

# Central Counterparties\*

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## Abstract

Central counterparties (CCPs) have increasingly become a cornerstone of financial markets infrastructure. We present a model where CCPs are necessary to implement efficient trade when trades are time-critical, liquidity is limited and there is limited enforcement of trades. We then show that – when collateral is sufficient to avoid default – profit-maximizing CCPs “overcollateralize” trades relative to user-oriented CCPs and, hence, are less efficient. However, when collateral is not covering all default exposure, user-oriented CCPs avoid default, but allow for less trade, while profit-maximizing CCPs yield a higher volume of trade despite allowing for some default. In such a situation, profit-maximizing CCPs can be efficient, provided overall default costs are not too high.

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# 1 Introduction

Since the 1990s, central counterparties (CCPs) have become more and more commonplace as a cornerstone of financial market infrastructures. One role of CCPs is to novate contracts. In the novation process, the original contract between a buyer and a seller is extinguished and replaced by two new contracts; one between the buyer and the CCP, and another one between the seller and the CCP. For example, clearinghouses that serve as CCP interpose themselves as the legal counterparty for trades carried out on formal security exchanges and more recently also in over-the-counter (OTC) markets.

In assuming responsibility for the terms of the trade CCPs become exposed to *replacement cost risk* - the obligation to fulfill the terms of a contract with sellers (respectively buyers) even though buyers (respectively sellers) default on their obligations.<sup>1</sup> Novation concentrates default risk in the hands of a single institution, the CCP. As a consequence, it has the potential to disrupt financial markets if this risk is not properly controlled for.<sup>2</sup>

The willingness of the CCP to take on risk depends on its governance structure. Currently CCPs operate under two main governance structures. The first structure is the mutual ownership of the CCP among members. We will refer to such institutions as user-oriented CCPs. The second type of institutions is operated on a for-profit basis, rather than optimizing the provision of services for the majority of its users. Traditionally CCPs were user-oriented institutions, but lately many CCPs have demutualized and switched their objective toward profit-maximization.

Our paper makes three main contributions. First, we explain why CCPs exist. Second, we provide an explanation for the recent shift in governance structures. To do so, we investigate optimal collateral policies and how these are influenced by the CCP's governance structure. Third, we point out that for-profit CCPs lead to more default than user-oriented CCPs, but may be desirable from the point of view of users.

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<sup>1</sup>According to the European Central Bank Glossary on Payments and Security, this is “the risk that a counterparty to an outstanding transaction for completion at a future date will fail to perform on the settlement date. This failure may leave the solvent party with an un-hedged or open market position or deny the solvent party un-realised gains on the position. The resulting exposure is the cost of replacing, at current market prices, the original transaction.”

<sup>2</sup>See Russo, et al. (2002). The Committee on Payment and Settlement Systems (CPSS) recently issued international principles for CCP risk management that address the three key issues for controlling systemic risk in this area: (i) the transparent and prudent way of employing risk management, (ii) the design of governance structures that balance the requirements of users and the public interest; (iii) the potential trade-off between efficiency and risk in a situation of increased competitive pressure (see CPSS (2004)).

We develop a framework where a CCP arises endogenously in order to implement the efficient level of trade. The model features agents with a random need to trade a security. The structure of markets and preferences of traders are such that (i) trades have to be carried out by a specific time (i.e., trades are time-critical), (ii) trades cannot be fully and immediately settled at that time (i.e., there is limited liquidity) and (iii) traders have an opportunity to renege on their obligations (i.e., there is a problem of enforcing the terms of the trade). We show that these elements rule out a delivery-vs.-payment (DvP) mechanism which can lead to the impossibility of trade.

We introduce a CCP as a technology that can hold collateral and can commit to its promises. As such the CCP is the ideal counterparty and it arises endogenously in response to trading imperfections. However, while the transaction enables trades, it creates a replacement cost risk for the CCP, as it guarantees the terms of the trade in the event of some trader's default. The CCP controls its risk through collateral policies. It can employ margin calls on individual transactions to secure its exposure. It can also require agents, independently of their trading needs, to participate in a default fund. Using the default fund as an insurance pool, the CCP can mutualize losses on transactions across agents participating in the CCP.

We show that collateral structures used by a CCP differ remarkably across both governance structure. If a CCP maximizes profits, it will prefer the default fund, thus maximizing revenue from obtaining collateral. To the contrary, a user-oriented CCP (maximizing the welfare of users) will prefer margin calls to impose the cost of managing default risk on traders, i.e. from whom activities emanates the risk. As there is never default in our model in the absence of active risk taking by its participants, this leads to profit-maximizing CCPs *over-collateralizing* trades, which is more costly for market participants.

Finally, we introduce an aggregate shock that increases the risk associated with the security, but decreases the amount of trading. The increase in risk introduces the possibility of default, since the collateral available to traders cannot cover all exposures from trading. Whenever trading involves default, the CCP has to use the default fund to cover its exposure.

The basic trade-off for a CCP stems then from enabling trade at the expense of incurring losses on users' contribution to the default fund. When the overall gains from trade are large relative to the overall losses from default for non-traders, it is efficient to allow for trades even if the aggregate shock occurs. A user oriented CCP maximizes the welfare of a majority of users. If non-traders are in the majority, the user-oriented CCP will avoid shifting losses to the default fund, since non-

traders would pay a cost. Hence, once the aggregate shock hits, the user-oriented CCP shuts down trade and, thus avoids the costs of default for the majority of its users (non-traders in this case). A profit-oriented CCP to the contrary has still an incentive to allow for trade, as its revenue is strictly increasing in collateral pledged and users bear the costs of default. A profit-oriented CCP thus avoids the problem that non-traders hold up traders.

This implies that under such circumstances only a profit-maximizing CCP can commit to implement the efficient volume of trade in case of the aggregate shock, while a user-oriented cannot. Even though there is default associated with trades, having a for-profit CCP can then be welfare maximizing. While profit-oriented CCPs follow a costly collateral policy in normal times, they enable efficient trades in risky times. Provided risky times are frequent enough, users might prefer membership in a for-profit CCP. We conclude that profit-oriented CCPs are more likely to operate in markets where relatively risky securities are traded.

The remainder of the paper is as follows. The next section lays out the basic environment. Then, we show that a CCP is necessary to obtain an efficient level of trade if liquidity is limited, trade is time-critical and there is limited enforcement. Section 4 derives the optimal collateral policies of user- and profit-oriented CCPs. Section 5 introduces risk-taking and explains how different governance structures shape the trade-off between trading volume and default risk. Section 6 concludes.

## 2 Basic Environment

There are four periods  $t = 0, 1, 2, 3$ . The economy is populated with a measure one of agents. Agents can be of three types, non-traders, buyers or sellers. At  $t = 1$  people learn their type. With probability  $\pi$  they are traders and conditional on being a trader, with equal probability they are sellers or buyers. Cash can be stored between periods.

All agents are endowed with  $x_0$  unit of an infinitely divisible good - cash - in period 0. Non-traders do not receive any additional endowment in period 1, 2 or 3. Their preferences are described by  $u(c_3)$  where  $c_3$  is the amount of cash they have in period 3. We assume that  $u$  is strictly increasing and strictly concave.

Sellers receive an indivisible security at  $t = 1$  with cash pay-off equal to  $x_h$  or  $x_\ell$  with equal probability at  $t = 2$ . The security's payoff is known in period 2 and is public information. Sellers' preferences are given by  $u(c_3)$ , where  $c_3$  is again the amount of cash consumed in period 3. For

short, sellers are risk averse and face a risky endowment stream.

Buyers receive an additional endowment of cash  $x_2$  in period 2. They value cash and are risk-neutral. That is they derive linear utility from cash holdings in period 3.

Last, we assume that there is limited commitment in the economy, i.e. it is impossible to (fully) enforce *intertemporal* trades, so that exchanges either take place as *spot* transactions or are supported by incentives.<sup>3</sup> In particular, nobody can commit to give up his initial endowment at a later stage. Sellers cannot commit to hand over the security at any stage.

Our environment can be interpreted as follows. Agents are financial market participants who have direct access to financial markets. They either trade for their own account or are intermediaries that trade for people outside the model. The difference between traders and non-traders captures that financial markets participants have random trading needs that are not initially known. In period 0, any institutional choice has to be in the interest of the average agent. The transaction between risk-averse and risk-neutral traders captures the essentials of a futures trade: sellers are hedgers, while buyers are speculators. Period 1 is the trading period of the contract/security, the contract matures in period 2. Finally, settlement will occur in period 3.

### 3 The role of a CCP

#### 3.1 Efficient Allocation of Risk and Impossibility of Trade

The main problem in this economy is to allocate risk efficiently between buyers and sellers. An allocation is a distribution of cash and security holdings across agents. The state is the security's payoff. Allocations are required to be *individually rational* at  $t = 0, 1$  and 2. That is they have to deliver a higher utility in period 0 and 1 than would autarky.

