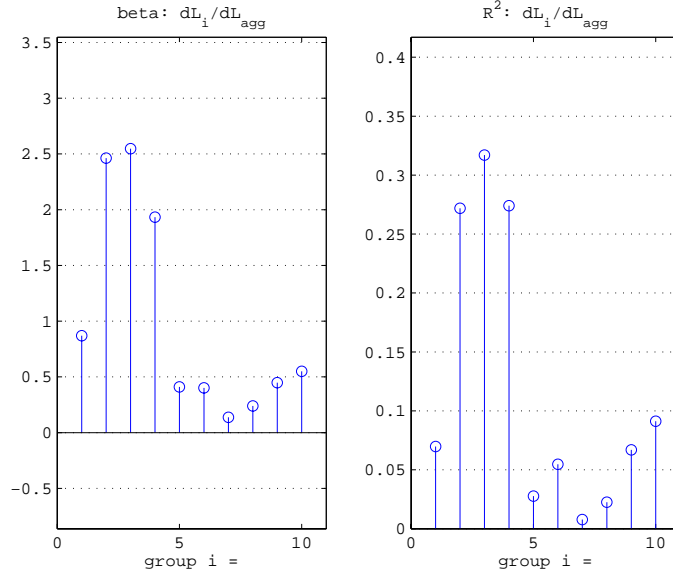


Figure 3: Regression coefficients β_i and associated R^2 for each decile i .

Group i	1	2	3	4	5	6	7	8	9	10
$\hat{\beta}$	0.87	2.46	2.55	1.93	0.41	0.40	0.14	0.24	0.45	0.55
$\hat{\sigma}$	0.23	0.29	0.27	0.22	0.17	0.12	0.11	0.11	0.12	0.12
$\hat{\sigma}_{Newey-West}$ (5 lags)	0.21	0.30	0.28	0.22	0.14	0.10	0.12	0.11	0.11	0.13
R^2	0.07	0.27	0.32	0.27	0.03	0.05	0.01	0.02	0.07	0.09
Durbin-Watson	1.66	1.78	1.90	1.96	1.62	1.81	2.01	1.66	2.18	1.95
$\mathbb{E}[n_i/N]$	0.02	0.37	0.80	1.09	1.27	1.29	1.29	1.29	1.29	1.29
$\mathbb{E}n_i$ (,000)	0.03	0.61	1.29	1.77	2.05	2.09	2.09	2.09	2.09	2.09
$\sigma(n_i)$ (,000)	0.02	0.09	0.08	0.05	0.02	0.00	0.00	0.00	0.00	0.00
$\mathbb{E}[d \log n_i]$ (,000)	na	0.77	0.03	-0.12	-0.08	-0.06	-0.06	-0.06	-0.06	-0.06
$\sigma(d \log n_i)$ (,000)	na	0.24	0.07	0.04	0.02	0.02	0.01	0.02	0.02	0.02

Table 3.5: Results of regression (3.2) for each group i (=decile of hours) on simulated data.

The model is described in Section 3.1. The point estimates $\hat{\beta}$ and R^2 are plotted in figure 3.

D. Quantitative implications for the aggregate labor supply elasticity

In this subsection, we use the simulated data to measure the aggregate elasticity of labor supply implied by our model and calibration. Table 3.6 gives estimates of β in the following time

series regression:

$$\Delta \log N_t = \beta \cdot \Delta \log z_t + u_t \tag{3.3}$$

We run this regression for monthly, quarterly and annual data. Since z is an exogenous shock to our infinitely-elastic labor demand, this regression identifies the elasticity of labor supply. We do not include an intercept, as our model is stationary.

For our calibration, the (quarterly) aggregate elasticity of labor supply is about 0.6: a one-percent increase in z induces an increase in labor supply by 0.6%. This is a fairly elastic labor supply, though it probably does not account for some macroeconomic facts. For instance, we know that the volatility of hours is roughly 50% greater than the volatility of wages, so if shocks to productivity were the only driving shock, we would need the elasticity to be greater than one. On the other hand, this 0.5% response is about the number of the RBC model of Figure 1, which corresponds to a rather high Frisch elasticity of labor supply of 3.08 (However, the persistence of wages is smaller in this case, hence the elasticity is smaller in this case).

These results are obviously sensitive to the calibration: the calibration determines the importance of the “marginal heterogeneity”. Our calibration tries to reproduce features of a PSID sample of family heads aged 25-65, which may not be representative of the whole pool of potential workers. We plan to take into account secondary earners and other age categories in future work.

