Accounting for the Heterogeneity in Retirement Wealth*

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Abstract

This paper studies a quantitative dynamic general equilibrium life-cycle model where parents and their children are linked by bequests, both voluntary and accidental, and by the transmission of earnings ability. This model is able to match very well the empirical observation that households with similar lifetime incomes hold very different amounts of wealth at retirement. Income heterogeneity and borrowing constraints are essential in generating the variation in retirement wealth among low lifetime income households, while the existence of intergenerational links is crucial in explaining the heterogeneity in retirement wealth among high lifetime income households.

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1 Introduction

Many papers document that households with similar characteristics, such as lifetime income, age, and family structure, hold very different amounts of wealth at retirement (see among others Hurst, Luoh and Stafford (1998), Venti and Wise (2001), Hendricks (2004) and Grafova, McGonagle and Stafford (2006)). Various economists (see for example Bernheim, Skinner and Weinberg (2001) and Hendricks (2004)) argue that this feature of the data is inconsistent with most models of life-cycle consumption-saving behavior, and thus constitutes a challenge to such theories and their policy implications.

The literature so far has examined the implications of various models for the cross-sectional distribution of wealth among people of all ages (see Quadrini and Rios-Rull (1997) and Cagetti and De Nardi (2005)). Some recent papers (Engen, Gale and Uccello (1999, 2004), Scholz, Seshadri, and Khitatrakun (2004), and Hendricks (2004)) have examined the implications of different models on wealth dispersions at retirement age. The standard life-cycle framework used to study wealth dispersion typically assumes ex-ante identical households, ex-post income shocks, and incomplete markets. In such models, households are ex-post heterogenous in the realization of income shocks and wealth holdings. This version of the model implies a tight relationship between lifetime earnings and wealth. Income-rich people are wealth-rich at retirement age, but wealth differences among households with similar earnings usually are small.

Hendricks (2004) builds a dynastic world where a worker has a constant probability of moving between "age" states, and is altruistic towards his/her descendants. He finds that while the qualitative implications of the model are in line with the data, the quantitative implications are not. In the data there is a positive but low correlation between earnings and wealth, and a large heterogeneity in retirement wealth among households with similar lifetime earnings. The observed wealth difference at retirement between earnings-rich and earnings-poor is small compared to the one predicted by Hendricks' version of the life-cycle model.

Engen, Gale and Uccello (1999, 2004) study the adequacy of household retirement saving and find that households at the median of the empirical wealth-lifetime earnings distribution are saving as much as or more than what the underlying model suggests is optimal, and households at the high end of the wealth distribution are saving significantly more than the model indicates. Scholz, Seshadri and Khitatrakun (2004) compare, household by household, wealth predictions that arise from a life-cycle model to data in Health and Retirement Survey (HRS) and find that the model is capable of accounting for more than 80 percent of the cross-sectional variation in wealth.

This paper explores the implications of a richer model of saving behavior, proposed by De Nardi (2004). In this model, households face uninsurable labor income risk, uncertain lifetimes and a borrowing constraint. Households save to self-insure against labor earning shocks and life-span risk, for retirement, and possibly to leave bequests to their children. The key feature of this model, compared with Engen, Gale and Uccello (1999, 2004), and Scholz, Seshadri and Khitatrakun (2004), is that members of successive generations are linked by bequests and by the children's inheritance of part of their parent's productivity. The key differences of this and Hendricks' model are that (i) households do not know the exact time and amount of inheritance, and (ii) they are not allowed to borrow against future inheritance.

In this model households are also ex-ante identical. Retirement wealth inequality arises because households differ in the timing of earnings over the life cycle and in the amount and timing of inheritance received. I find that while differences in the timing of income shocks and borrowing constraints can generate large heterogeneity in wealth at retirement for households at lower lifetime income deciles, differences in the timing and amount of inheritance help to generate large heterogeneity in retirement wealth for households at higher lifetime income deciles.

The existence of a borrowing constraint prevents households from smoothing consumption intertemporally. Two households may have the same lifetime earnings, but one may have positive earning shocks when young and negative earning shocks when old while the other has negative earning shocks when young and positive earning shocks when old. At retirement, these households will hold amounts of wealth that differ substantially.

Inheritance adds another source of wealth heterogeneity among households with similar lifetime earnings. Some earnings-poor households hold a large amount of wealth at retirement because they have inherited a large amount of assets. Some earnings-rich households receive no inheritance and thus own less wealth.

I also compare the benchmark economy with one without intergenerational links. This comparison indicates that heterogeneity of inheritance does not play a big role for the lower and middle income deciles, but does play an important role at generating wealth heterogeneity for the higher income deciles. Modeling bequests as luxury goods is key to generate a skewed distribution of inheritances and a large wealth heterogeneity among households with similar high lifetime income.

This paper is also related to the literature that attempts to account for the skewness of wealth distribution. Laitner (2001) uses an overlapping generations model with both life-cycle saving and altruistic bequest to successfully match the high degree of wealth concentration. Castañeda et al. (2003) show that a model of earnings and wealth inequality, based on ex-ante identical households facing uninsured idiosyncratic shocks to their endowments of efficiency labor units, accounts for the U.S. earnings and wealth inequality well. De Nardi (2004) constructs a model in which parents and children are linked by accidental and voluntary bequests and by earnings ability and shows it can explain the emergence of large estates and the long upper tail of the wealth distribution. Cagetti and De Nardi (2003) use higher marginal returns to business investment and the bequest motive to reproduce the high concentration of wealth at the top of the distribution. This paper goes one step further and explains the wealth distribution conditional on age and lifetime income.

The paper is organized as follows. In Section 2, I present some empirical results from Venti and Wise (2001) and Hendricks (2004) documenting heterogeneity of wealth at retirement among households with similar lifetime earnings. In Section 3, I present the model and define the equilibrium. The calibration of the model is presented in Section 4. In Section 5, I present the quantitative results of the benchmark model. Section 6 investigates the quantitative importance of income heterogeneity and inheritance heterogeneity. Brief concluding remarks are provided in Section 7. Technical discussions about the computational algorithm are provided in the Appendix.

2 Empirical Findings

When we claim that households are "similar" with respect to their lifetime income, a fundamental question is whether we are using a good measure of lifetime income. Recent work by Venti and Wise (2001) and Hendricks (2004) is particularly careful about measuring this important variable.

Venti and Wise (2001) use data from the HRS for households whose head is between age 51 to 61 in 1992. They use wealth of the household and lifetime income measured by historical earnings reported to Social Security Administration. They find that at all levels of lifetime earnings there is a large dispersion in the accumulated retirement wealth. They argue, informally, that the dispersion of retirement wealth must be attributed to differences in the amount that households choose to save. They also find that investment choices matter little in determining the dispersion of retirement wealth.

Hendricks (2004) uses data from the Panel Study of Income Dynamics (PSID)[1] on lifetime income and wealth, where the latter corresponds to the average wealth reported between the age of 50 and 65, discounted to age 60.

Figure 2.1 shows the scattered plot of log retirement wealth and log lifetime earnings from Hendricks (2004) and displays the correlation between retirement wealth and lifetime income. At all levels of lifetime earnings there is large dispersion in the accumulated retirement wealth: a significant fraction of high income households save very little and a significant fraction of low income households save a lot.

Figure 2.2 displays the Gini coefficient of retirement wealth for each lifetime income decile from Venti and Wise (2001) and Hendricks (2004). The PSID narrow income measure consists of only wages and salaries received by the household head and the spouse. The broad income covers most forms of income other than interest and dividends. We observe that controlling for age and lifetime earnings, there is still large wealth inequality: the Gini

[1] SCF has a better coverage of high-earner and high-wealth households than PSID and HRS. However, as is shown in Cagetti (2003) and Juster, Smith and Stafford (1999), for lower quartiles, those datasets give similar information.

coefficients are all above 0.4. The degree of wealth inequality declines with lifetime income decile.

Figure 2.3 displays the median retirement wealth normalized by mean lifetime income at each income decile from Venti and Wise (2001) and from Hendricks (2004). This is meant to measure how large the wealth differences are between earnings-rich and earnings-poor households. We observe that the earnings-rich households save more relative to their lifetime earnings than the earnings-poor households. The ratio of median wealth to lifetime earnings more than doubles between the 2nd and the 9th earning deciles.

Table 2.1 shows some statistics summarizing the relationship between retirement wealth and lifetime earnings computed by Hendricks (2004) from PSID. From the baseline sample we notice that:

1. Retirement wealth is strongly correlated with lifetime earnings: The correlation coefficient between lifetime earnings and retirement wealth (C_WE) is 0.48, and the correlation coefficient between log lifetime earnings and log retirement wealth (C_LWE) is 0.51.

2. Controlling for age and lifetime earnings reduces wealth inequality. The average of the Gini coefficients in wealth across lifetime earning deciles is 0.60, compared with 0.70 to 0.75 in the full sample, depending on the year.