Clearly, it is efficient to redistribute as much risk as possible across buyers and sellers, once preferences are known. That is all sellers should sell the security for cash to buyers at or after  $t = 1$ . This could either be achieved through a long-term contract or a spot trade.

Note first that trade at  $t = 2$  cannot allocate risk anymore. Then everybody knows the security's pay-off. As there is no uncertainty left, all agents value the security at its cash payoff. Hence, all trades have to take place prior to  $t = 2$ . In other words, trading is *time-critical*.

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<sup>3</sup>Note that we chose to dispense with reputation issues by having only 3 periods.

At  $t = 1$ , sellers would like to sell their security as long as its price  $p$  satisfies

$$u(x_0 + p) > \frac{1}{2}[u(x_0 + x_h) + u(x_0 + x_\ell)], \quad (3.1)$$

while buyers would agree to buy the security as long as

$$x_0 - p + x_2 + \frac{1}{2}(x_h + x_\ell) > x_0 + x_2 \quad (3.2)$$

or

$$\frac{x_h + x_\ell}{2} > p. \quad (3.3)$$

There are two possibilities to trade the security. First, a spot trade where the security is exchanged against cash only at  $t = 1$  and, second, a long-term contract. Sellers cannot promise to hand over the security at  $t = 2$ . Hence, buyers will never enter such a contract. This implies that sellers have to hand over the security at  $t = 1$  due to *limited commitment*. In a pure spot trade, buyers can at most pay their endowment of cash  $x_0$  at  $t = 1$ . However sellers require a payment of  $p > x_0$  to enter into a trade at  $t = 1$  if  $u(2x_0) < \frac{1}{2}[u(x_0 + x_h) + u(x_0 + x_\ell)]$ . Limited commitment then rules out long-term contracts and spot trades are impossible, as buyers do not have sufficient cash to fully pay for the security at  $t = 1$ . We summarize this discussion in the next proposition.

**Proposition 3.1.** *Let  $(x_h + x_\ell)/2 > x_0$ . If  $u(2x_0) \geq (1/2)[u(x_0 + x_h) + u(x_0 + x_\ell)]$ , the security is traded at  $t = 1$  at some price  $p \leq x_0$ . Otherwise, there is no trade at  $t = 1$  and the only feasible allocation is autarky.*

From now on we will assume that spot trades at  $t = 1$  with partial settlement in cash at  $t = 2$  are impossible. This boils down to the following assumption.

**Assumption 3.1.**  $u(2x_0) < (1/2)[u(x_0 + x_h) + u(x_0 + x_\ell)]$

Note that we do not make any assumption about the price, except  $p \in \mathcal{P} = (p_{min}, \frac{x_h + x_\ell}{2})$  where  $p_{min} > x_0$  is defined as the price that makes sellers indifferent between trading at  $t = 1$  or not or by the solution to

$$u(x_0 + p_{min}) = \frac{1}{2}[u(x_0 + x_h) + u(x_0 + x_\ell)]. \quad (3.4)$$

All prices in  $\mathcal{P}$  lead to an efficient allocation of risk with the surplus distributed across sellers and buyers according to  $p$ .

### 3.2 Central Counterparties - A Collateral Facility

Suppose now that a planner has access to a (potentially costly) collateral technology. The planner can take in and pay out cash while being able to commit to his actions. The planner can therefore require that (some or all) agents post collateral  $f$  at  $t = 0$  and collateral  $m$  at  $t = 1$ . We call  $f$  a *default fund* and  $m$  a *margin call*. The difference is that  $m$  can condition on an agent's type (buyer or seller) while  $f$  cannot. Collateral bears a fee  $\phi \geq 0$  per unit posted at any date charged to the agent. We take this fee as exogenously given here reflecting some underlying cost of collateral. The planner cannot seize the security from sellers.

To establish efficiency, the planner has to solve the commitment problem for buyers and sellers. He does so by requiring sellers to give up collateral against the promise to receive cash  $p$  at  $t = 3$ . Buyers are required to post collateral while being promised to obtain the security at  $t = 3$  against the payment of  $p$ . If there is no default all collateral is returned to the person that has posted it minus  $\phi$  at  $t = 3$ . If there is default the planner keeps all the collateral, but still has to fulfill the obligations of the trade against any party that has not defaulted. In other words, the planner novates the trade: he takes on all the existing default risk in exchange for collateral.

The incentives to default are as follows. Sellers need an incentive to give up the security at  $t = 3$  against cash  $p$ . Note that for  $x_\ell$  sellers *receive* cash in the settlement stage as  $p > x_\ell$ , so they have no incentive to default. If the state  $\sigma = h$  they do not default if and only if

$$u(x_0 + p - \phi(m + f)) \geq u(x_0 + x_h - (m + f)) \quad (3.5)$$

or

$$(1 - \phi)(m + f) \geq x_h - p. \quad (3.6)$$

Similarly, if  $\sigma = h$  buyers *receive* cash from sellers, so they have no incentive to default. If  $\sigma = \ell$ , they will not default at  $t = 3$  as long as

$$x_0 + x_2 + x_\ell - p - \phi(m + f) \geq [x_0 + x_2 - (m + f)] \quad (3.7)$$

which is equivalent to

$$(1 - \phi)(m + f) \geq p - x_\ell. \quad (3.8)$$

Hence sellers and buyers have identical default constraints. The incentives to trade at  $t = 1$  are the same as before taking into account that collateral is potentially costly and that having posted  $f$  for

the default fund is a sunk cost. We have

$$u(x_0 + p - \phi(m + f)) \geq \frac{1}{2}[u(x_0 + x_h - \phi f) + u(x_0 + x_\ell - \phi f)] \quad (3.9)$$

for sellers and

$$\frac{x_h + x_\ell}{2} - \phi(m + f) > p - \phi f. \quad (3.10)$$

for buyers at  $t = 1$ , where total collateral is restricted by liquid funds before and at the time of trading, or, equivalently,  $m + f \leq x_0$ .

Non-traders cannot be required to post any collateral  $m$  at  $t = 1$ .<sup>4</sup> Finally, given any  $m$ , agents will post initial collateral  $f$  only if they prefer to trade at  $t = 1$ , having posted  $m + f$  in total collateral. The incentives to post initial collateral  $f$  at  $t = 0$  are then described by

$$\frac{1}{2}\pi \left[ u(x_0 + p - \phi(m + f)) + (x_0 + x_2 + \frac{x_h + x_\ell}{2} - p - \phi(m + f)) \right] + (1 - \pi)u(x_0 - \phi f) \geq V_{aut} \quad (3.11)$$

where  $V_{aut} = (1 - \pi)u(x_0) + \frac{1}{2}\pi [E[u(x_0 + x_\sigma)] + (x_0 + x_2)]$ .

Consider first the case where posting collateral is not costly ( $\phi = 0$ ). As long as there is no default at  $t = 3$ , people prefer to trade at some price  $p \in \mathcal{P}$  at  $t = 1$ . Also, agents prefer to participate and post collateral  $f$  at  $t = 0$  as there are strictly positive expected gains from trade.

This implies that trade is possible if agents have enough cash to secure the trade with collateral at  $t = 1$ . Set  $x_0 = m + f$ . Since by Assumption 3.1,  $E[x_\sigma] \geq p$ , from equations (3.6) and (3.8) it follows that whenever sellers do not have an incentive to default buyers do not have an incentive either. The incentives for sellers to default are the smallest at  $p = \frac{x_h + x_\ell}{2}$ . Hence, as long as  $x_0 > \frac{x_h - x_\ell}{2}$  there will be some price  $p \in \mathcal{P}$  such that there is no default. The next proposition follows then immediately from this argument.

**Proposition 3.2.** *(First-best) Let  $x_0 > \frac{x_h - x_\ell}{2}$ . Then for  $\phi = 0$ , it is optimal to require collateral equal to  $m + f = x_0$  which enables trade at  $t = 1$  at some price  $p \in (p_{min}, \frac{x_h + x_\ell}{2})$  and rules out default at  $t = 3$ .*

Consider now the case of strictly positive costs of collateral ( $\phi > 0$ ). Then a first-best cannot be achieved anymore. However, as long as these costs are small enough, a collateral facility can still enable trade if it is beneficial ex-ante. In such a case, it is optimal, of course, to choose the least costly collateral policy.

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<sup>4</sup>If  $\phi = 0$  they are indifferent and we assume they do not post collateral.