As we increase the degree of time aggregation, the elasticity decreases (This feature seems robust across simulations and parameters). Moreover, the elasticity may vary with the state of the business cycle, but here the sign seems to be less robust. The last two columns of our Table 3.6 give the elasticity when we restrict the sample to the highest or lowest quartile of z . There are at least two possible explanations for this state-dependent elasticity. Perhaps in bad times, borrowing constraints make agents less able to substitute work over time.. Or perhaps the marginal agents to draw into the labor force are more homogeneous in good times than in bad times. Future work will explore these possibilities in the model and in the data.

	monthly	quarterly	annual	lowest quartile z	higher quartile z
β	0.75	0.60	0.53	0.56	1.02
$\sigma_{Newey-West}$	0.08	0.09	0.11	0.18	0.12
R^2	0.04	0.06	0.08	0.02	0.09

Table 3.6: Estimates of the Aggregate Elasticity of Labor Supply in Data Simulated from our calibrated model. The regression is (3.3), estimated without an intercept.

Newey-West standard errors computed with 5 lags for annual data,

3 lags for monthly and quarterly data .

4 Empirical Evidence from the PSID, 1967-1996

In this Section, we present empirical evidence that workers with different characteristics react differently to changes in aggregate hours. Our model contends that intermediate productivity workers should be more sensitive to aggregate fluctuations. By contrast, fluctuations of the *aggregate* wage rate z should have no effect on hours worked for individuals at both ends of the reservation wage distribution, if idiosyncratic shocks and reservation wages are sufficiently persistent. In this Section we interpret θ as encompassing an individual’s taste for leisure as well as his productivity at home (household production) relative to this market productivity (which we proxy by observed wages).

In this section, we check this prediction, which was derived in Sections 2 and 3; moreover, we investigate whether the cyclical groups are numerous and responsive enough to account for a sizeable fraction of aggregate fluctuations in hours.

Our main empirical strategy is to measure the sensitivity of hours worked to fluctuations in average hours, sorting workers along various indicators of their position on the reservation wage distribution.¹⁹ We try to capture an individual’s reservation wage $z^*(\theta)$ by fixed or variable characteristics. Reservation wages certainly vary over time: a worker’s productivity on the market, at home, his disutility of labor are all subject to idiosyncratic shocks. As a result, even though we fall short of a proper estimation of reservation wages, we proxy for them by age, education,

¹⁹We measure sensitivities to fluctuations in average hours worked, not to wages: first we believe that individual wages reflect some risk-sharing considerations, and thus may fail to fluctuate as much as the marginal product of labor; second, there is no obvious measure of z , the macroeconomic wage rate, in particular because the average wage also incorporates composition effects of the workforce.

current hours worked, current wage or current income.

In section A we describe the PSID data that we use. Then in section B, we run some regressions with cross-terms to identify the effect of some characteristics on the sensitivity to average hours; finally, in section C, we construct groups of workers based on some characteristics and examine if these groups have different sensitivities. These last results are comparable to the ones we show in Section 3 on simulated data.

A. The dataset

The Panel Study of Income Dynamics (PSID) collects data annually on demographics, income and hours worked for a sample of US households. The low frequency of the data is a limitation from our point of view, since this leads to an underestimation of the amount of intertemporal fluctuations in labor.

We use the different samples, that is, the main sample, the survey of economic opportunities, and the Latino sample (PSID sampling weights are used). We investigate only hours worked by heads of family units. This excludes married women. We keep only workers aged 25 to 65. We also drop observations with missing data for hours worked or for labor income, with negative family income, or with negative taxable income. Our final sample covers 30 years, from 1967 to 1996.²⁰ This leaves us with a total of 154,867 observations for 17,221 households, meaning that each household is surveyed on average for 9 years.

The PSID dataset is widely used for studies on employment and hours worked, and several studies confirm that the hours data is of good quality. Figure 4 shows that the aggregate measures of hours worked inferred from PSID are highly correlated with the hours measures constructed by the BLS from the Current Population Survey.

Figure 5 depicts the cumulative distribution of hours worked in 1995. (The shape is similar for other years.) There are approximately 20% of respondents that do not work and 38% that work between 1,700 and 2,300 hours this year (i.e. between 33 and 45 hours per week for 52 weeks). Those working positive amounts, yet less than 1,700 hours per year, have probably had discrete shifts in their labor supply over the months, entering or exiting the labor market: annual data induce time aggregation that hides the importance of the indivisibility of hours work. However, around 25% of all respondents report working more than 2,300 hours, meaning that they work much more than the usual hours at one job. They might even have several jobs or be entrepreneurs.

²⁰The PSID surveyed households only every two years after 1997.