3. Earnings-rich households hold more wealth relative to lifetime earnings: the ratio of retirement wealth relative to lifetime earnings for the 9th versus the 2nd lifetime earnings decile (R_90/20) is $1.78 - 3.0$.

These papers' findings thus indicate that households with similar lifetime incomes hold diverse amounts of wealth at retirement age, even when samples are restricted to exclude sources of wealth heterogeneity that are not related to income. These features of the data constitute a challenge to our theories of saving behavior.

3 The Model

The economy is a discrete-time overlapping generation world with an infinitely lived government. There are idiosyncractic income shocks. There are no state contingent markets for the household specific shocks. The only financial instrument is a one-period bond. The members of successive generations are linked by bequests and the children's inheritance of part of their parent's productivity. At age 20, each agent enters the model and starts consuming, working, and paying labor and capital income taxes. At age 30, the agent procreates. After retirement the agent no longer works but receives interest from accumulated assets and social security benefits from the government. The government taxes labor earnings, capital income and estates, pays pensions to the retirees and provides government consumption.

3.1 Demographics

During each model period, which is 5 years long, a continuum of people is born. Since there are no inter-vivos transfers, all agents start their working life with no assets. I denote age $t = 1$ as 20 years old, age $t = 2$ as 25 years old, and so on. At the beginning of period 3, the agent's children are born, and four periods later (when the agent is 50 years old) the children are 20 and start working. The agents are retired at $t = 10$ (when they are 65 years old) and die for sure by the end of age $T = 12$ (before turning 80 years old). From $t = 10$ (when they are 65 years old), each person faces a positive probability of dying given by $(1 - p_t)$. The probability of dying is exogenous and independent of other household characteristics. The population grows at rate n. Since the demographic patterns are stable, agents at age t make up a constant fraction of the population at any point in time. Figure 3.1 illustrates the demographics in the model.

3.2 Technology

There is one type of goods produced according to the aggregate production function $F(K; L)$ where K is the aggregate capital stock and L is the aggregate labor input. I assume a standard Cobb-Douglas functional form and a single representative firm. The final goods can be either consumed or invested into physical capital. Let C denote the aggregate consumption, I the aggregate investment on physical capital goods, G the government spending, and δ is the depreciation rates on physical capital. The aggregate resource constraints are:

$$C + I + G = F(K; L) = AK^{\alpha}L^{1-\alpha} \tag{1}$$

$$K' = I + (1 - \delta) K. \tag{2}$$

Households rent capital and efficient labor units to the representative firm each period and receive rental income at the interest rate r and wage income at the wage rate w.

3.3 Timing and Information

At the beginning of each period, households observe their idiosyncratic earning shocks and possibly receive some inheritance from their parents. Then labor and capital are supplied to firms and production takes place. Next, the households receive factor payments and make their consumption and asset allocation decisions. Finally, uncertainty about early death is revealed.

The idiosyncratic labor productivity status and assets holding are private information and the survival status is public information. I assume that children can observe their parent's productivity when their parent is 50 and the children are 20.

3.4 Consumer's Maximization Problem

3.4.1 Preferences

Individuals derive utility from consumption and from bequest transferred to their children upon death. Preferences are assumed to be time separable, with a constant discount factor β. The momentary utility function from consumption is of the constant relative-risk aversion class given by

$$U(c) = \frac{c^{1-\eta} - 1}{1 - \eta}. \tag{3}$$

Following De Nardi (2004), the utility from bequest is denoted by

$$\phi(b) = \phi_1(1 + b/\phi_2)^{1-\eta}. \tag{4}$$

The term ϕ_1 reflects the parent's concern about leaving bequests to her children, while ϕ_2 measures the extent to which bequests are luxury goods.

Note that this form of 'impure' bequest motive implies that an individual cares about the bequests left to his/her children, but not about consumption of his/her children.

3.4.2 Labor Productivity

In this economy all agents of the same age face the same exogenous age-efficiency profile ϵ_t, which recovers the fact that productive ability changes over the life cycle. Workers also face stochastic shocks to their productivity level. The Markov process of worker's stochastic productivity, Q_y, is given by:

$$\ln y_t = \rho_y \ln y_{t-1} + \mu_t,\ \ \mu_t \sim N(0, \sigma_y^2).$$

This Markov process is the same for all households. This implies that there is no aggregate uncertainty over the aggregate labor endowment although there is uncertainty at the individual level. The total productivity of a worker of age t is given by the product of the worker's stochastic productivity in that period and the worker's deterministic efficiency index at the same age: $y_t\epsilon_t$.

The parent's productivity shock at age 50 is transmitted to children at age 20 according to the following transition function Q_{yh}:

$$\ln y_1 = \rho_{yh} \ln y_{h,7} + \nu_1,\ \ \nu_1 \sim N(0, \sigma_{yh}^2).$$

What the children inherit is only their first draw; from age 20 on, their productivity y_t evolves stochastically according to Q_y.

For simplicity, I assume that children cannot observe directly their parent's assets, but only their parent's productivity when they are 20 and their parent is 50. Children infer the size of the bequests they are likely to receive based on this information.

3.4.3 The Household's Recursive Problem

In the stationary equilibrium, the household's state variables are given by (t, a, y, yp). The first three variables denote the agent's age, financial assets carried from the previous period, and the agent's productivity, respectively. The last term yp denotes the value of the agent's parent's productivity at age 50 until the agent inherits and zero thereafter. When yp is positive, it is used to compute the probability distribution on bequests that the household expects from the parent. When the agents have already inherited, yp is set to be 0.

According to the demographic transitions, there are four cases.

(i) From $t = 1$ to $t = 3$ (from 20 to 30 years of age), the agent survives for sure until next period and does not expect to receive a bequest because his/her parent is younger than 65. Since the law of motion of yp is dictated by the death probability of the parent, for these sub periods $yp' = yp$.

$$V(t, a, y, yp) = \max_{c,a'} U(c) + \beta E(V(t+1, a', y', yp)) \tag{5}$$

subject to

$$c + a' = (1 - \tau_l)w\varepsilon y + (1 + r(1 - \tau_a))a \tag{6}$$

$$a' \geq 0, \; c \geq 0. \tag{7}$$

At any subperiod, the agent's resource are derived from asset holdings, a, labor endowment, εy. Asset holdings pay a risk-free rate r and labor receives a real wage w. The evolution of y is described by the transition function Q_y. Government taxes labor income at the rate τ_l and interest income at the rate τ_a.

(ii) From $t = 4$ to $t = 6$ (from 35 to 45 years of age), the worker survives for sure until the next period. However, the agent's parent is at least 65 years old and faces a positive probability of dying at any period; hence, a bequest might be received at the beginning of the next period. The conditional distribution of bequest a person of state x expects in case of parental death is denoted by $\mu_b(x; :)$. In equilibrium this distribution must be consistent with the parent's behavior. Since the evolution of the state variable yp is dictated by the death process of the parent, yp' jumps to zero with probability $1 - p_{t+6}$. Let $I_{yp>0}$ be the indicator function for $yp > 0$; it is one if $yp > 0$ and zero otherwise.

$$V(t, a, y, yp) = \max_{c,a^+} U(c) + \beta E(V(t + 1, a', y', yp')) \tag{8}$$

subject to (7) and

$$c + a^+ = (1 - \tau_l)w\varepsilon y + (1 + r(1 - \tau_a))a \tag{9}$$

$$a' = a^+ + b' I_{yp>0} I_{yp'=0}, \tag{10}$$

where a^+ denotes the financial assets at the end of the period before receiving bequest.

(iii) The sub periods $t = 7$ to $t = 9$ (from 50 to 60 years of age) is the periods before retirement, during which no more inheritances are expected because the agent's parent is already dead by that time. Therefore yp is not in the state space any more. The agent does not face any survival uncertainty.

$$V(t, a, y) = \max_{c,a'} U(c) + \beta E(V(t + 1, a', y')) \tag{11}$$

subject to (6) and (7).

(iv) From $t = 10$ to $t = 12$ (from 65 to 75 years of age), the agent does not work and does not inherit any more, but faces a positive probability of dying. In case of death, the agent derives utility from bequeathing his/her assets.

$$V(t, a) = \max_{c,a'} U(c) + \beta p_t(V(t + 1, a')) + (1 - p_t)\phi(b) \tag{12}$$

subject to (7) and

$$\begin{aligned} c + a' &= (1 + r(1 - \tau_a))a + p \\ b &= a' - \tau_b * \max(a' - ex_b, 0). \end{aligned}$$

Households receive pension P. For simplicity, I assume the pension level is independent of household's income history[2]. Government taxes bequests at the rate τ_b for the proportion above the exemption level ex_b.

3.4.4 Definition of Stationary Equilibrium

I focus on an equilibrium concept where factor prices are constant over time. In addition, the age-wealth distribution is stationary over time. Agents differ in term of their age, assets holding t, and idiosyncratic labor productivity y and also parent's labor productivity yp at age 50. Each agent's state is denoted by x. An equilibrium is described as follows.