**Proposition 3.3.** (Second-best) *There exists  $\bar{\phi}$  such that for all  $\phi \in [0, \bar{\phi}]$  there exists a collateral policy  $(m, f)$  where all agents participate at  $t = 0$ , there is trade at  $t = 1$  at some price  $p \in (p_{min}, \frac{x_h + x_\ell}{2})$  and no default at  $t = 3$ .*

*Proof.* Let the collateral policy be given by  $f = 0$  and

$$m = \begin{cases} \frac{1}{1-\phi}(x_h - p) & \text{for sellers} \\ \frac{1}{1-\phi}(p - x_\ell) & \text{for buyers.} \end{cases} \quad (3.12)$$

There exists  $\phi_1$  such that for  $\phi < \phi_1$ ,  $m \leq x_0$  for sellers (and, hence, for buyers). For such a collateral policy neither sellers nor buyers default and collateral costs are minimized. Since  $u$  is concave there also exists  $\phi_2$  such that for all  $\phi \in [0, \phi_2]$  equations (3.9) and (3.10) both are satisfied. Hence, traders prefer to trade for all  $\phi \leq \phi_2$ . Finally, all agents participate at  $t = 0$  for such  $\phi$ , since  $f = 0$ . Setting  $\bar{\phi} = \min\{\phi_1, \phi_2\}$  completes the proof.  $\square$

The collateral facility can thus be interpreted as a Central Counterparty Clearinghouse. The planner enables a long-term contract between trades by taking on the replacement cost risk. Since traders' positions are collateralized, it enables a delivery-vs-payment (DvP) mechanism on each side of the trade at the settlement stage. Note that trades on the buyers' side are secured partly with collateral and partly through the fraction of cash that can be seized – a feature that proxies for reputation and leads to some direct cash settlement beyond netting obligations with collateral posted earlier.

The remainder of the paper has now two objectives. We first explain some observed differences in collateral schemes across two different governance structures for CCPs and discuss the implications of these governance structures for welfare. There, we also show that under some weak assumptions (monotonicity of the coefficient of absolute risk aversion), there always exists a feasible collateral policy if and only if  $\phi$  is relatively low.

## 4 Governance Structure

In this section, we consider the effect of different governance structures - user-orientation or profit maximization - on collateral policies. There are two aspects of governance structures to account for. First, CCPs can have different objective functions. Second, CCPs can differ in their ownership structure, i.e., who holds claim to its profits. It can either be the users or some outside owner. In light of these two aspects, we interpret our framework as follows. The parameter  $\phi$  summarizes

the exogenous costs of pledging collateral *for users*. These costs are going as revenue to *owners* of the CCP, whether it is user-oriented or profit-oriented, to cover the CCP's operational costs. This reflects a strict separation of ownership from governance structure. The governance structure is then simply given by the objective function and can be interpreted as instructions by owners to a manager that runs the CCP on behalf of its owners.

We assume that a CCP maximizes its objective function with respect to its collateral policy taking trading prices as given. Therefore, given the price, the CCP's collateral policy might affect the level of participation.<sup>5</sup>

For now, we rule out the possibility of the CCP to default itself and abstract from the possibility to cross-subsidize default among participants. It follows from equation (3.6) and (3.8) that any default causes a loss for the CCP on the collateralized trade. Hence, being unable to cross-subsidize default, it is never in the interest of any type of CCP to allow for default.

#### 4.1 User-oriented CCP

We consider first the problem of a user oriented CCP. The problem of a user-oriented CCP is given by

$$V = \max_{(m_s, m_b, f)} \frac{1}{2} \pi [u(x_0 + p - \phi(m_s + f)) + (x_0 + x_2 + E[x_\sigma] - p - \phi(m_b + f))] + (1 - \pi)u(x_0 - \phi f)$$

subject to

$$x_0 \geq m_s + f \geq \frac{1}{1 - \phi} [x_h - p] \quad (4.1)$$

$$x_0 \geq m_b + f \geq \frac{1}{1 - \phi} [p - x_l] \quad (4.2)$$

$$x_0 + x_2 + E[x_\sigma] - p - \phi(m_b + f) \geq x_0 + x_2 - \phi f \quad (4.3)$$

$$u(x_0 + p - \phi(m_s + f)) \geq E[u(x_0 + x_\sigma - \phi f)] \quad (4.4)$$

$$V \geq V_{aut} \quad (4.5)$$

where  $V_{aut} = (1 - \pi)u(x_0) + \frac{1}{2}\pi [E[u(x_0 + x_\sigma)] + (x_0 + x_2)]$ . Inequalities (4.1) and (4.2) are default constraints for sellers and buyers respectively. Interim participation constraints are given by (4.3) and (4.4). The inequality (4.5) is the (ex-ante) participation constraint.

For given  $\phi$  and  $p$  no collateral policy might be feasible. We therefore restrict the analysis to the

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<sup>5</sup>In our static environment, CCPs rationally expect what the price will be and set its collateral policy accordingly. An extension is to consider price uncertainty, but we leave this to future work.

space of parameters such that there exists some feasible collateral policy. Proposition 3.3 shows that this set is non-empty for all  $p \in \mathcal{P}$ .

Using the objective function for a user-oriented CCP, it is obvious that it would like to set  $f = 0$  and  $m_i$   $i = s, b$  to the lowest value that satisfies the default constraints. This is the cheapest collateral policy. However, it might not be feasible if - given  $\phi$  and  $p$  - some interim participation constraint binds. Then, it may be necessary to use contributions to the default fund. This is exactly the intuition given in the literature describing the usefulness of the default fund. The next proposition expresses this formally.

**Proposition 4.1.** *A user-oriented CCP sets  $(m_s^*, m_b^*, f^*)$  such that*

$$m_s^* + f^* = \frac{1}{1 - \phi}(x_h - p) \quad (4.6)$$

$$m_b^* + f^* = \frac{1}{1 - \phi}(p - x_l). \quad (4.7)$$

$f^* > 0$  if and only if at least one interim participation constraint is binding.

*Proof.* A user oriented CCP would like to minimize  $m_s, m_b$  and  $f$ . Since the default fund bears a higher weight than margin calls in its objective function, it would like to increase  $f$  only as a last resort to enable trade. Therefore the default constraint will always bind. Otherwise, as  $f + m_i > 0$  it is possible to lower  $m_i$ , for  $i = s, b$ . This relaxes the participation constraints and increases the objective function of the CCP.

Now suppose  $f > 0$ , but no interim participation constraint is binding. Then,  $f - \epsilon$  and  $m_i + \epsilon$  for  $i = s, b$  is feasible for  $\epsilon > 0$  sufficiently small and increases utility ex-ante. This is a contradiction.

Suppose next some interim participation constraint is binding, but  $f = 0$ . Since  $f = 0$ , interim participation constraints are given by

$$E[x_\sigma] - p \geq \phi m_b \quad (4.8)$$

$$u(x_0 + p - \phi m_s) \geq E[x_0 + x_\sigma] \quad (4.9)$$

with some strict equality. But since default constraints hold with equality, this implies that the participation constraints do not bind. A contradiction.  $\square$

In words, we obtain the intuitive result that a user-oriented CCP always uses as little collateral as possible. Note that we do not restrict margin calls to be positive. Hence, for some parameters, it may be the case that  $f$  is so high that margin calls are negative for some agents. We characterize the

optimal collateral policy further. First, we show that there will always be trade at the least costly collateral policy for a user-oriented CCP at a given price  $p$ , if collateral costs are sufficiently low.

**Proposition 4.2.** *Let  $p \in \mathcal{P}$ . Then there exists a cut-off value  $\bar{\phi}(p)$  such that  $f^* = 0$  if and only if  $\phi \in [0, \bar{\phi}(p)]$ .*

*Proof.* Fix  $p \in \mathcal{P}$ . Set  $f = 0$  and let  $m_i$  be defined by the binding default constraints. Define  $\bar{\phi}$  as the highest value of  $\phi$  that satisfies

$$E[x_\sigma] - p - \phi m_b \geq 0 \quad (4.10)$$

$$u(x_0 + p - \phi m_s) \geq E[u(x_0 + x_\sigma)] \quad (4.11)$$

$$\frac{1}{2}\pi [u(x_0 + p - \phi m_s) + E[x_\sigma] - p - \phi m_b] \geq \frac{1}{2}\pi E[u(x_0 + x_\sigma)]. \quad (4.12)$$

If  $\phi > \bar{\phi}$ , either no collateral policy is feasible or any feasible collateral policy has  $f > 0$ . If  $\phi \leq \bar{\phi}$ , there is a feasible collateral policy with  $f = 0$ .  $\square$

Note that  $f$  is determined by (4.3), (4.3) or (4.5) depending on which binds first. But this in turn depends on the degree of risk aversion of sellers. For simplicity, we assume now that the coefficient of absolute risk aversion is monotone. Recall, that the certainty equivalent is increasing in the wealth level if and only if the coefficient of absolute risk aversion is increasing. Then, we can fully characterize the optimal collateral policy for the interval  $[0, \phi_{max}(p)]$  which describes all values for a given  $p$  such that a feasible collateral policy exists. We have the following proposition with the understanding that  $\bar{\phi}$  could be  $\phi_{max}$ .