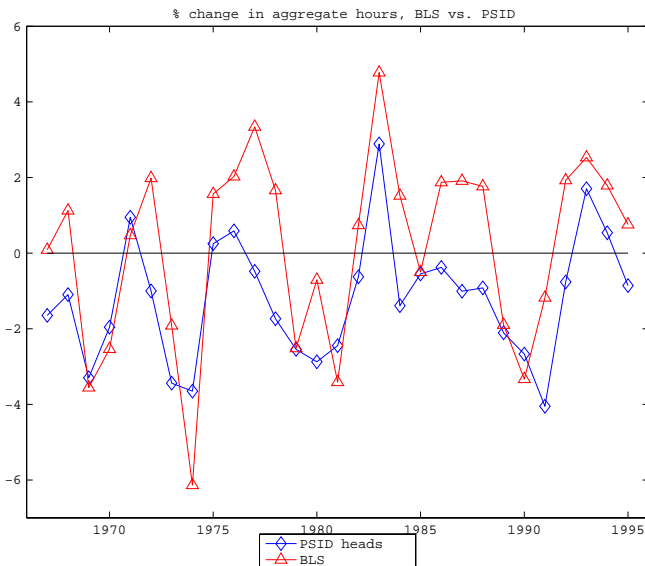


Figure 4: A comparison of the aggregate measures of hours worked per person from the BLS (CPS) and the PSID aggregates for heads, 1967-1996 (in growth rates).

We do not attempt to model this behavior.

B. Some Regressions on Individual Data

Our first specification to measure how individual sensitivities to aggregate fluctuations in labor are affected by some characteristics is the following:²¹

$$dn_{i,t} = c + \beta_1 \cdot dN_t + \beta_2 \cdot (x_{i,t}^* \cdot dN_t) + \beta_3 \cdot x_{i,t}^*. \quad (4.1)$$

Here $n_{i,t}$ is hours worked by respondent i over year t , N_t is the average of hours worked over the whole PSID population, and $x_{i,t}^*$ is a characteristic (e.g. income) of the household. We regress first differences (of the levels) of $n_{i,t}$ on first differences (of the levels) of N_t .²² We normalize $x_{i,t}^*$ as follows: $x_{i,t}^* = (x_{i,t} - \bar{x}_t) / \sigma_t$, where \bar{x}_t and σ_t are the cross-sectional mean and standard deviation of $x_{i,t}$ at time t . With this normalization, β_2 measures the effect of a one standard deviation increase in x on the overall coefficient on dN_t . Table 4.1 reports coefficients $\hat{\beta}_2$ for

²¹This is a pooled panel data regression. This allows us to use the PSID weights as in all our regressions. Fixed-effects would be partly redundant with the cross-terms.

²²We use first differences (rather than growth rates) because this allows us to include respondents with $h_{i,t} = 0$. Also, for people with low hours at t the correlation with aggregate variations in hours would be magnified in growth rates relative to first differences (low denominator); computing first differences makes all workers comparable in terms of their impact on changes in aggregate hours.

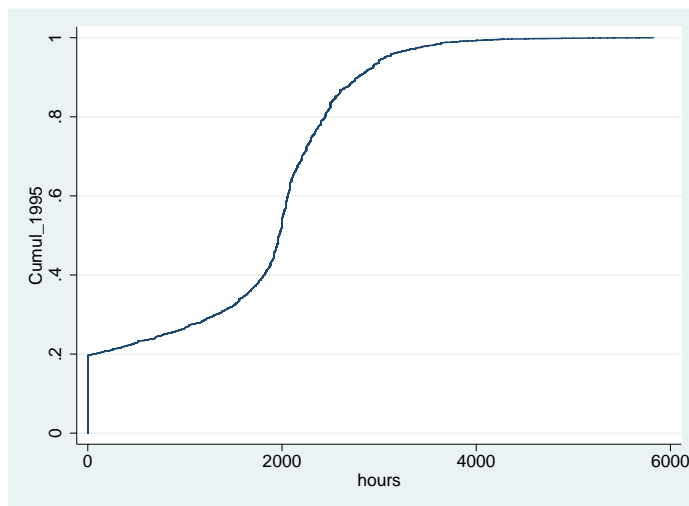


Figure 5: The cumulative distribution of hours worked in 1995 for the household head, PSID.

different $x_{i,t}$. (Estimates of β_1 all stand between .95 and 1.05.)

Cross-term $x_{i,t}^* \cdot dN_t$	$\hat{\beta}_2$	$\hat{\sigma}(\hat{\beta}_2)$
current hours worked: $(n_{i,t}^* \cdot dN_t)$	-0.12	0.0013
current wage: $(w_{i,t}^* \cdot dN_t)$	-0.19	0.0011
age: $(A_{i,t}^* \cdot dN_t)$	-0.14	0.0013
family income: $(I_{i,t}^* \cdot dN_t)$	-0.17	0.0011

Table 4.1: Point estimates and OLS standard error for the cross-term in the specification (4.1), for four possible characteristics. PSID Data, 1967-1996.