Definition 1 *A stationary equilibrium is given by government tax rates and transfers* $(\tau_l, \tau_a, \tau_b, ex_b, P)$*; an interest rate* r *and a wage rate* w*; value functions* $V(x)$*, allocations* $c(x)$*,* $a'(x)$*; a family of probability distributions for bequests* $\mu_b(x; :)$ *for a person with state* x*; and a constant distribution of people over the state variables* x*:* $m^*(x)$*, such that the following conditions hold:*

(i) Given government tax rates and transfers, the interest rate, the wage, and the expected bequest distribution, the functions $V(x)$*,* $c(x)$ *and* $a'(x)$ *solve the above described maximization problem for a household with state variables* x*.*

(ii) m^* *is the invariant distribution of households over the state variables for this economy*[3]*.*

[2] A more realistic assumption is that the level of social security benefits is a concave function of the accumulated contribution over the working ages. However, under this assumption the accumulated contribution becomes a state variable, tremendously raising the computational time. Assigning everyone the same pension makes high income group save more, low income group save less than otherwise, and has only minor effect on the heterogeneity of retirement wealth for households with similar lifetime incomes.

[3] I normalize m^* so that $m^*(X) = 1$, which implies that $m^*(\chi)$ is the fraction of people alive that are in a state χ.

(iii) All markets clear.

$$C = \int cm^*(dx), \; K = \int am^*(dx), \; L = \int \epsilon ym^*(dx)$$

$$C + (1+n)K - (1-\delta)K + g = F(K;L)$$

(iv) In equilibrium the price of each factor is equal to its marginal product:

$$r = F_1(K,L) - \delta, \; w = F_2(K,L).$$

(v) $\mu_b(x;:)$ *is consistent with the bequests that are actually left by the parents.*

(vi) Government budget constraint is balanced at each period.

4 Calibration

The model has nineteen parameters. I pick fifteen of them from other empirical studies and choose the remaining four parameters so that the model matches the bequest-output ratio, the capital-output ratio, and the ratio of average bequest left by people in the lowest 95th bequest percentile, and government spending[4].

I set the rate of population growth, n, to the average value of population growth from 1950 to 1997 from Economic Report of the President (1998). The p_t's are the vectors of conditional survival probabilities for people older than 65 and is set to the mortality probabilities of people born in 1965 provided by Bell, Wade, and Goss (1992).

The deterministic age-profile of labor productivity ϵ_t is taken from Hansen (1993). Since I impose mandatory retirement at the age of 65, I take $\epsilon_t = 0$ for $t > 9$. The logarithm of the productivity process is assumed to be an AR(1) process with persistence ρ_y and variance σ_y^2. These two parameters ρ_y, and σ_y^2 are estimated from PSID data and aggregated over five years (Altonji and Villanueva (2002)). The logarithm of the productivity inheritance process is also assumed to be an AR(1) process with persistence ρ_{yh} and variance σ_{yh}^2. I take ρ_{yh} from Zimmerman (1992), and take σ_{yh}^2 to match a Gini coefficient of 0.44 for after-tax

[4] Since one period in this model corresponds to 5 years in real life, I adjust parameters accordingly.

earnings.

I take α, the share of income that goes to capital, to be 0.36 (Prescott (1986), Cooley and Prescott (1995)). I take depreciation to be 6% (Stokey and Rebelo (1995)). Given the calibration for the production function, the before-tax interest rate on capital net of depreciation r, is 6%.

The capital income tax τ_a is set to be 20% (Kotlikoff, Smetters and Walliser (1999)). The rate τ_b is the tax rate on estates that exceed the exemption level ex_b. I choose these parameters from De Nardi (2004) who matches the observed ratio of estate tax revenues to GDP, and the proportion of estates that pay estate taxes, 1.5%. The social security replacement rate P is chosen to be 40%, a number commonly used in the social security literature. G is total government expenditure and gross investment, excluding transfers and is chosen to be 18% of GDP (Council of Economic Advisors (1998) for 1996). The labor income tax τ_l is chosen to balance government budget.

I take risk aversion coefficient, η, to be 1.5, from Attanasio et al. (1999) and Gourinchas and Parker (2002), who estimate it from consumption data. This value is in the commonly used range (1-5) in the literature.

I choose the discount factor, β, to match the capital-output ratio of 3. I use ϕ_1 to match bequest output ratio of 2.65% in the US simulation (Gale and Scholz(1994)). ϕ_2 is chosen to match the ratio of average bequest left by single decedents at the lowest 95th percentile (Hurd and Smith (2001)).

5 Numerical Results

This section examines to what extent the quantitative life-cycle model, with income heterogeneity, inheritances heterogeneity and borrowing constraints, can account for the relationship between retirement wealth and lifetime earnings and the observed large wealth difference among households with similar lifetime earnings. To answer this question, I first solve for the equilibrium, and then simulate 600,000 households starting from age 20. Each household's actual inheritance is drawn from the bequest distributions calculated in the equilibrium. Details about computation are in the Appendix. I define retirement wealth to be the wealth at age 65, the year before retirement, and lifetime earnings to be earnings from age 20 to 60, discounted to age 20 using an after-tax interest rate of 4.8% as in the model.

5.1 Benchmark

5.1.1 Distribution of Lifetime Earnings

The distribution of lifetime earnings, in the data and in the model, normalized by average lifetime earnings is plotted in Figure 5.1. This data comes from table 2 in Venti and Wise (2001). Lifetime earnings measured by the present value of social security earnings are surprisingly evenly distributed. The extreme low lifetime income for the lower two deciles is caused by the fact that some persons in these deciles were employed in sectors not covered by the Social Security system and thus reported zero social security earnings. The model does a good job in matching lifetime earnings for each decile. Since this is not one of the features matched by construction, it can be seen as evidence of the ability of the model to replicate the realistic distribution of lifetime earnings.

5.1.2 Wealth Distribution

Table 5.1 reports values for the wealth distribution for the benchmark economy. I present shares for the quintiles, the 80-95%, the 95-99%, the top 1%, and the Gini coefficient for wealth. The U.S. data on the wealth distribution is from De Nardi (2004) who uses the 1989 Survey of Consumer Finances (SCF) and refers to households 25 years of age and older. Wealth includes owner-occupied housing, other real estate, cash, financial securities, unincorporated business equity, insurance and pension cash surrender value, and is net of mortgages and other debt.

In the data wealth is highly unevenly distributed with a Gini coefficient of 0.78. The top 1% of the households hold 29% of the total wealth and the 95-99% of the households hold 24% of the total wealth. The model generates a skewed wealth distribution that is comparable with the data except for the top 1% of the wealth. The fraction of wealth held by the richest 1% is 21% in the model, compared with 29% in the data. There are more persons with non-positive wealth in the model than in the data. This may be caused by the lack of inter-vivos transfer in the model.

5.1.3 Bequest Distribution

Table 5.2 reports values for the inheritance distribution implied by the benchmark economy and for the data from the SCF (Hendricks (2004)). All inheritances received by either spouse are deflated and discounted to the year where the head is 50 years old both in the model and in the data. In the data inheritances are highly unevenly distributed: 70% of the households receive very little or no inheritance during their life time. The top 2% of the households receive 69% of all the inheritances and the top 5% of the households receive 81% of all the inheritances. This table shows that the model generates a skewed inheritance distribution that is comparable with the data. 70% of the households aged 50 receive no inheritance during their life time. The top 2% of the households receive 70% of the total inheritance and the top 5% of the households receive 83% of the total inheritance.

Modeling bequests as luxury goods is essential to match the observed skewness in the inheritance distribution. The intuition is that, the marginal utility from bequeathing is positive at zero bequest so wealth-poor households may not leave any bequest at the last stage of their life cycle. Some large inheritances are transmitted across generations because of the voluntary bequests. The richest households have strong bequest motives to save and keep some assets to leave to their children even when very old. Their offspring are more likely to be earnings-rich and thus tend to leave more wealth to their offspring, thus generating skewed inheritance distribution.

Figure 5.2 compares the cumulative distribution of estates at any given time both in the model and in the actual data. The U.S. data on the estate distribution comes from Hurd and Smith (2001) who use the Asset and Health Dynamics Among the Oldest Old (AHEAD) data exit interview of 771 deceased between 1993-1995[5]. The distribution of

[5] I use distribution for single decedents instead of the one for all decedents (which turns out to be 1-2 times bigger) because typically a surviving spouse inherits a large share of the estate, which will be partly consumed before finally being left to the couple's children.

the bequest is very concentrated both in the data and in the model: 30% of the deceased AHEAD respondents had an estate of no value[6]. The mean estate was $104,500 but the median was much lower ($62,200). Some respondents leave relatively large estates: 30% are in excess of $120,000 and 5% are $300,000 or more. Only 3% of the estates were valued in excess of $600,000. The estate distribution generated by the model actually matches very well with the AHEAD data.