**Proposition 4.3.** *Suppose the coefficient of absolute risk aversion is monotone. Then, for all  $\phi \in [\bar{\phi}(p), \phi_{max}(p)]$ , we have  $m_s > 0$  and  $f < x_0$ .*

*Proof.* Consider first the case when the coefficient of absolute risk aversion is non-increasing. We first show that

$$u(x_0 + p - \frac{\phi_{max}}{1 - \phi_{max}}(x_h - p)) \geq E[u(x_0 + x_\sigma - \frac{\phi_{max}}{1 - \phi_{max}}(x_h - p))]. \quad (4.13)$$

Suppose not. Since  $m_s + f \geq \frac{1}{1 - \phi}(x_h - p)$ , we cannot increase the LHS by lowering collateral. Furthermore, setting  $f > \frac{1}{1 - \phi}(x_h - p)$  to lower the RHS increases the difference, since this decreases the wealth level before risk ( $x_0 + \frac{\phi_{max}}{1 - \phi_{max}}(x_h - p)$ ) and we have non-increasing absolute risk aversion. Hence, at  $\phi_{max}$  there does not exist a feasible collateral policy. A contradiction.

Suppose now  $\phi \in (\bar{\phi}, \phi_{max})$ . This increases the wealth level before risk and by non-increasing absolute risk aversion increases the difference. Hence, for all these values of  $\phi$  the constraint is slack for  $f > 0$  and  $m_s = 0$ . Since

$$\frac{1}{1-\phi}(x_h - p) > \frac{1}{1-\phi}(p - x_\ell) \quad (4.14)$$

we have that the optimal  $f$  can at most be equal to the RHS. Hence,  $m_s > 0$ .

Consider next the case where the coefficient of absolute risk aversion is non-decreasing. Suppose  $f = x_0$  is optimal. Since  $u(x_0 + p) > E[u(x_0 + x_2)]$  for all  $p \in \mathcal{P}$ , we also have that

$$u((1-\phi)x_0 + p) > E[u((1-\phi)x_0 + x_\sigma)] \quad (4.15)$$

for all  $\phi \in [0, 1]$ . But then, reducing  $f = x_0$  by  $\epsilon > 0$  small enough and increasing  $m_i$  by the same amount is feasible. Hence,  $f = x_0$  cannot be optimal. A contradiction.  $\square$

## 4.2 For-profit CCP

We turn now to the case where CCPs maximize profits. CCPs have then a different objective function from user-oriented CCPs given by  $\phi f + \frac{1}{2}\phi\pi[m_s + m_b]$ . The constraint set of a for-profit CCP is identical to the one of a user-oriented CCP.

Given any price  $p \in \mathcal{P}$ , when collateral costs are sufficiently low,  $f = x_0$  is feasible. Such a policy maximizes the CCPs possible revenue, but it is inefficient since the CCP requires more collateral than necessary. Furthermore, all CCP members have to post collateral and not only traders.

**Proposition 4.4.** *For all  $p \in \mathcal{P}$ , there exists  $\underline{\phi}(p) > 0$  such that a profit-oriented CCP sets  $f = x_0$  and  $m_s = m_b = 0$  for all  $\phi \in [0, \underline{\phi}(p)]$ . Furthermore, if the coefficient of absolute risk aversion is monotone,  $0 < f < x_0$  for all  $\phi \in [\underline{\phi}(p), \phi_{max}(p))$ , where  $\underline{\phi}(p) < \bar{\phi}(p)$  for all  $p \in \mathcal{P}$ .*

*Proof.* If  $f = x_0$  the buyer's default and interim participation constraints never bind. There are at most three constraints that can be violated: the seller's default constraint, the seller's interim and ex-ante participation constraints:

$$x_0 \geq \frac{1}{1-\phi}[x_h - p] \quad (4.16)$$

$$u((1-\phi)x_0 + p) \geq E[u((1-\phi)x_0 + x_\sigma)] \quad (4.17)$$

$$\frac{1}{2}\pi [u((1-\phi)x_0 + p) + (x_0 + x_2 + E[x_\sigma] - p - \phi x_0)] + (1-\pi)u((1-\phi)x_0) \geq V_{aut}. \quad (4.18)$$

The first and the third are fulfilled if and only if  $\phi$  lies in a positive interval from 0. For the second one, note that

$$u(x_0 + p) > E[u(x_0 + x_\sigma)]. \quad (4.19)$$

For the second statement, suppose first the coefficient of absolute risk aversion is non-increasing. Define  $\tilde{\phi}$  such that

$$u((1 - \tilde{\phi})x_0 + p) = E[u((1 - \tilde{\phi})x_0 + x_\sigma)] \quad (4.20)$$

If there is no interior value of  $\phi$  that satisfies this equation we define  $\tilde{\phi} = 1$ . For  $\phi$  larger than  $\tilde{\phi}$ , the inequality is violated. Finally, suppose the coefficient of absolute risk aversion is non-decreasing. Then,  $u((1 - \phi)x_0 + p) > E[u((1 - \phi)x_0 + x_\sigma)]$  for all  $\phi$ . Hence, there also exists an interval such that this inequality is satisfied.

Finally, suppose  $f = 0$ . Then, the ex-ante participation constraint does not bind for any  $\phi < \phi_{max}$ . Hence, it is feasible to set  $f = \epsilon > 0$  for  $\epsilon$  sufficiently small.  $\square$

### 4.3 Comparison of Collateral Policies

We are now in a position to compare the welfare achieved by different governance structures. The main result in this section is that a user-oriented CCP strictly dominates a profit-oriented CCP in welfare terms for any cost of collateral  $\phi$  and for any price  $p$  guaranteeing some surplus from trade at  $t = 1$ .

**Lemma 4.1.** *The ex-ante participation constraint never binds for a user-oriented CCP for  $\phi \in (0, \phi_{max})$ . However, the ex-ante participation constraint is binding for a profit-oriented CCP, whenever  $f < x_0$ .*

*Proof.* Suppose  $\phi \in (0, \phi_{max})$ . Let  $(f^*, m_s^*, m_b^*)$  be an optimal collateral policy for a user-oriented CCP. If  $f^* = 0$ , the interim participation constraints are not binding. Since there is a strictly positive surplus from trade for any  $p \in \mathcal{P}$ , the ex-ante participation constraint has then a strict inequality.

Let  $f^* > 0$  and suppose that the ex-ante participation constraint holds with strict equality. Some of the interim participation constraint must be binding, since otherwise one could lower  $f^*$  by a sufficiently small  $\epsilon > 0$  and increase  $m_i^*$   $i = s, b$  by the same amount. Suppose first, the buyer's participation constraint is binding. Then,  $m_b > 0$  as  $E[x_\sigma] > p$ . Consider now  $\phi + \epsilon > \phi$ . Then,  $m_b$  must be lower and, hence, the default fund  $f$  has to increase in order to satisfy the default constraint of buyers. Then, the ex-ante participation constraint must be violated for  $\pi > 0$  unless

$\phi = \phi_{max}$ , which is not possible. Suppose next that the seller's participation constraint binds at  $\phi$ . Consider  $\phi + \epsilon > \phi$ . In order to fulfill the ex-ante participation constraint,  $\phi f$  has to decrease. By the corollary, the seller's default constraint always binds and, hence,  $m_s + f$  have to increase. This implies that for any feasible  $\tilde{m}_s$  and  $\tilde{f}$  at  $\phi + \epsilon$  we have

$$E[u(x_0 + x_2 + (\phi + \epsilon)\tilde{f})] > u(x_0 + p - (\phi + \epsilon)(\tilde{m}_s + \tilde{f})). \quad (4.21)$$

Hence,  $\phi + \epsilon$  is not feasible or, equivalently,  $\phi + \epsilon > \phi_{max}$ . A contradiction.

We now prove the second statement. First, let  $m_i^* + f^* \leq x_0$  for all  $i = s, b$  with  $f^* < x_0$  be the optimal collateral policy. Suppose that the ex-ante participation constraint does not bind. If none of the interim participation constraint bind, we can lower  $m_i^*$  by  $\epsilon > 0$  sufficiently small and increasing  $f^*$  by the same amount is feasible, since it relaxes the interim participation constraint. If  $m_i^* = 0$  for some  $i$ , we can increase  $f^*$  directly. Since this increases the objective function, we obtain a contradiction.