This table shows that working more, earning more per hour, being older, or earning a higher income makes a worker's labor less cyclical: for instance, a one standard deviation increase in wages is associated with a 0.19 decrease in the sensitivity of hours worked to average hours, i.e. instead of being about one, the sensitivity is about 0.81. These regressions are however silent on the hump-shape pattern of sensitivities.²³

Another way to see differential sensitivities to the business cycle is to directly group workers by hours worked last period. We now run for each group of hours the regression:

$$dn_{i,t} = c + \beta \cdot dN_t + \varepsilon_t. \quad (4.2)$$

²³These regressions proved unstable when squared terms were included. For this reason we examine the hump-shape pattern in the next subsection.

Group	$\hat{\beta}$	$\hat{\sigma}(\hat{\beta})$	\hat{c}
$n_{i,t} = 0$	0.83	0.003	179
$0 < n_{i,t} < 1,000$	2.07	0.007	466
$1,000 \leq n_{i,t} < 1,700$	1.13	0.004	167
$1,700 \leq n_{i,t} < 2,300$	0.62	0.001	-4
$2,300 \leq n_{i,t}$	0.89	0.003	-215

Table 4.2: Point Estimates and Standard Errors of the regression (4.2).

Regression done for each level of hours. PSID Data, 1967-1996.

Table 4.2 provides the results, which illustrate the hump-shaped pattern of responsiveness to aggregate hours: the estimate of β first grows with hours worked then decreases. In particular, people with 0 hours are less responsive than people with $0 < n_{i,t} < 1,000$. Respondents who worked more than 1,700 hours at t are also much less responsive than the second group.

C. Group regressions

Forming groups, and Computing Sensitivities to Aggregate Fluctuations

We now sort PSID respondents into different groups and examine the sensitivity of each group to aggregate fluctuations. The groups are formed as follows. Each year, we create deciles based on one characteristic (e.g. income or hours worked).

For each group or decile, we measure average hours worked at t and $t + 1$ for individuals that belong to that group at t . We compute the change in hours (the first difference) between hours at t and $t + 1$ for these individuals. This allows us to build a time series for each group. We then run time-series regressions to check the sensitivity of changes in average hours worked for each group to changes in aggregate hours worked. Our specification is in first-difference or growth rates::

$$dn_{i,t} = \alpha^i + \beta^i dN_t + \varepsilon_t^i,$$

where $dN_t = \frac{1}{\text{Card}(J)} \sum_{i \in J} \omega_{i,t} dn_{i,t}$ if J is the whole set of PSID respondents who report hours both in t and $t + 1$; and

$$dn_t^i = \frac{1}{\text{Card}(I_t^i)} \sum_{h \in I_t^i} \omega_{h,t} dn_{h,t+1},$$

where I_t^i is the set of people who are in group i at time t , and $\omega_{h,t}$ denotes PSID sampling weights.

Some of these characteristics are fixed (or predictable, e.g. age or education), so that the group composition is fixed or moves slowly. But some other characteristics (e.g. income or hours)

are volatile, so that agents move between groups. Groups are defined by deciles of the distribution of these variables at t , except for age and education, where we choose some thresholds to sort people.²⁴

The results are presented in figures 6 to 10 (in appendix). For each regression that we run (based on some sorting criterion), the left panel plots $\widehat{\beta}^i$ for each group i (on the x-axis), and the right panel plots the associated R^2 , which measures the association of the change in hours for group i and the change for the aggregate economy. Below each duet of graphs, a table reports regression statistics, where each column is a group i , as well as some characteristics of the group.²⁵ We also tried more specifications, which are available in an additional appendix available on request.²⁶

Comments on the results

Figure 6 displays the results when sorting on family income: a clear hump-shaped pattern of point estimates and R^2 statistics emerges. The third and fourth decile are significantly more

²⁴When sorting on current hours and wages, we have only people with $h_{i,t} = 0$ in the first group, so the 9 other groups are an equal division of the remaining respondents. Our procedure is otherwise equivalent to that in section 3.C. on our simulated sample.

When sorting on age, our cutoffs are 30, 40, 50, 60, hence 5 age groups.

For education, we have 6 categories until 1993, and follow afterwards (since it is a fixed characteristic, even if not surveyed after 1993). Groups are as follows: **(1)** less than 5 grades. **(2)** 6th to 8th grade. **(3)** 9th to 11th grade. **(4)** High School degree. **(5)** Non academic training or some college but no degree **(6)** College degree and/or more.

²⁵Each table reports the following statistics for each of the 10 groups: $\widehat{\beta}_i$; the OLS standard error of the coefficient $\widehat{\beta}_i$; the Newey-West standard error of $\widehat{\beta}_i$ (with 3 lags); the R^2 ; the Durbin-Watson statistic; hours worked by group i as a fraction of average hours worked; average hours worked by group i in thousands; standard deviations of hours worked by group i in thousands; average growth rate of hours worked by group i ; and standard deviation of growth rates of hours worked by group i ; and finally the average weight of group i in the population, which is not the same when we sort on age and education.