To compare the size of the inheritances with lifetime income, I present in Table 5.3, the ratio (in percentage) of lifetime inheritances to lifetime household earnings, both discounted to age 50. For the majority of households in the PSID, inheritances account for only a small fraction of lifetime resources. In the benchmark economy, for 95% percent of the households, inheritances account for less than 4.7% of lifetime resources, compared with 3% in the PSID. If we looked at the 98th percentile, the number goes up to 7.5%, while in the data it is 10.7%. The model does a very good job of matching this feature as well.

5.1.4 Wealth Inequality and Lifetime Earnings

Figure 5.3 shows the scattered plot of log retirement wealth and log lifetime earnings generated by the model, where I normalize values using 1992 dollars. The model implies a positive correlation of retirement wealth with lifetime income and a large dispersion in accumulated retirement wealth at all levels of lifetime earnings. In the benchmark model as in the data, a substantial fraction of high income households save very little and a significant fraction of low income households save a lot.

Figure 5.4 compares the Gini coefficients for wealth for each decile in the benchmark economy and in the actual data. We notice two important features. First, we observe that controlling for age and lifetime earnings, there is still large wealth inequality in the model: the Gini coefficients in all income deciles in the benchmark economy are all above 0.35. Second, the degree of wealth inequality declines as lifetime income increases, as is observed in the data. For example, in the model the Gini coefficient is 0.64 for the 2nd income decile and is 0.40 for the 5th decile. The model economy matches the wealth inequality for

[6] 30% households report leaving no bequest in AHEAD but 70% households report reveiving no inheritance in SCF and PSID. One reason is that estates are often divided among several children.

the lower and higher deciles quite well but underestimates wealth inequality for the middle deciles a bit.

Figure 5.5 shows the ratio of median retirement wealth to mean lifetime earnings for each lifetime income decile. In the model as in the data, earnings-rich households hold more wealth relative to lifetime earnings. The model does a very good job in matching median retirement wealth to lifetime earnings ratios observed in the data at the various deciles.

To better gauge the amount of wealth dispersion at retirement generated by the model, Figures 5.6 a-c compare the retirement wealth distributions for the 2nd, 5th and 9th lifetime earning deciles in the model with those in the data, where wealth is normalized by household earnings. The model successfully replicates the fact that households with similar lifetime incomes hold diverse amounts of wealth. A large fraction of households in the 2nd income decile hold almost no wealth by the time they have attained age 51-61. The model also replicates the fact that households in the lower wealth deciles hold very little wealth while households in the higher wealth deciles hold much more wealth[7]. The model generates skewed wealth distribution comparable to the data for the 2nd and 5th lifetime earning deciles. Among households in the 9th lifetime earning decile in the model, most people but the richest hold more wealth than in the data.

Table 5.4 compares some statistics summarizing the relationship between retirement wealth and lifetime earnings in PSID with those in the benchmark economy. We find that in the benchmark economy the correlation between retirement wealth and lifetime earnings (0.41) is positive and of the same magnitude as in the data (0.48). Controlling for age and lifetime earnings reduces wealth inequality: the average Gini coefficient of retirement wealth within lifetime earning deciles is 0.46, compared with 0.60 in the data. In the model, earnings-rich households hold more wealth relative to lifetime earning (7.4 times) than in the data (3 times). On the one hand, this is caused by the fact the model underpredicts the median retirement wealth for the first two decile and overpredicts the median retirement

[7]In particular, the ratio of median retirement wealth to average household earnings is close to the data. From Figure 5.5 we see that the ratio of median retirement wealth to mean lifetime earnings for the 2nd lifetime income decile is much lower that in the data. These two seemly contradicting findings could be explained by the fact that the mean lifetime earnings for the 2nd lifetime income decile in the model is higher than in the data.

wealth for the 9th decile due to the equalizing social security system. On the other hand, as we noticed in Figure 5.1, the model overpredicts lifetime earning for the first two deciles. This explains, in part, why R_90/20 is much higher in the model. If I adjust for the difference of lifetime earnings at the 2nd and the 9th deciles between the model and the data, then R_90/20 drops to about 4.9 in the model.

Compared with the joy-of-giving model in Hendricks (2004), the benchmark economy generates much larger heterogeneity in retirement wealth holding among households with similar lifetime earnings. The are two important reasons for the difference in findings. First, in Hendricks (2004), households know the exact time and amount of inheritance and are allowed to borrow against future inheritances when young, which relaxes the borrowing constraints for young agents. As a consequence, a large part of inherited wealth is consumed before retirement. Second, since bequests are not modeled as a luxury good, inheritance distribution is not as skewed as in the data and households at the bottom 70% receive 38% of the total inheritance, which is counterfactual. In this model, households cannot borrow against expected bequests, which generates more heterogeneity in retirement wealth among households with similar lifetime earnings.

5.2 Households Without Inheritance

While the benchmark model does a good job in generating heterogeneity in retirement wealth among household with similar lifetime earnings, let us now try to understand the key features of the model by comparing 65-year-old households that did and did not inherited for each income decile. This comparison can shed light on what role income shocks and borrowing constraints play in generating the heterogeneity of retirement wealth.

5.2.1 Wealth Inequality and Lifetime Earnings

Figures 5.7 shows the relationship between retirement wealth and lifetime earnings in the model among households who never inherited. Compared with all households in the model (Figure 5.3), those who never inherited, on average, has lower level of retirement wealth, and almost all the wealthiest households inherit large amounts of wealth from the previous generations.

Figure 5.8 displays Gini coefficients for wealth for each decile. The subsample of households that never inherited has similar wealth inequality for the lowest 7 deciles but has lower wealth inequality for the highest 3 deciles than for the whole sample. This indicates that bequests do not play a major role at generating wealth heterogeneity for the lower and middle income deciles but they are crucial in explaining wealth heterogeneity for the higher income deciles.

Figures 5.9 a-c show the retirement wealth distributions for the 2nd, 5th and 9th lifetime earning deciles. The distribution of retirement wealth among the subsample of households who never inherit is similar to that in the whole sample. In the model, those who never inherit hold less wealth that the whole sample, and the difference increases as wealth decile increases. Again this comparison indicates that inheritance heterogeneity plays a more important role for the higher income deciles.

Table 5.5 provide further evidence along these lines. We find that, in the subsample, the correlation between retirement wealth and lifetime earnings (0.86) is stronger than in the whole sample(0.40)[8]. The average Gini coefficient of retirement wealth within lifetime earning deciles is 0.42, compared with 0.46 among all households.

5.2.2 Intuition

A simple life-cycle model without earning uncertainty and without borrowing constraints predicts a perfect correlation between lifetime earnings and retirement wealth. Adding earning uncertainty and borrowing constraints breaks the prefect correlation since the timing of positive or negative shocks differ among household with identical lifetime earnings.

Consider two households with the same lifetime earning but one has positive earning shocks when young and negative earning shocks when old, the other has the reverse.

A household with positive earning shocks when young would save more in the earlier age to buffer against negative earning shocks later. When he/she suffers from negative earning

[8]The correlation of retirement wealth and lifetime earnings among households never inherited in the model is higher than this observed in the data (0.51). However, the low correlation of retirement wealth and lifetime earnings in the data among households in the PSID inherited less than $1000 should be interpreted with care since PSID households tend to underreport inheritances (Hendricks (2004)).

shocks, he/she uses assets to finance consumption, resulting in low level of retirement wealth.

A household with negative earning shocks when young anticipates high income in the future and would like to borrow to finance consumption but cannot. When he/she gets positive earning shocks, he/she saves most of them for retirement, and ends up holding a relatively large amount of wealth at retirement.

For example, a household may have the following realizations of sequences of income shocks

$$3 \quad 2 \quad 3 \quad 4 \quad 4 \quad 3 \quad 2 \quad 2 \quad 1 \cdot$$

The numbers indicate the corresponding grid points in the Markov process. He/she has the same discounted lifetime income of \$146,100 (in 1992 dollars) with another household with the following realizations of sequences of income shocks

$$2 \quad 1 \quad 1 \quad 1 \quad 2 \quad 3 \quad 3 \quad 6 \quad 6 \cdot$$

Their parents have income level of 2 at age of 50 so their expectations of bequests are the same. Neither of them received any bequests. But because of the timing of income, the second household has retirement assets of \$588,600, 62 times bigger than the first household has (\$9,500).

6 Decomposition

To understand the quantitative importance of intergenerational links, I run several experiments. First, to see how much wealth inequality can be generated by the life-cycle structure when only earnings uncertainty is activated, I turn off all intergenerational links and assume accidental bequests are equally redistributed among 50-year-old people. I recalibrate β and τ_l accordingly. Next I activate the intergenerational transfer of productivity. Finally I look at a model where parents care about bequests but there is no intergenerational transfer of productivity.

6.1 No Intergenerational Links

6.1.1 Wealth Distribution

Table 6.1 compares the benchmark economy with one without intergenerational links. This confirms findings in De Nardi (2004), that a model without intergenerational links cannot generate a skewed wealth distribution comparable with the data. The Gini coefficient of wealth is only 0.64, compared with 0.72 in the benchmark economy and 0.78 in the data. The fraction of wealth held by the richest 1% is only 7% in the model, compared with 21% in the benchmark model and 29% in the data.