Next, suppose the seller's interim participation constraint binds. Then, the same argument as before applies. Finally, suppose that the buyer's PC at  $t = 1$  binds. Then,  $m_b^* > 0$ . Again the argument for the other cases applies.  $\square$

Lemma 4.1 implies immediately that a for-profit CCP always requires more collateral per user (in ex-ante terms) than a user-oriented CCP, unless  $\phi(p) = \phi_{max}(p)$  when the ex-ante constraint binds with  $f = 0$  and  $m_i$  such that the default constraints are fulfilled. Since collateral is costly, we have the following result.

**Proposition 4.5.** *The user-oriented CCP yields a strictly higher utility for users than the profit-oriented CCP for all  $\phi \in (0, \phi_{max})$ , since a profit-oriented CCP sets  $2f + m_b + m_s$  and/or  $f$  strictly higher than a user-oriented CCP for any  $\phi \in (0, \phi_{max}(p))$  and all  $p \in \mathcal{P}$ .*

*Proof.* This follows directly from Lemma 4.1 and the fact that  $f^* = x_0$  only for  $\phi < \underline{\phi}$  for the profit-oriented CCP. In the latter case, we always have  $f^* = 0$  for the user-oriented CCP, since  $\underline{\phi} < \bar{\phi}$ .  $\square$

To summarize this section, as revenue is increasing in the collateral posted, we find that a profit-oriented CCP “over-collateralizes” relative to a user-oriented CCP thus lowering welfare. There are two distinct channels for this result. First, profit-oriented CCPs require more collateral if collateral

costs are small. Second, independent of collateral costs, a profit-oriented CCP always requires non-traders to contribute to the default fund upfront. To the contrary, a user-oriented CCP relies on the default fund only if necessary, i.e if prices are sufficiently skewed to erase surplus from trading. In other words the sunk cost feature of the default fund enables trade.

These results depend on the fact that default never occurs in this basic environment. Hence, having more collateral than necessary to prevent default cannot be beneficial. In the next section, we extend the model and allow for default. We show that the new feature can overturn our conclusion that user-oriented CCPs are preferred from a welfare point of view.

## 5 Risk, Default and the Efficient Volume of Trading

In the basic environment, risk in the economy arises from the uncertain payoff of the security. We introduce now an aggregate shock that increases the extent of this risk. With increased risk, the available collateral for traders will not be enough anymore to cover all exposures. Hence, the aggregate shock introduces default on trades.

We assume now that the economy is safe with probability  $(1 - \eta)$  and the security returns  $x_\ell$  or  $x_h$  with probability  $1/2$ . This is essentially the case studied so far. However, with probability  $\eta$  the economy is hit by an aggregate shock. First, the security returns  $\hat{x}_\ell$  and  $\hat{x}_h$  with probability  $1/2$ . There is an increase in risk, since  $\hat{x}_h - \hat{x}_\ell > x_h - x_\ell$ .<sup>6</sup> Second, there are less traders in the risky economy, i.e.,  $\hat{\pi} \leq \pi$  with probability  $\eta$ . We assume that  $\hat{\pi} < 1/2$ . Furthermore, we assume that the payoff is such that the price is not modified. To give some substance to the notion of risk, we impose the following assumption on the payoff structure.

**Assumption 5.1.**  $\hat{x}_h - p > (1 - \phi)x_0$  and  $p - \hat{x}_\ell > (1 - \phi)x_0$ .

Given these assumptions, whenever the aggregate shock hits, sellers will default, if the security returns  $\hat{x}_h$ , and buyers will default, if the security returns  $\hat{x}_\ell$ . However when buyers default, sellers do not and inversely. We assume that the state of the economy (risky or safe) is public information. Given the uncertainty on the state ex-ante, the default fund will be identical across states. However, margin calls may differ, since the uncertainty will be realized at the time of pledging it.

We now analyze the outcome for user-oriented and for-profit CCPs. We will denote  $\hat{f}$  the amount

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<sup>6</sup>Although we would still require  $E[\hat{x}_\sigma] = E[x_\sigma]$ .



participants are required to pledge in the default fund (independently of the aggregate shock) and  $\hat{m}_i$  the margin call in case of the aggregate shock.

## 5.1 User Oriented CCP

Since Assumption 5.1 holds, a CCP cannot avoid default once it allows for trade. When there is default, a CCP has to cover its losses. In order to do so, we assume that the default fund is mutualized. After revenue from collateral the default fund available to finance the incurred loss is equal to  $(1 - \phi)\hat{f}$ .

When the state is high in the risky economy the loss per defaulting seller is given by  $\Gamma_s = \hat{x}_h - p - (1 - \phi)(\hat{f} + \hat{m}_s)$ , while if the state is low, the loss per defaulting buyer is  $\Gamma_b = p - \hat{x}_\ell - (1 - \phi)(\hat{f} + \hat{m}_b)$ . These losses have to be compensated from the default fund. As is current practice, we assume that only the default fund can be used to finance the default, and not the margin call of other parties engaged in trades.<sup>7</sup>

The CCP can either charge traders and non-traders symmetrically. However, this would be acting too much in our favor since non-traders do not benefit from being in the risky economy. Hence, we assume that the CCP first charges as much as possible to traders (i.e.,  $(1 - \phi)\hat{f}$  to a mass  $\hat{\pi}/2$  of agents) and if that does not suffice to cover losses, the CCP charges  $\delta_i = \max\{0, \frac{\hat{\pi}}{2}[\Gamma_i - (1 - \phi)\hat{f}]\}$  to non traders. Under the assumption that  $(1 - \phi)\hat{f}\hat{\pi}/2$  does not suffice to cover losses, the payoff for a user oriented CCP in a risky economy is given by

$$\begin{aligned} \hat{V}_u &= \frac{1}{2}\hat{\pi} \left[ \frac{1}{2}u(x_0 + p - \phi(\hat{m}_s + \hat{f}) - (1 - \phi)\hat{f}) + \frac{1}{2}u(x_0 + \hat{x}_h - (\hat{m}_s + \hat{f})) \right] \\ &+ \frac{1}{2}\hat{\pi} \left[ \frac{1}{2}(x_0 + x_2 + \hat{x}_h - p - \phi(\hat{m}_b + \hat{f}) - (1 - \phi)\hat{f}) + \frac{1}{2}(x_0 + x_2 - (\hat{m}_s + \hat{f})) \right] \\ &+ (1 - \hat{\pi}) \left[ \frac{1}{2}u(x_0 - \phi\hat{f} - \delta_b) + \frac{1}{2}u(x_0 - \phi\hat{f} - \delta_s) \right] \end{aligned}$$

Given the CCP acts in the interest of the majority of its users, it minimizes the cost of default born by non-traders by setting  $\hat{m}_i$  such that  $\hat{f} + \hat{m}_i = x_0$  for traders. Then the share of the default fund of non-traders  $(1 - \hat{\pi})\hat{f}$  should cover the loss from default after seizing the traders default funds or

$$\delta = \max\{0, \max_i \frac{\hat{\pi}}{2}[\Gamma_i - (1 - \phi)\hat{f}]\}. \quad (5.1)$$

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<sup>7</sup>In this paper we will not explain why CCPs only use the default fund and not the margin calls of other parties when there is default.

We let  $\Gamma = \max_i \Gamma_i$ . Given the CCP wishes to minimize the cost for non-traders, the CCP will set  $\hat{f}$  so that the non-traders' participation in the default fund is just enough to cover the maximum amount lost. Otherwise, it would be optimal to reduce the default fund slightly. Therefore,  $\hat{f}$  will be either 0, if  $\delta = 0$ , or such that

$$\left[ (1 - \hat{\pi}) + \frac{\hat{\pi}}{2} \right] (1 - \phi) \hat{f} = \frac{\hat{\pi}}{2} \Gamma \quad (5.2)$$

or

$$\hat{f} = \frac{\hat{\pi}/2}{(1 - \phi)(1 - \hat{\pi}/2)} \Gamma. \quad (5.3)$$

Using this result in the expression for  $\delta$ , we find that  $\delta > 0$ .

Note that we haven't checked ex-ante and interim participation constraints. However, if they are not satisfied for this value of  $\hat{f}$ , then  $\hat{f}$  will be higher or there cannot be trade. Also, it is useful to note that if  $f < \frac{\hat{\pi}/2}{(1 - \phi)(1 - \hat{\pi}/2)} \Gamma$  then  $\hat{f} = \frac{\hat{\pi}/2}{(1 - \phi)(1 - \hat{\pi}/2)} \Gamma$  is the most efficient collateral policy available to the CCP, if  $\eta$  is relatively small. Indeed, all the available collateral fund  $(1 - \hat{\pi})(1 - \phi)\hat{f} + \frac{\hat{\pi}}{2}(1 - \phi)\hat{f}$  is used to cover losses. If this what not the case, the CCP could reduce  $\hat{f}$  slightly and increase utility in the case where the economy is safe, which occurs with a higher probability. Note that given  $\hat{x}_h - p > (1 - \phi)x_0$  and  $p - \hat{x}_\ell > (1 - \phi)x_0$ , we have  $\hat{f} < x_0$  not binding. We can summarize the above discussion in the following lemma.