²⁶We also tried growth-rate specifications as in

$$\frac{dn_{i,t}}{n_{i,t}} = \alpha^i + \beta^i \frac{dN_t}{N_t} + \varepsilon_t^i.$$

For each sorting criterion, there are four types of results, depending on whether the data are in growth rates, or in first differences; and depending on whether the right-hand side variables dN_t are computed from the PSID sample, or from the BLS series. However for conciseness, we present them selectively. In particular, we only present regressions in first differences, because regressions in growth rates are more difficult to interpret: groups with low hours have always higher growth rates since the denominator is lower, without necessarily contributing a lot to aggregate fluctuations in hours worked. Also, we mostly show results where the regressor dN_t is the PSID average, rather than the BLS average: the BLS average leads to coefficients that differ more importantly, thus making the heterogeneity more visible. However, standard errors are also larger.

cyclical, in terms of labor supply, than the higher or lower ones; and they are also much more correlated with the macroeconomic fluctuations (the R^2 is 70% versus 20% for the higher groups). Similarly, figure 7 shows that hours worked are nearly insensitive to macroeconomic fluctuations for workers with college degrees, or for workers who didn't reach the sixth grade. By contrast, workers with intermediate schooling are more cyclical.

Consider now sorting workers on their labor market outcomes at t . Figure 8 sorts on wages, and shows sensitivities for the first group ($i = 1$) which did not work at t , and the nine other groups which are quantiles of the distribution of hourly wage rates. It is striking to observe that the second quantile has a much higher sensitivity to macro fluctuations than all other groups (1.85 versus 1.26 for the second highest). When sorting on hours worked at t , in figure 9, the difference is even larger: the second group has a sensitivity twice larger than the other groups. This group works 750 hours on average in year t . These results are very close to our numerical simulations of table 4.2 in Section 3. As a robustness test, we repeat this exercise using the BLS measure of the difference in average hours worked as a right-hand-side variable, rather than the PSID measure (see figure 10). The coefficients display the same pattern, with wider fluctuations but larger standard errors too. The R^2 coefficients are on the order of 45% for the first two quantiles in the distribution of hours worked, and less than 10% for most other quantiles

Regarding the aggregate impact of heterogeneity, consider figure 9, and the associated table. Group 2, the quantile with the lowest positive hours $n_{i,t}$, has an average variation in hours more than twice the macro average fluctuation. This group of the workforce worked on average 750 hours, i.e. 40% of the average. Since there are nine quantiles (group 1 has zero hours), this group represents 11% of the working population; and it thus accounts typically for about 4.4% of aggregate hours. Yet this group is responsible for more than 22% of the aggregate variation in hours worked.

This empirical section shows that the heterogeneity is quantitatively significant: some workers are several times more responsive than others to macro fluctuations. This heterogeneity may affect macroeconomics: we identify some types of workers that account for a much larger share of hours fluctuations than of hours averages. We probably underestimate the phenomenon since we sort only on some simple variables and not on several at once. In future work, we will try and estimate a reservation wage distribution.

With indivisible labor and heterogeneous reservation wages, different workers exhibit different labor supply elasticities: workers with either low or high reservation wages are insensitive to aggregate fluctuations, while workers who are nearly indifferent between working and not are highly sensitive to aggregate fluctuations. Moreover, the shape of the reservation wage distribution around this “marginal worker” determines the aggregate elasticity of labor supply. These implications are robust to the market structure (complete or incomplete markets).

Our empirical work supports strongly our cross-sectional implication. Using the PSID, we show that people with intermediate income, wage, hours worked or education are on average more sensitive to aggregate fluctuations. In some cases, a relatively small group accounts for a large share of aggregate fluctuations.

We build a simple partial-equilibrium incomplete market model and calibrate it to match cross-sectional statistics on hours worked. We find that it can reproduce our empirical findings from the PSID. We next examine the aggregate implications of the model: it predicts a more elastic labor supply than microeconomic estimates would suggest. This step is however still work in progress.

There are two natural lines to improve this paper, and we are currently exploring both.

- The first line is to make the moment-matching exercise of Section 3 more challenging by adding moments and simultaneously extending the model and especially the distribution of reservation wage. Indeed, one major limitation of our work is the highly parametric assumptions (in particular, the distribution of θ is log-normal), which is in sharp contrast to the theoretical results which emphasize a local hazard rate. Relaxing these assumptions would allow us to draw more precise implications for the aggregate elasticity of labor supply. (In this line of research, we could continue using the PSID and our procedure to confront the monthly model with the yearly data.)

- The second, more ambitious line is to estimate a dynamic discrete choice model of labor supply, and try to recover with little parametric assumptions the distribution of the reservation wage. For this model, it may be helpful to use higher frequency data (e.g. the NLSY).