6.1.2 Wealth Inequality and Lifetime Earnings

Table 6.2, row three, shows statistics summarizing the relationship between retirement wealth and lifetime earnings in the model without intergenerational links. The correlation between retirement wealth and lifetime earnings is stronger than in the benchmark economy, and once we control for age and lifetime earnings, there is much less wealth inequality than in the data. The average Gini coefficient of retirement wealth within lifetime earning deciles is 0.37, compared with 0.46 in the benchmark.

Figure 6.1 shows the relationship between retirement wealth and lifetime earnings in the model without intergenerational links. The correlation of retirement wealth and lifetime earnings in the model is much stronger than in the benchmark economy (Figure 5.3). Compared with the benchmark economy, households in this economy have, on average, much less retirement wealth. This comparison shows that a model without intergenerational links has trouble generating large wealth holdings.

Figure 6.2 compares the Gini coefficients of wealth and Figure 6.3 shows the ratio of median retirement wealth to mean lifetime earnings, for each lifetime income decile. The model without intergenerational links generates a realistic amount of wealth inequality and a realistic ratio of retirement wealth to earnings for the lower and median deciles but not for the higher deciles. This confirms that the heterogeneity of inheritance plays a role only for the higher income deciles.

Figures 6.4 a-c show the retirement wealth distributions for the 2nd, 5th and 9th lifetime earning deciles. The model generates skewed wealth distribution comparable to the data for the 2nd and 5th lifetime earning deciles. Households in the 9th lifetime earning decile in this model hold less wealth than in the benchmark economy. Households with higher lifetime income will still save more than households with lower lifetime income since pension is independent of lifetime income. But without an operative bequest motive, the saving for retirement motive alone is not strong enough for income-rich households (for example, households in the 9th income decile) to generate a high saving rate and a high retirement wealth comparable to the data.

6.2 No Bequest Motives

I now look at the model where parents do not care about leaving bequests to their children but there is intergenerational transfer of productivity. Accidental bequests are inherited by the children of the deceased. Table 6.2, row four, reports the relevant statistics. In this case, the correlation between retirement wealth and lifetime earnings is close to the case without intergenerational links. The average Gini coefficient of retirement wealth within lifetime earning deciles is 0.37, compared with 0.46 in the benchmark.

This comparison shows that the unequal distribution of involuntary bequests and intergenerational transfer of human capital are not sufficient to generate the observed heterogeneity of retirement wealth. One reason is that accidental bequests are not enough to generate skewed inheritance distribution. Table 6.3, row three, shows the Lorenz curve of inheritance distribution. Without voluntary bequest motive, more than 80% of the households do not inherit any bequests, compared with 70% in the benchmark model. The top 2% of the households receive 37% of the total inheritance, compared with 70% in the benchmark economy. The other reason is that, without an operative bequest motive, those who have inherited large estates from their parents will consume a large part of their inheritances by the age of 65.

Figure 6.5 displays the Gini coefficient of wealth for each income decile. Compared with the whole sample, the subsample of households that never inherited has only slightly smaller wealth inequality for all income deciles. Thus the additional heterogeneity of retirement wealth resulting from the endogenous heterogeneity of inheritance, is small compared with the benchmark model.

In summary, the comparison of models with and without bequest motives shows that the unequal distribution of involuntary bequest and intergenerational transfers of human capital are not quantitatively important.

6.3 No Productivity Transfers

I now look at the model where parents care about leaving bequests to their children but there is no intergenerational transfer of productivity (Table 6.2 row five). In this case the correlation between retirement wealth and lifetime earnings is 0.78, compared with 0.40 in the benchmark economy and 0.89 in the model without intergenerational links. The average Gini coefficient of retirement wealth within lifetime earning deciles is 0.45, compared with 0.46 in the benchmark, and 0.37 in the model without intergenerational links.

Table 6.3, row four, shows the Lorenz curve of inheritance distribution. The top 2% of the households receive 34% of the total inheritance, compared with 70% in the benchmark economy, and 37% in a model without bequest motive. However, with an operative bequest motive, those who inherited large estates from their parents will consume only a small part of their inheritances by the age of 65. Thus the heterogeneity of inheritance adds a lot to the heterogeneity of retirement wealth.

Figure 6.6 shows the Gini coefficient of wealth for each income decile. Compared with the whole sample, the subsample of households that never inherited has a much smaller wealth inequality for all income deciles. This differs from the benchmark with productivity transfer (Figure 5.8) where inheritance heterogeneity only adds wealth inequality for high income deciles. In a model without intergenerational transfer of productivity, low productivity households can also inherit large fortune from their parents, resulting in large heterogeneity of retirement wealth among low income groups.

In summary, the comparison of models with and without a productivity link shows that the intergenerational transfers of human capital is crucial in generating the heterogeneity of retirement wealth for high income deciles.

7 Conclusions

Empirical studies using micro data find that there is a large heterogeneity in retirement wealth among households with similar lifetime earnings, and raise doubts about the ability of a standard life-cycle model of saving behavior to reproduce the observed facts.

I use a quantitative, incomplete-markets, life-cycle, general equilibrium model in which parents and their children are linked by voluntary bequests and by the transmission of earnings ability. I show that the two intergenerational links I consider generate an amount of heterogeneity in retirement wealth comparable to that in the data. This suggests that a properly specified life-cycle model with intergenerational transfers of human capital and bequests captures the fundamental determinants of households saving and wealth accumulation.

I also investigate the quantitative relevance of income heterogeneity, borrowing constraints, and intergenerational links, respectively, in causing heterogeneity in retirement wealth. I find that, while income heterogeneity and borrowing constraints are essential to generate the heterogeneity in retirement wealth among low lifetime income households, the existence of intergenerational links is crucial to explain the heterogeneity in retirement wealth among high lifetime income households.

Finally, I discuss the likely implications of some of my assumptions. One important assumption is that there are no inter-vivos transfers, while in the data, parents give money to children when they need it the most (i.e. when they are young). Data from the HRS suggests that these transfers are fairly small (see Cardia and Ng (2000)). Given the small size of observed inter-vivos monetary transfers, I doubt that this inclusion would much affect the quantitative predictions of our model.

I make the restrictive assumption that children only observe parents' productivity at certain ages. This assumption is for computational reasons. For example, allowing children to observe parents productivity at two periods adds one more state variable and also increases substantially the time needed to iterate over the bequest distributions. Moreover, income in the calibration is very persistent, so an observation of one year of income is likely to be not much less informative than two. In a model in which the parent's assets and income are observable by the children, the saving behavior of households aged before 40 is likely to be similar as in this model, due to the uncertainty of inheritance time. For households aged 45 whose parents die for sure next period, those with poor parents will save more and those with rich parents will save less in the current period, making the effect of inheritance heterogeneity on retirement wealth variation smaller than in this model. Since inheritance heterogeneity only affects retirement wealth variation for the high income deciles, the Gini coefficient of the highest three income deciles may be lower than in this model.

This paper assumes ex-ante identical households and lets ex-post income shocks account for the observed heterogeneity in lifetime earnings. In the data, we also observe a very large inequality in wealth holdings by race (see for example Smith (1995) and Altonji and Doraszelski (2005)) and by education (Hubbard, Skinner and Zeldes (1995), Cagetti(2003)). It would be interesting to extend this model to allow for ex-ante heterogeneity in the earnings process, to study wealth inequality among different social groups.

This paper abstracts from entrepreneurship. Entrepreneurship is an important source of wealth inequality (Quadrini (2000); Cagetti and De Nardi (2003)). Since, with incomplete markets, self-employment gives households an additional incentive to save, entrepreneurs and workers have different saving behaviors even with similar lifetime income. Adding entrepreneurship in an otherwise standard model could help to generate more heterogeneity of retirement wealth. This is left for future research.

This paper also abstracts from housing. Housing is the single largest investment made by consumers over their life time. The median household owns a house valued about twice its annual income. As it is shown in Yang (2005), abstracting from housing may bias the study of life-cycle consumption and assets accumulation behavior. It will be interesting to extend this model to look at the effect of income heterogeneity and bequest heterogeneity on wealth heterogeneity in an environment with housing.