**Lemma 5.1.** *If a user-oriented CCP allows for trade with the aggregate shock, it sets a default fund  $x_0 > \hat{f} \geq \frac{\hat{\pi}/2}{(1 - \phi)(1 - \hat{\pi}/2)} \Gamma > 0$ .*

Hence, non-traders lose part of their collateral when there is trade with the aggregate shock. Since, we have assumed that  $\hat{\pi} < 1/2$ , non-traders are in the majority and, hence, the CCP shuts down trade whenever the aggregate shock occurs.

**Proposition 5.1.** *A user-oriented CCP will set  $\hat{f} = f$  and  $\hat{f} + \hat{m}_i > x_0$  for all  $i$ . Hence, there is no trade whenever the aggregate shock occurs.*

## 5.2 Profit Maximizing CCP

The analysis for a profit maximizing CCP is easier since we already saw that this type of CCP will charge as high a default fund as possible. For simplicity, we will restrict our attention here to the case where  $\phi$  is low enough such that  $f = x_0$ . In this case we want to determine first whether there will be trade in the risky economy. We denote by  $\delta$  the share of the default fund that will be used

to compensate for losses due to default by one type of trader. Since a CCP cannot make losses, it must be the case that

$$\delta_i[(1 - \hat{\pi})(1 - \phi)\hat{f} + \frac{\hat{\pi}}{2}(1 - \phi)\hat{f}] \geq \frac{\hat{\pi}}{2}\Gamma. \quad (5.4)$$

As  $f = x_0$ , we will also have  $\hat{f} = x_0$ , if this is feasible. Hence,

$$\delta_i = \frac{\hat{\pi}/2}{(1 - \hat{\pi}/2)(1 - \phi)x_0}\Gamma_i. \quad (5.5)$$

**Lemma 5.2.** *Suppose a profit-oriented CCP sets  $\hat{f} = f = x_0$  and  $\hat{\pi}$  is sufficiently low. Then there will be trade whenever the aggregate shock occurs.*

*Proof.* Since  $\hat{f} = x_0$ , buyers prefer to trade if

$$x_0 + x_2 - \phi x_0 < \frac{1}{2}(x_0 + x_2 + \hat{x}_h - p - \phi x_0 - (1 - \phi)\delta_s x_0) + \frac{1}{2}x_2, \quad (5.6)$$

which is equivalent to

$$\frac{1}{2}(1 - \phi)x_0 < \frac{1}{2}(\hat{x}_h - p - (1 - \phi)\delta_s x_0). \quad (5.7)$$

Replacing the expression for  $\delta_s$  and  $\Gamma_s$  we obtain buyers prefer to trade if

$$[(1 - \phi)x_0 - (\hat{x}_h - p)][1 - \frac{\hat{\pi}/2}{(1 - \hat{\pi}/2)}] < 0. \quad (5.8)$$

Hence, by assumption 5.1 buyers always prefer to trade when the aggregate shock occurs.

Sellers will prefer to trade whenever

$$\frac{1}{2}u((1 - \phi)x_0 + \hat{x}_h) + \frac{1}{2}u((1 - \phi)x_0 + \hat{x}_\ell) \leq \frac{1}{2}u((1 - \phi)x_0 + p - \frac{\hat{\pi}/2}{1 - \hat{\pi}/2}\Gamma_b) + \frac{1}{2}u(\hat{x}_h). \quad (5.9)$$

Since we have a mean-preserving spread

$$E[u((1 - \phi)x_0 + x_\sigma)] \geq E[u((1 - \phi)x_0 + \hat{x}_s)]. \quad (5.10)$$

Furthermore, we have that  $p > \hat{x}_\ell$  and that  $\hat{x}_h > (1 - \phi)x_0 + p$ . Hence, sellers will prefer trade as long as  $\hat{\pi}$  is sufficiently low.  $\square$

### 5.3 Ex-ante Welfare Comparison

Denote by  $\hat{V}$  the aggregate value of trading when the shock hits and a profit-oriented CCP allows for trade. This value is given by

$$\begin{aligned} \hat{V} = & \frac{1}{2}\hat{\pi} \left[ \frac{1}{2}u \left( (1 - \phi)x_0 + p - \frac{\hat{\pi}/2}{1 - \hat{\pi}/2}\Gamma_b \right) + \frac{1}{2}u(\hat{x}_h) \right] \\ & \frac{1}{2}\hat{\pi} \left[ \left( (1 - \phi)x_0 + x_2 + \hat{x}_h - p - \frac{\hat{\pi}/2}{1 - \hat{\pi}/2}\Gamma_s \right) + \frac{1}{2}x_2 \right] \\ & (1 - \hat{\pi}) \left[ \frac{1}{2}u \left( (1 - \phi)x_0 + \frac{\hat{\pi}/2}{1 - \hat{\pi}/2}\Gamma_s \right) + \frac{1}{2}u \left( (1 - \phi)x_0 + \frac{\hat{\pi}/2}{1 - \hat{\pi}/2}\Gamma_b \right) \right]. \end{aligned} \quad (5.11)$$

The value of no trade – and, hence, of a user-oriented CCP – when the aggregate shock occurs is given by

$$V_{aut} = \frac{\hat{\pi}}{4} [u(x_0 + \hat{x}_h) + u(x_0 + \hat{x}_\ell)] + \frac{\hat{\pi}}{2} [x_0 + x_2] + \frac{1 - \hat{\pi}}{2} u(x_0). \quad (5.12)$$

This implies that trading in the risky economy is beneficial *from an ex ante* point of view at  $t = 0$  whenever  $\hat{V} > V_{aut}$ . We give examples below where this inequality is fulfilled for a range of the parameter  $\hat{\pi}$ . This, however, does not imply that at  $t = 0$  prefer a profit-oriented CCP. The reason is that without the aggregate shock the collateral policy of a profit-oriented CCP is too costly and, hence, inefficient.

The ex-ante value of a user-oriented CCP,  $V_u$  is given by

$$V_u = (1 - \eta) \left[ \frac{1}{2} \pi u(x_0 + p - \frac{\phi}{1 - \phi} (x_h - p)) + \frac{1}{2} \pi (x_0 + x_2 + E(x_\sigma) - p - \frac{\phi}{1 - \phi} (p - x_\ell)) + (1 - \pi) u(x_0) \right] + \eta V_{aut}$$

while the value of a profit-maximizing CCP,  $V_p$ , is given by the expression

$$V_p = (1 - \eta) \left[ \frac{1}{2} \pi u((1 - \phi)x_0 + p) + \frac{1}{2} \pi ((1 - \phi)x_0 + x_2 + E(x_\sigma) - p) + (1 - \pi) u((1 - \phi)x_0) \right] + \eta \hat{V}.$$

For  $\eta$  high enough when  $\hat{V} > V_{aut}$ , agents will prefer to become member of a profit-oriented CCPs. We resort next to some numerical examples to show that profit-oriented CCPs can indeed welfare dominate user-oriented CCPs due to the hold-up problem outlined in this section. In these examples, we will express the gains from profit-oriented CCPs as functions of the two shock parameters  $(\eta, \hat{\pi})$  for different values of risk when there is default ( $\hat{x}_\sigma$ ) and different degrees of risk aversion. To do so, we choose a CRRA utility function which is parameterized by the coefficient of risk aversion  $\sigma \in (0, \infty)$ . All other parameters are kept fixed and are chosen as shown in the following table.

$\pi$	$\phi$	$x_0$	$x_2$	$x_h$	$x_\ell$	$p$
0.9	0.01	0.65	1	1.5	0.5	0.9

Table 1: Parameter Values

For the two examples we present one can verify that a user-oriented CCP sets optimally  $f = 0$  and does not allow for trade once the aggregate shock has been realized. To the contrary, in these

examples, it is optimal for a profit-oriented CCP to set  $f = x_0$  as collateral costs are sufficiently low. Hence, the welfare differences among CCPs when the aggregate shock hits are maximized and the value for  $\pi$  is relatively high.

### 5.3.1 Example 1:

The first example uses  $\sigma = 0.5$  and the mean-preserving spread is given by  $(\hat{x}_h, \hat{x}_l) = (2.5, -0.5)$  in the benchmark case. In Figure 1, we show below three graphs that exhibit (i) the net gains from choosing a profit-orientated CCP at  $t = 0$  as a function of  $\eta$  and  $\hat{\pi}$ ; (ii) the critical value  $\eta_{crit}$  for a given degree of heterogeneity  $\hat{\pi}$  such that for all  $\eta > \eta_{crit}$  it is optimal to choose profit-orientation; and (iii) the gains from profit-orientation for the extreme case that  $\eta = 1$ .