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6 Appendix for Section 4: Group Regression Results

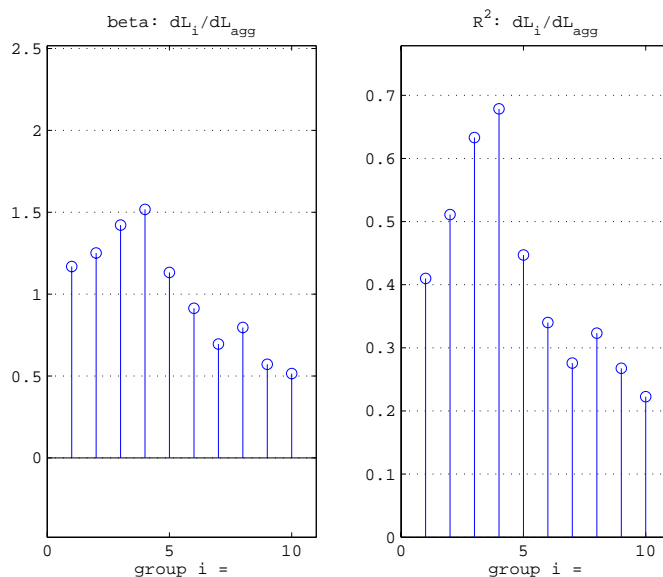


Figure 6: Sorting on family income, PSID aggregates, regression in first differences.

The groups are the income distribution deciles.

Group $i =$	1	2	3	4	5	6	7	8	9	10
$\hat{\beta}$	1.17	1.25	1.42	1.52	1.13	0.91	0.70	0.80	0.57	0.52
$\hat{\sigma}$	0.26	0.23	0.20	0.20	0.24	0.24	0.21	0.22	0.18	0.18
$\hat{\sigma}_{Newey-West}$	0.21	0.26	0.26	0.12	0.22	0.20	0.20	0.17	0.13	0.17
R^2	0.41	0.51	0.63	0.68	0.45	0.34	0.28	0.32	0.27	0.22
Durbin-Watson	1.55	1.93	2.33	1.50	2.12	1.54	2.36	2.21	2.82	2.23
$\mathbb{E}[n_i/N]$	0.39	0.78	0.93	1.00	1.05	1.10	1.14	1.16	1.19	1.27
$\mathbb{E}n_i$ (,000)	0.73	1.47	1.76	1.89	1.99	2.08	2.16	2.20	2.26	2.40
$\sigma(n_i)$ (,000)	0.10	0.10	0.09	0.09	0.09	0.08	0.07	0.06	0.06	0.05
$\mathbb{E}[d \log n_i]$ (,000)	0.16	0.00	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02	-0.03	-0.03
$\sigma(d \log n_i)$ (,000)	0.09	0.04	0.03	0.03	0.03	0.02	0.02	0.02	0.01	0.01

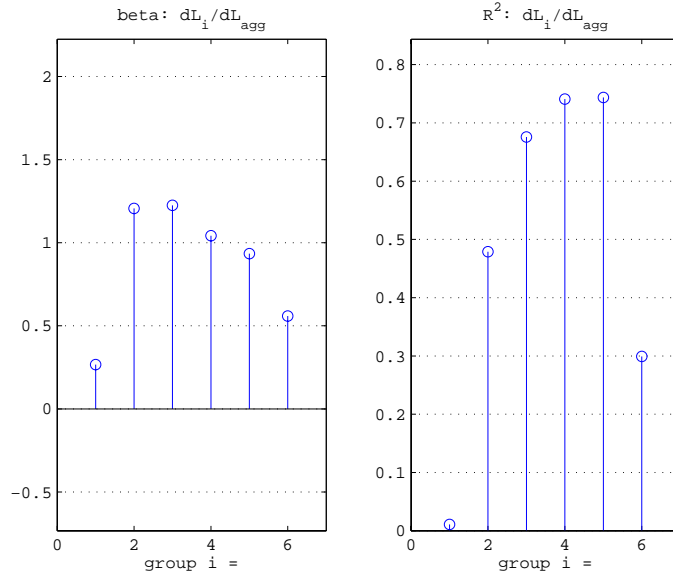


Figure 7: Sorting on education, PSID aggregates, regression in first differences.