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8 Appendix: Computation of the Model

I discretize both the productivity and the productivity inheritance processes to six-state Markov chains according to Tauchen and Hussey (1991). Since I want the possible realizations for the initial inherited productivity level to be the same as the possible realizations for productivity during the lifetime, I choose the quadrature points jointly for the two processes. The resulting grid points for the productivity process y are [0.2114 0.4135 0.7495 1.3342 2.4187 4.7309]. The transition matrix Q_y is given by

$$\begin{bmatrix} 0.5529 & 0.3619 & 0.0789 & 0.0062 & 0.0002 & 0.0000 \\ 0.1877 & 0.4398 & 0.2970 & 0.0702 & 0.0053 & 0.0001 \\ 0.0315 & 0.2286 & 0.4208 & 0.2627 & 0.0540 & 0.0025 \\ 0.0025 & 0.0540 & 0.2627 & 0.4208 & 0.2286 & 0.0315 \\ 0.0001 & 0.0053 & 0.0702 & 0.2970 & 0.4398 & 0.1877 \\ 0.0000 & 0.0002 & 0.0062 & 0.0789 & 0.3619 & 0.5529 \end{bmatrix}$$

The transition matrix Q_{yh} is given by

$$\begin{bmatrix} 0.3341 & 0.5791 & 0.0853 & 0.0015 & 0.0000 & 0.0000 \\ 0.0368 & 0.4857 & 0.4336 & 0.0435 & 0.0004 & 0.0000 \\ 0.0016 & 0.1315 & 0.5798 & 0.2738 & 0.0132 & 0.0000 \\ 0.0000 & 0.0132 & 0.2738 & 0.5798 & 0.1315 & 0.0016 \\ 0.0000 & 0.0004 & 0.0435 & 0.4336 & 0.4857 & 0.0368 \\ 0.0000 & 0.0000 & 0.0015 & 0.0853 & 0.5791 & 0.3341 \end{bmatrix}$$

The transition matrices Q_y and Q_{yh} also induce an initial distribution of earnings.

The state space for asset holdings is discretized. Using this grid I can store the value functions and the distribution of households as finite-dimensional arrays.

I solve the approximated optimal consumption and saving plans recursively. Households surviving to the last period T has an easy problem to solve. Based on the period T policy functions, I solve the consumption and saving decisions that maximize the period $T-1$ value function. The same procedure is carried back until decision rules in the first period are computed for a large number of states.

I solve for the steady state equilibrium as follows:

1. Given an initial guess of τ_l.

2. Given an initial guess of interest rate r, use the equilibrium conditions in the factor markets to obtain the wage rate w.

3. Set the interval for assets.

4. Guess an initial bequest distribution.

5. Solve the value function for the last period of life for each of the points of the grid.

6. By backward induction, repeat the steps 5 until the first period in life.

7. Compute the associated stationary distribution of households.

8. Given the stationary distribution and policy functions, compute the bequest distribution. If the bequest distributions converges, go to step 9; otherwise go to step 4.

9. Check if the distributions of assets do not have a large mass at the maximum levels. If so, increase the maximum level and go back to step 3. If not, continue to step 10.

10. Check if all markets clear. If not, go to step 2 and update interest rate r.

11, If the government budget is balanced, an equilibrium is found. If not, go to step 1 and update τ_l.

After I solve the equilibrium, I then simulate 600,000 households starting from age 20 drawn from the initial stochastic earnings distribution. All households start life without any assets. In the case that the parents die, each household's actual inheritance is drawn from the bequest distributions calculated in the equilibrium.

	C_WE	C_LWE	Mean Gini	R_90/20	N
Baseline sample	0.48	0.51	0.60	1.78-3.0	1466

Note: from Hendricks (2004)
C_WE: correlation coefficient between lifetime earnings and retirement wealth.
C_LWE: correlation coefficient between log lifetime earnings and log retirement wealth.
Mean Gini: average Gini coefficient of retirement wealth within lifetime earnings deciles.
R_90/20: ratio of retirement wealth relative to lifetime earnings for the 9th v.s. the 2nd lifetime earnings decile.

Table 2.1: PSID summary statistics

Parameters		Calibrations
Demographics		
n	population growth	1.2%
p_t	survival probability	see text
Technology		
α	capital share in National Income	0.36
δ	depreciation rate of capital	0.06
Endowment		
ϵ_t	age-efficiency profile	see text
ρ_y	AR(1) coefficient of income process	0.85
σ_y^2	innovation of income process	0.30
ρ_{yh}	AR(1) coefficient of income inheritance process	0.67
σ_{yh}^2	innovation of income inheritance process	0.37
Government policy		
τ_l	tax on labor income	29%
τ_a	tax on capital income	20%
τ_b	tax on bequest	10%
ex_b	exemption level on bequest tax	40
P	social security income	0.40
G	government spending	0.18
Preference		
η	risk aversion coefficient	1.5
β	discount factor	0.96
ϕ_1	weight of bequest in utility function	-13.7
ϕ_2	shifter of bequest in utility function	11.6

Table 4.1: Parameters used in the benchmark model

	Top 1%	95-99%	80-95%	60-80%	40-60%	20-40%	Wealth≤ 0(%)	Gini
SCF	29	24	27	13	5	2	5.8-15	0.78
Benchmark	21	18	34	18	6	2	10	0.72

Note: Data from De Nardi (2004)

Table 5.1: Percentage wealth held in the wealth distribution

Percentage class	70	80	90	95	98	100
SCF	0.0	1.8	9.4	18.9	30.8	100
Benchmark	0.0	0.01	6.13	16.89	29.51	100

Note: Data from Hendricks (2004)

Table 5.2: Lorenz curve of inheritance distribution

Percentage class	70	80	90	95	98	100
PSID	0.0	0.2	1.3	3.2	10.7	105
Benchmark	0.0	0.45	2.86	4.73	7.59	228.59

Note: Data from Hendricks (2004)

Table 5.3: Size distribution of inheritances relative to own lifetime earnings (%)

	C_WE	C_LWE	Mean Gini	Gini	R_90/20
PSID	0.48	0.51	0.60	0.70	1.78-3.0
Benchmark	0.41	0.79	0.46	0.64	7.45
Hendricks (2004)					
Deterministic aging	0.93	0.79	0.46		587.93
No bequest	0.91	0.66	0.47		18.02
Accidential bequest	0.91	0.68	0.45		15.18
Joy-of-giving	0.91	0.82	0.39		6.72
Strong altruism	0.90	0.68	0.49		19.25

Table 5.4: Relationship between retirement wealth and lifetime earnings

	C_WE	C_LWE	Mean Gini	Gini	R_90/20
PSID	0.48	0.51	0.60	0.70	1.78-3.0
Benchmark	0.41	0.79	0.46	0.64	7.45
Benchmark (never inherited)	0.86		0.42		

Table 5.5: Relationship between retirement wealth and lifetime earnings

	Top 1%	95-99%	80-95%	60-80%	40-60%	20-40%	Wealth≤ 0(%)	Gini
SCF	29	24	27	13	5	2	5.8-15	0.78
Benchmark	21	19	34	18	6	2	10	0.72
No links	7	22	41	21	7	2	17	0.64

Table 6.1: Percentage wealth held in the wealth distribution

	C_WE	C_LWE	Mean Gini	Gini	R_90/20
PSID	0.48	0.51	0.60	0.70	1.78-3.0
Benchmark	0.41	0.79	0.46	0.64	7.45
No links	0.90	0.82	0.37	0.53	6.44
No bequest motive	0.89	0.81	0.37	0.53	4.45
No productivity transfer	0.77	0.71	0.45	0.55	4.15

Table 6.2: Relationship between retirement wealth and lifetime earnings

Percentage class	70	80	90	95	98	100
SCF	0.0	1.8	9.4	18.9	30.8	100
Benchmark	0.00	0.01	6.13	16.89	29.51	100
No bequest motive	0.00	0.00	11.14	35.02	62.70	100
No productivity transfer	0.00	0.61	16.99	40.06	65.63	100