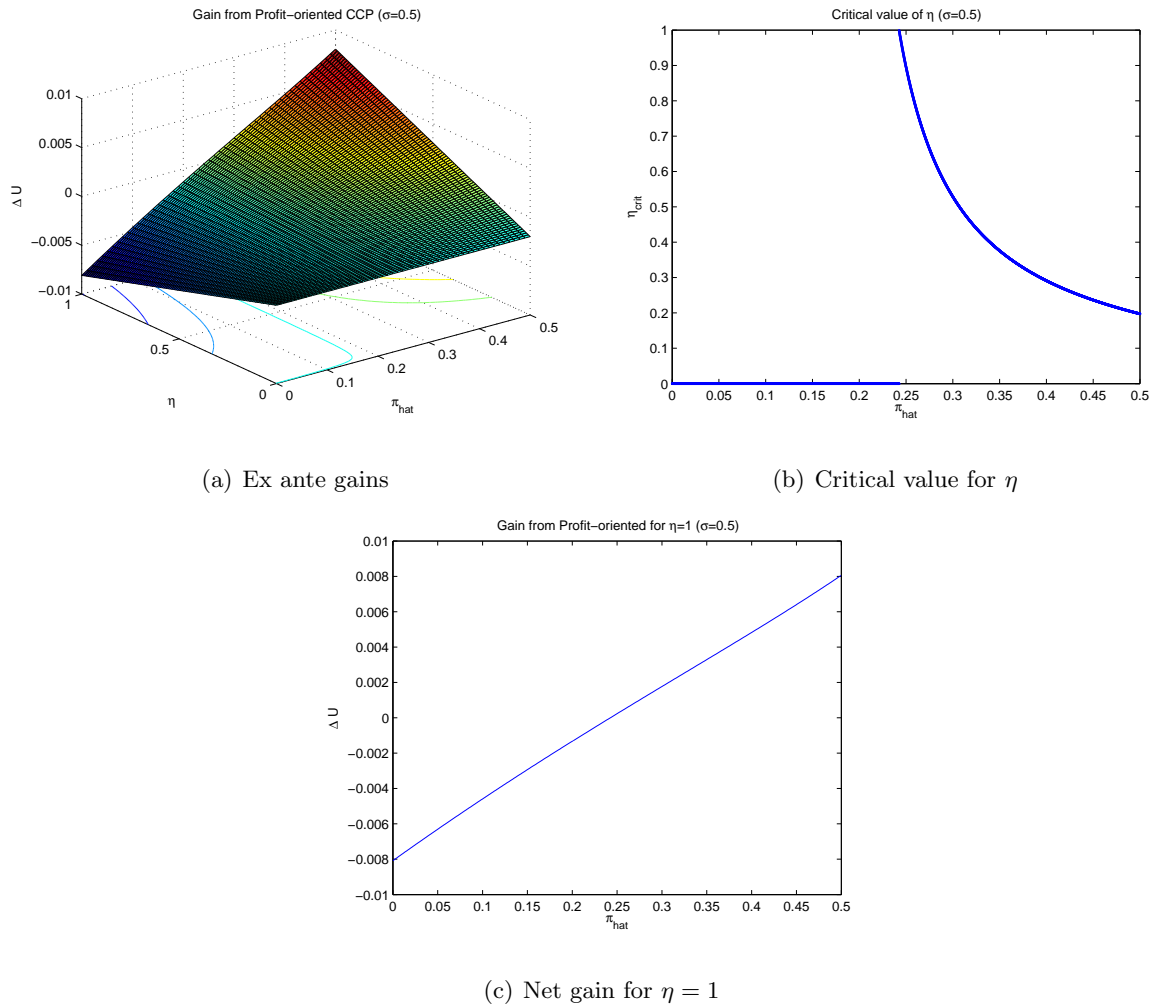


Figure 1: Benchmark case:  $\sigma = 0.5$

We do not show any graphs here for different values of  $\hat{x}_\sigma$ . However, as risk (in form of the mean-

preserving spread) decreases, the critical value  $\eta_{crit}$  increases and the middle graph shifts monotonically to the right, eventually rendering profit-orientation sub-optimal irrespective of parameters  $(\eta, \hat{\pi})$ . The next example, however, shows that such a comparative statics need not be monotone.

### 5.3.2 Example 2:

In this example, we increase the coefficient of risk aversion to  $\sigma = 2$  and analyze how the gains from profit-orientated change as the mean-preserving spread  $\hat{x}_\sigma$  increases from 2.8 to 3. Figure 2, shows that for the lower end of the spread we consider profit-orientation is never optimal.

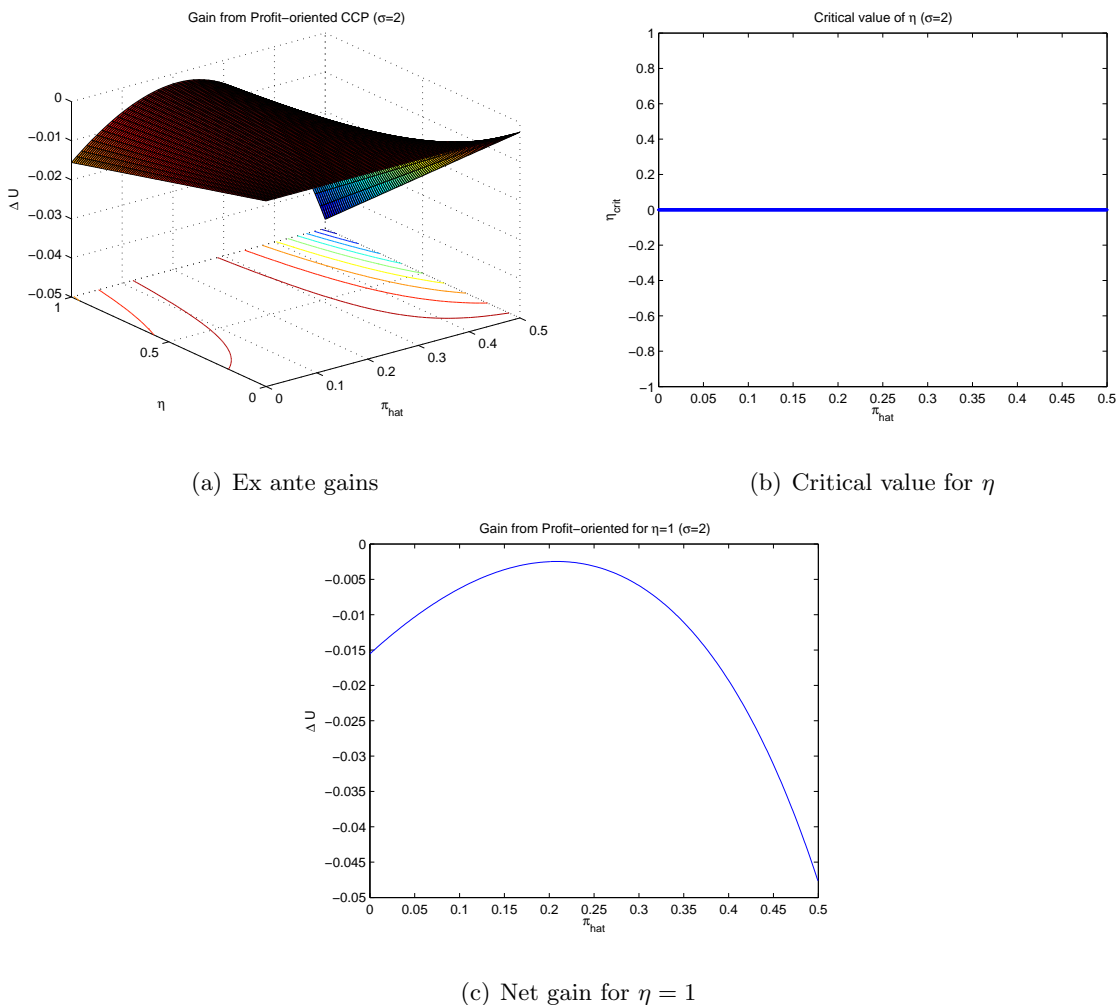


Figure 2: Mean-preserving Spread 2.8 with  $\sigma = 0.5$

Once the spread increases, profit-orientation is more attractive, the more likely the aggregate shock occurs. However, the critical value  $\eta_{crit}$  is *not* monotone in  $\hat{\pi}$ . When  $\hat{\pi}$  is close to 0.5, the costs from the buyers' default when  $\sigma = \ell$  are high for sellers relative to their surplus from trading. Hence, the

benefits from profit-orientation are declining. This is shown for the special case of  $\eta = 1$  in Figure 3(c).