Group $i =$	1	2	3	4	5	6
$\hat{\beta}$	0.27	1.21	1.22	1.04	0.93	0.56
$\hat{\sigma}$	0.48	0.24	0.16	0.12	0.10	0.16
$\hat{\sigma}_{Newey-West}$	0.34	0.24	0.11	0.12	0.11	0.14
R^2	0.01	0.48	0.68	0.74	0.74	0.30
Durbin-Watson	2.06	2.03	2.70	2.40	2.49	1.60
$\mathbb{E}[n_i/N]$	0.58	0.75	0.86	1.01	1.06	1.14
$\mathbb{E}n_i$ (,000)	1.11	1.43	1.63	1.91	2.01	2.15
$\sigma(n_i)$ (,000)	0.34	0.24	0.14	0.12	0.08	0.05
$\mathbb{E}[d \log n_i]$ (,000)	-0.08	-0.04	-0.02	-0.01	-0.01	-0.00
$\sigma(d \log n_i)$ (,000)	0.11	0.04	0.03	0.02	0.02	0.01
mean weight	0.03	0.09	0.16	0.22	0.29	0.22

Groups are as follows: (1) less than 5 grades. (2) 6th to 8th grade. (3) 9th to 11th grade. (4) High School degree. (5) Non academic training or some college but no degree (6) College degree and/or more.

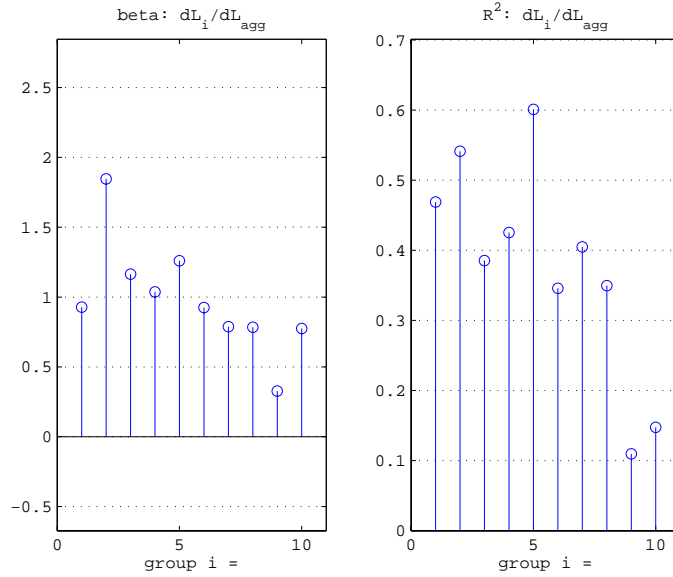


Figure 8: Sorting on current wage, PSID aggregates, regression in first differences.

Group $i =$	1	2	3	4	5	6	7	8	9	10
$\hat{\beta}$	0.93	1.85	1.16	1.04	1.26	0.93	0.79	0.78	0.33	0.77
$\hat{\sigma}$	0.19	0.32	0.28	0.23	0.19	0.24	0.18	0.20	0.18	0.35
$\hat{\sigma}_{Newey-West}$	0.19	0.23	0.32	0.16	0.15	0.17	0.20	0.16	0.15	0.28
R^2	0.47	0.54	0.39	0.43	0.60	0.35	0.40	0.35	0.11	0.15
Durbin-Watson	1.85	2.31	1.55	1.82	2.13	1.37	1.85	1.98	1.21	1.36
$\mathbb{E}[n_i/N]$	0.00	1.03	1.10	1.13	1.14	1.14	1.13	1.12	1.11	1.07
$\mathbb{E}n_i$ (,000)	0.00	1.95	2.08	2.14	2.16	2.15	2.15	2.12	2.09	2.03
$\sigma(n_i)$ (,000)	0.00	0.09	0.09	0.09	0.06	0.05	0.04	0.04	0.05	0.07
$\mathbb{E}[d \log n_i]$ (,000)	na	-0.08	-0.05	-0.03	-0.02	-0.02	-0.01	-0.01	0.01	0.03
$\sigma(d \log n_i)$ (,000)	na	0.04	0.03	0.02	0.02	0.02	0.02	0.02	0.01	0.03

The first group has zero hours worked, hence no observation for current wages, the latter groups are 9 quantiles in the distribution of hourly wage.

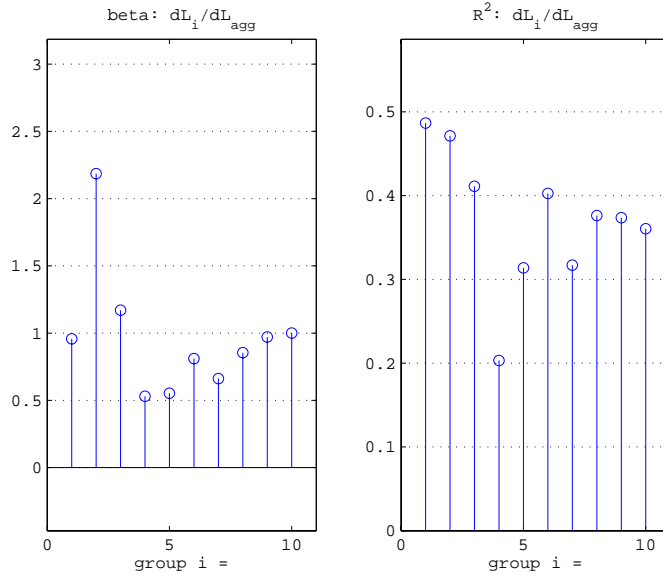


Figure 9: Sorting on current hours worked, PSID aggregates, regression in first differences.