Table 6.3: Lorenz curve of inheritance distribution

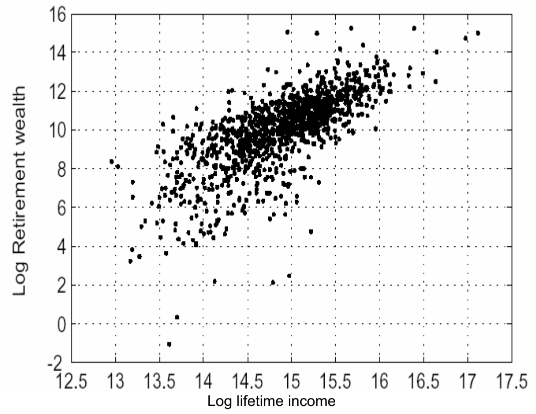

Figure 2.1: Retirement wealth and lifetime earning in PSID

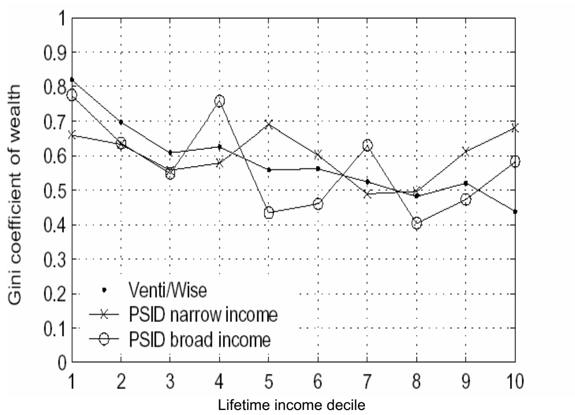

Figure 2.2: Gini coefficient of retirement wealth for each income decile

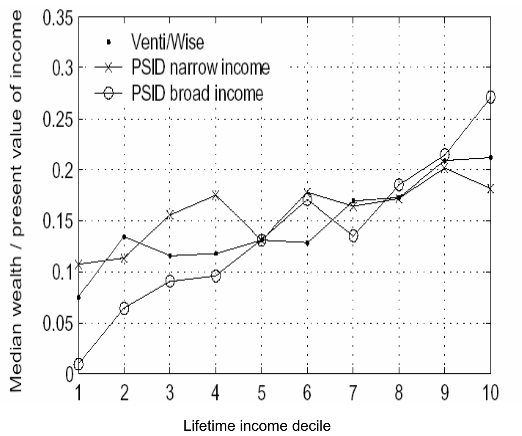

Figure 2.3: Median of retirement wealth for each income decile

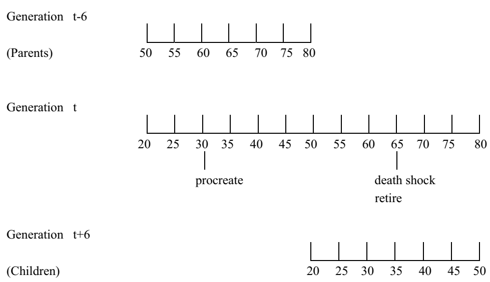

Figure 3.1: Demographics

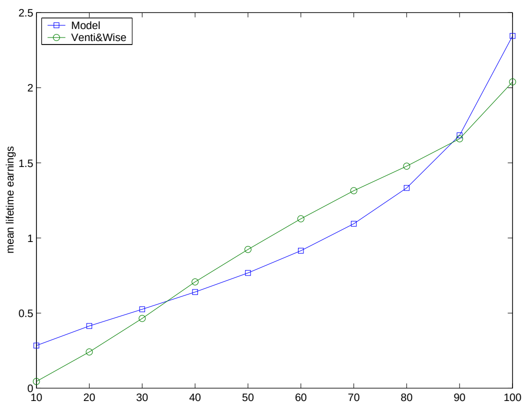

Figure 5.1: Mean of lifetime earnings for each income decile

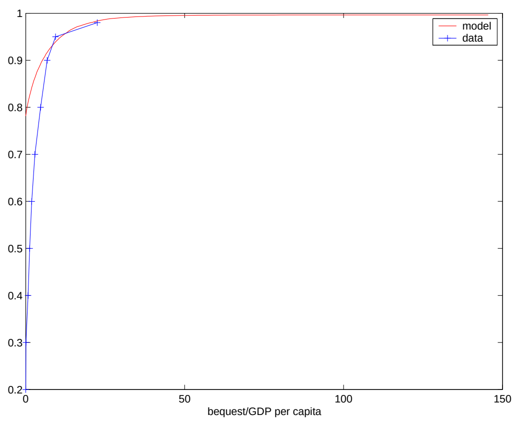

Figure 5.2: Cumulative distribution of bequest

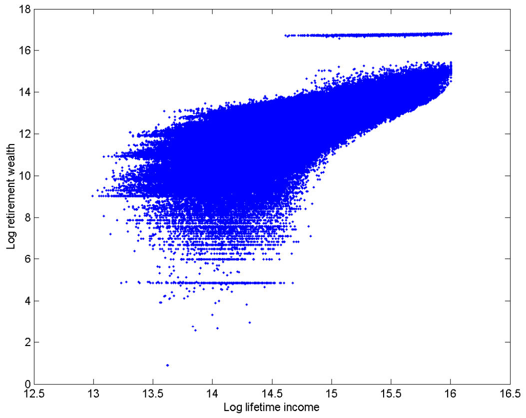

Figure 5.3: Retirement wealth and lifetime earning (benchmark model)

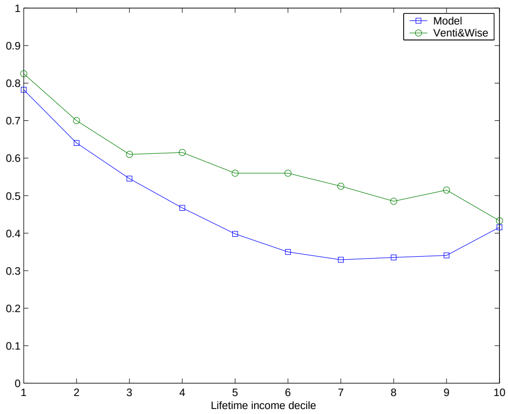

Figure 5.4: Gini coefficient of retirement wealth for each income decile (benchmark model)

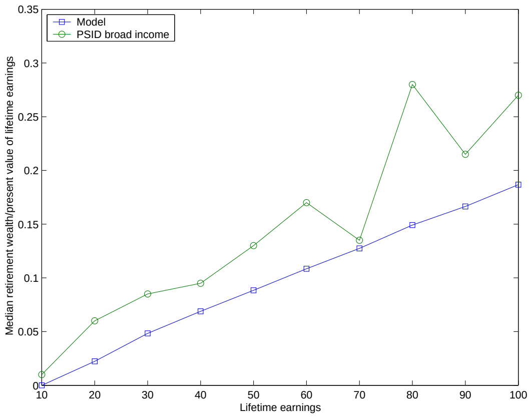

Figure 5.5: Median of retirement wealth for each income decile (benchmark model)

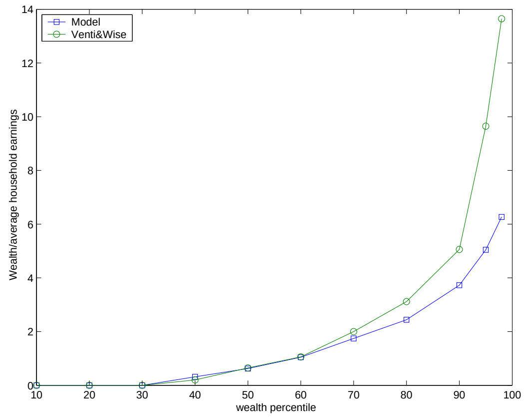

Figure 5.6 a: Distribution of wealth, 2nd income decile (benchmark model)

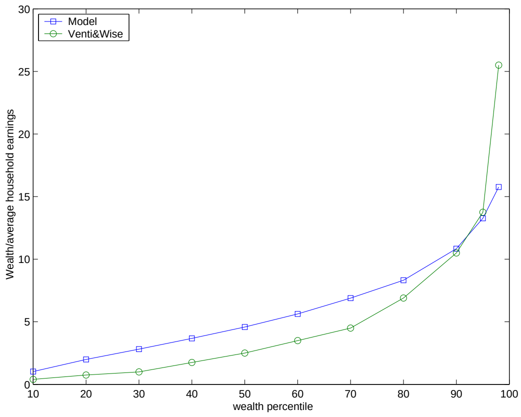

Figure 5.6 b: Distribution of wealth, 5th income decile (benchmark model)

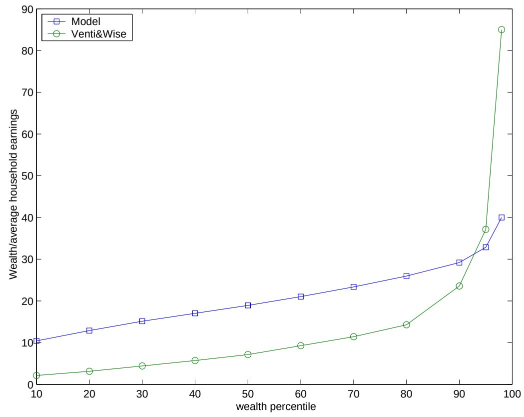

Figure 5.6 c: Distribution of wealth, 9th income decile (benchmark model)

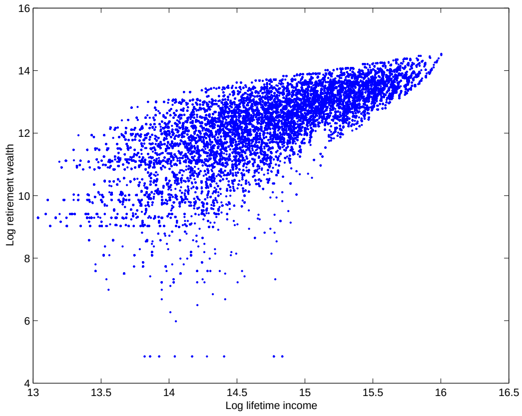

Figure 5.7: Retirement wealth and lifetime earning (benchmark model, households never inherited)

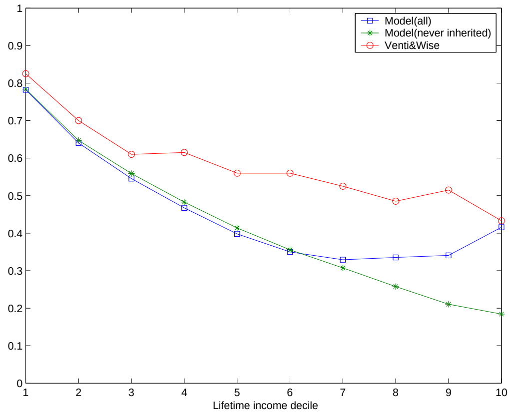

Figure 5.8: Gini coefficient of retirement wealth for each income decile (benchmark model, households never inherited)