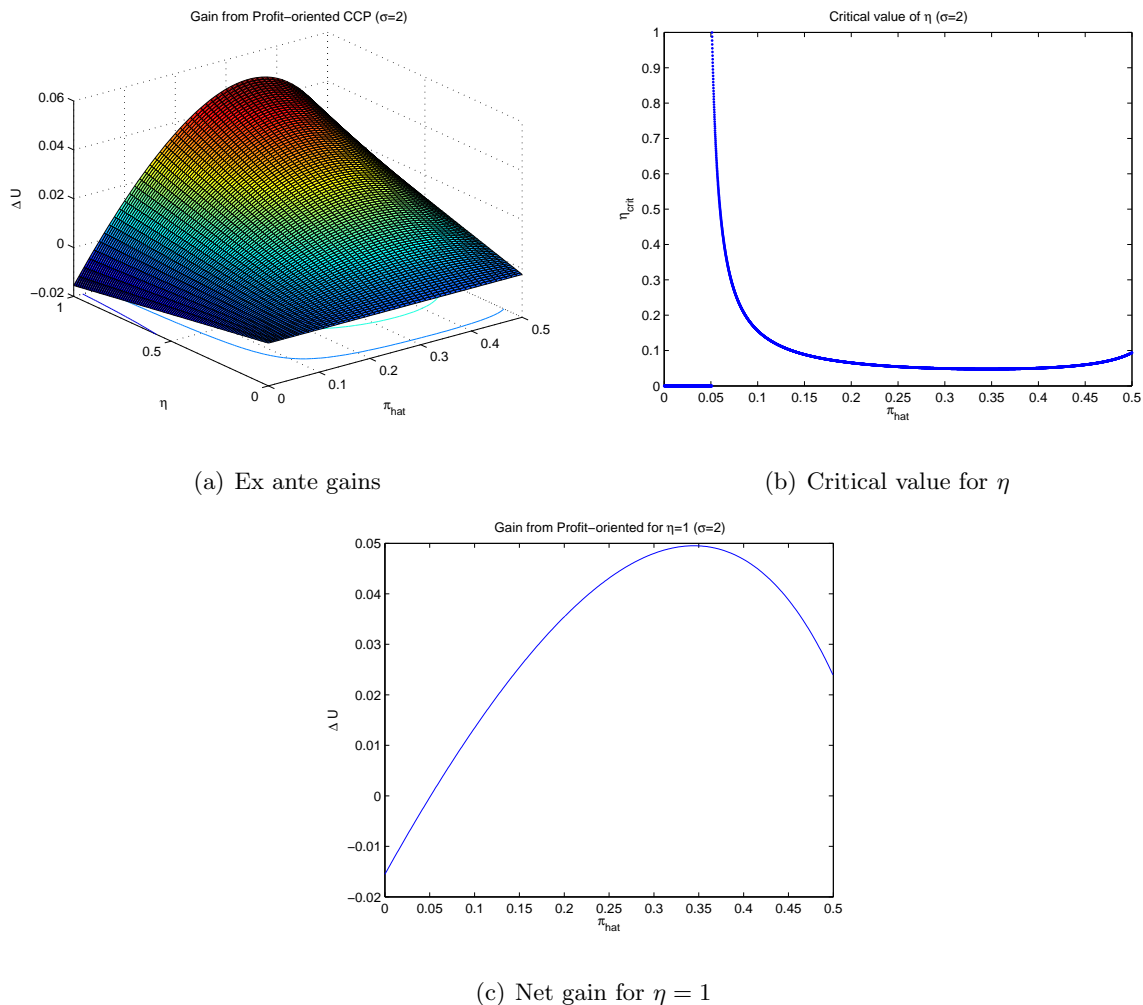


Figure 3: Mean-preserving Spread 2.9 with  $\sigma = 2$

This effect diminishes, if the spread increases further as shown in the last figure. As before, if the number of traders converges to zero, there are no ex-ante benefit of profit-orientation anymore. This is the case as the probability of being a non-trader when the aggregate shock materializes is too high. Bearing even small costs of default with high probability in this state renders the expected surplus from trading unimportant.

These examples demonstrate the trade-offs involved in deciding on the governance structures. Keeping preferences and technological parameters (such as collateral costs and the riskiness associated with the security) fixed, an increase in the likelihood of aggregate shocks increases the likelihood of encountering a profit-oriented rather than a user-oriented CCP. Similarly, for intermediate values

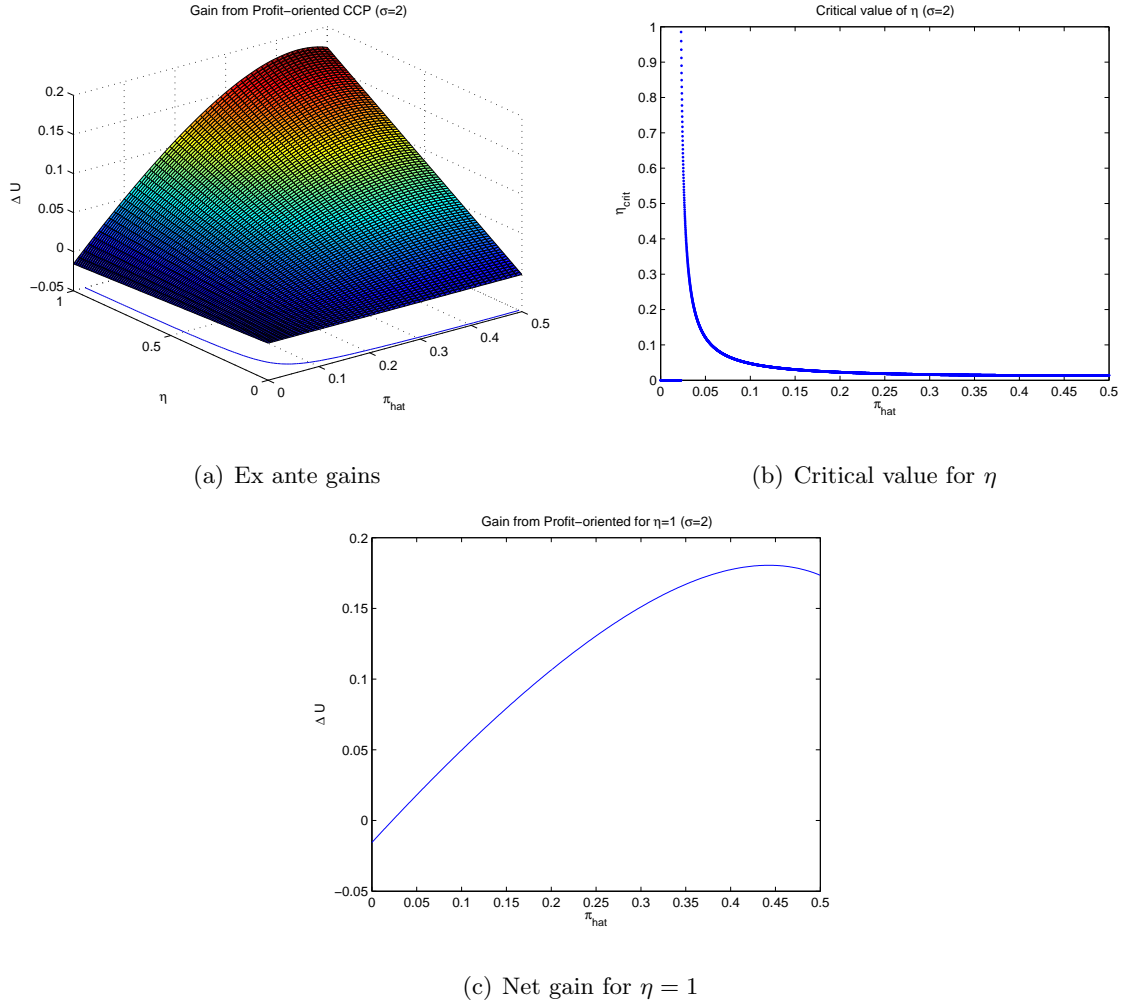


Figure 4: Mean-preserving Spread 3 with  $\sigma = 2$

of heterogeneity (i.e., intermediate values of  $\hat{\pi}$ ), we should encounter profit-oriented CCPs as the hold-up problem causes the largest welfare losses.

These are testable implications. In a next step, we intend to use information on the market environment, governance structures and risk management instruments of CCPs to test our results empirically.

## 6 Conclusion

In this paper we present a model of Central Counterparties and how they manage replacement cost risk depending on their corporate governance. We find that profit oriented CCPs are more likely to operate in markets where risky securities are traded. While user-oriented CCPs are more likely to



operate when relatively safe securities, such as cash derivatives, are traded.