Group $i =$	1	2	3	4	5	6	7	8	9	10
$\hat{\beta}$	0.96	2.19	1.17	0.53	0.55	0.81	0.66	0.85	0.97	1.00
$\hat{\sigma}$	0.19	0.44	0.26	0.20	0.15	0.19	0.18	0.21	0.24	0.25
$\hat{\sigma}_{Newey-West}$	0.18	0.26	0.26	0.16	0.14	0.17	0.19	0.14	0.27	0.29
R^2	0.49	0.47	0.41	0.20	0.31	0.40	0.32	0.38	0.37	0.36
Durbin-Watson	1.84	1.24	2.00	1.97	1.93	1.53	1.73	1.13	1.92	2.77
$\mathbb{E}[n_i/N]$	0.00	0.39	0.84	0.99	1.05	1.09	1.16	1.27	1.41	1.79
$\mathbb{E}n_i$ (,000)	0.00	0.75	1.59	1.87	1.98	2.06	2.20	2.40	2.67	3.39
$\sigma(n_i)$ (,000)	0.00	0.08	0.08	0.04	0.02	0.03	0.04	0.05	0.07	0.09
$\mathbb{E}[d \log n_i]$ (,000)	na	0.49	0.05	-0.00	-0.01	-0.01	-0.02	-0.03	-0.06	-0.14
$\sigma(d \log n_i)$ (,000)	na	0.19	0.04	0.02	0.02	0.02	0.02	0.02	0.02	0.01

The first group has zero hours worked, hence no observation for current wages, the latter groups are 9 quantiles in the distribution of current hours worked.

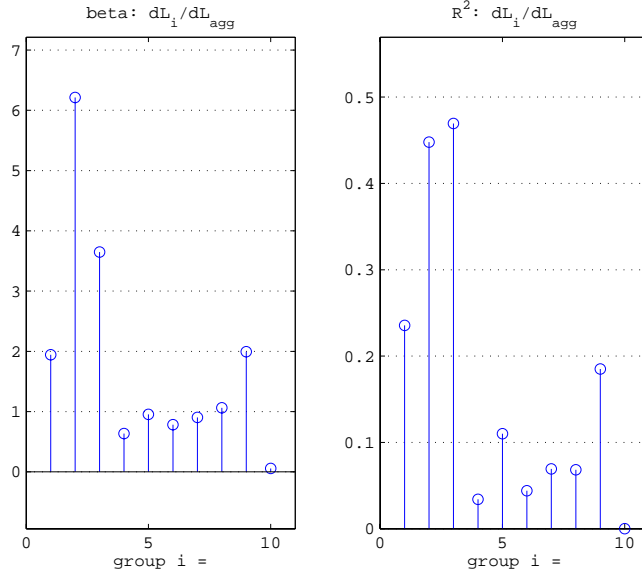


Figure 10: Sorting on current hours worked, BLS aggregates, regression in first differences.

Group $i =$	1	2	3	4	5	6	7	8	9	10
$\hat{\beta}$	1.94	6.21	3.65	0.63	0.95	0.78	0.90	1.06	1.99	0.06
$\hat{\sigma}$	0.66	1.30	0.73	0.64	0.51	0.69	0.63	0.74	0.79	0.92
$\hat{\sigma}_{Newey-West}$	0.75	1.32	0.73	0.60	0.50	0.55	0.54	0.76	1.00	0.87
R^2	0.24	0.45	0.47	0.03	0.11	0.04	0.07	0.07	0.19	0.00
Durbin-Watson	2.19	1.87	1.51	1.99	1.78	1.49	1.26	1.08	1.66	2.36
$\mathbb{E}[n_i/N]$	0.00	0.39	0.84	0.99	1.05	1.09	1.16	1.27	1.41	1.79
$\mathbb{E}n_i$ (,000)	0.00	0.75	1.59	1.87	1.98	2.06	2.20	2.40	2.67	3.39
$\sigma(n_i)$ (,000)	0.00	0.08	0.08	0.04	0.02	0.03	0.04	0.05	0.07	0.09
$\mathbb{E}[d \log n_i]$ (,000)	na	0.49	0.05	-0.00	-0.01	-0.01	-0.02	-0.03	-0.06	-0.14
$\sigma(d \log n_i)$ (,000)	na	0.19	0.04	0.02	0.02	0.02	0.02	0.02	0.02	0.01

The first group has zero hours worked, hence no observation for current wages, the latter groups are 9 quantiles in the distribution of current hours worked.