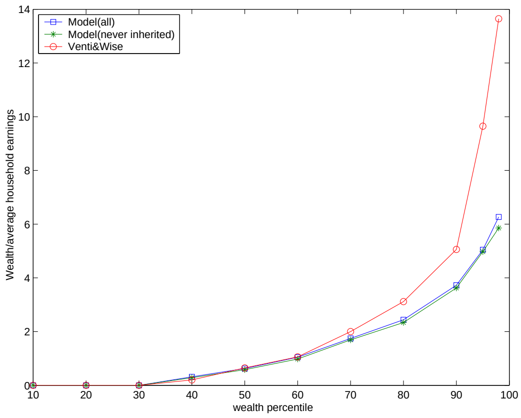

Figure 5.9 a: Distribution of wealth, 2nd income decile (benchmark model, households never inherited)

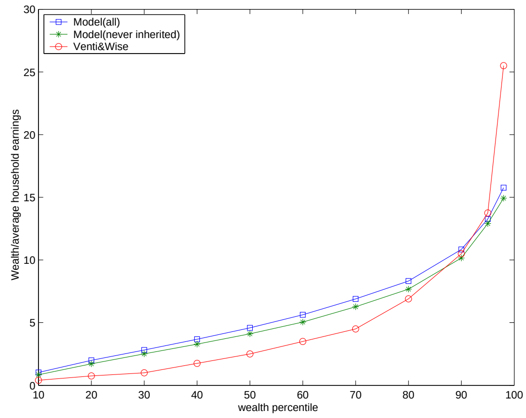

Figure 5.9 b: Distribution of wealth, 5th income decile (benchmark model, households never inherited)

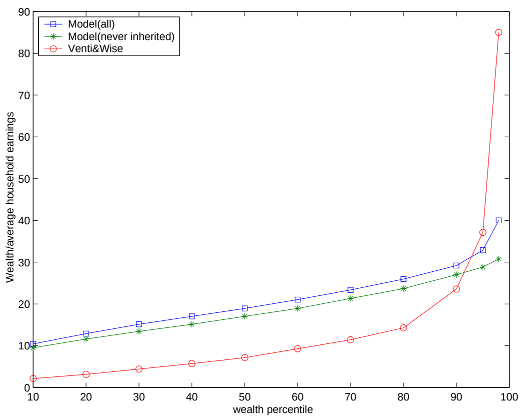

Figure 5.9 c: Distribution of wealth, 5th income decile (benchmark model, households never inherited)

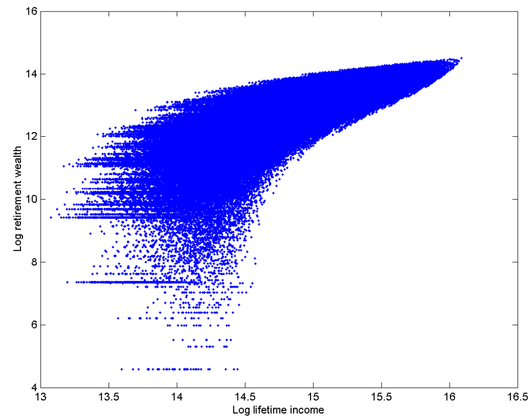

Figure 6.1 : Retirement wealth and lifetime earning in the model (no links)

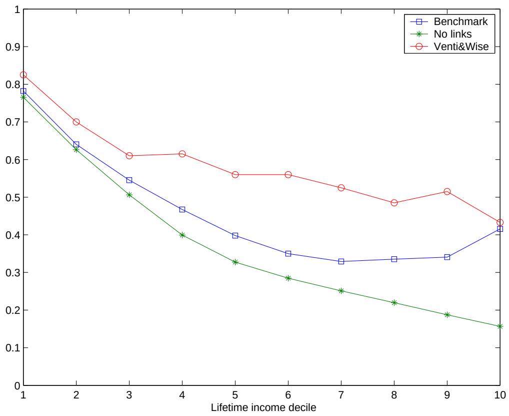

Figure 6.2: Gini coefficient of retirement wealth for each income decile (no links)

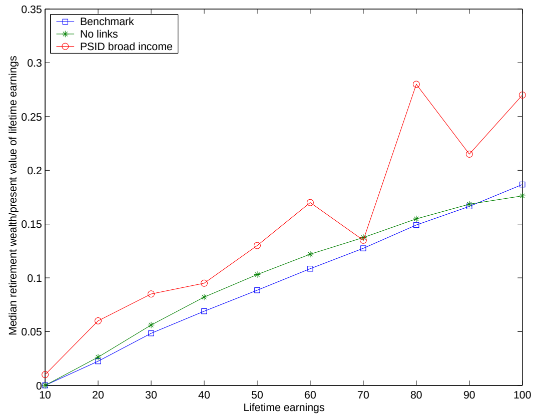

Figure 6.3: retirement wealth and lifetime earnings (no links)

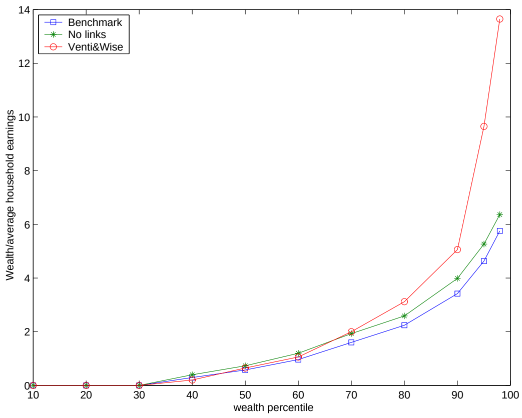

Figure 6.4 a: Distribution of wealth, 2nd income decide (no links)

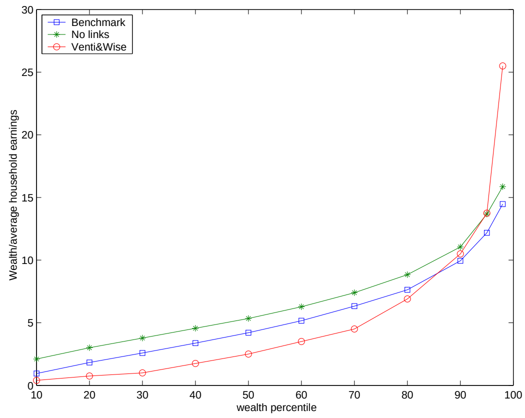

Figure 6.4 b: Distribution of wealth, 5th income decide (no links)

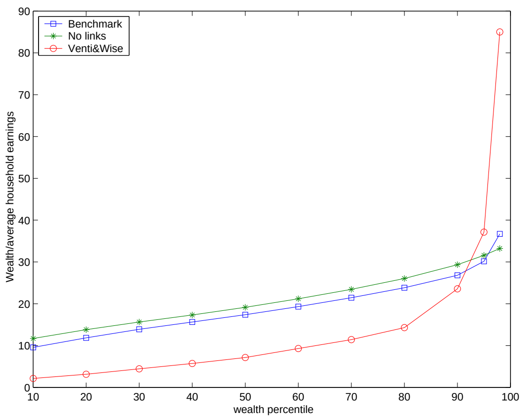

Figure 6.4 c: Distribution of wealth, 9th income decide (no links)

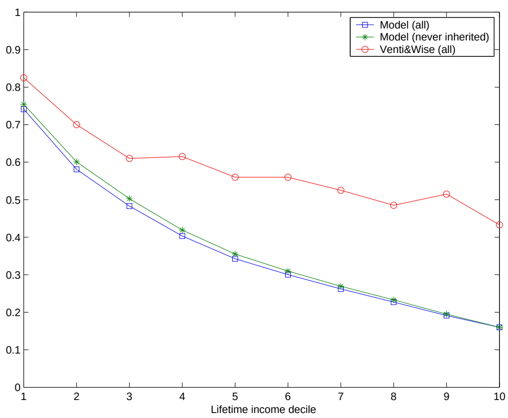

Figure 6.5: Gini coefficient for each income decile (no bequest motive)

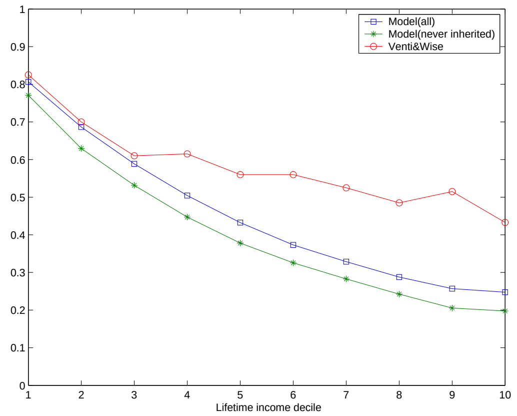

Figure 6.6: Gini coefficient for each income decile (no productivity transfer)