# Debt and Maturity without Commitment* 

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#### Abstract

I analyze how lack of commitment affects the maturity structure of sovereign debt. Ex post, the government trades off the gains from default induced redistribution against the cost of defaulting. Ex ante, the government issues debt of various maturities to raise an exogenous revenue requirement. The ex-post incentive compatibility constraints introduce a role for gross financial positions, rendering financial structure non-neutral although markets are complete and taxes non-distorting. The optimal maturity structure minimizes the expected costs due to opportunistic behavior ex post. It matches the maturity of government assets (tax collections) and liabilities (debt redemption) and avoids dilution as well as debt rollovers.


Keywords: Debt; maturity structure; no commitment; default.
JEL Classification Code: E62, F34, H63.

## 1 Introduction

Sovereign borrowers with a history of default exert considerable effort to structure their debt maturities optimally. This observation is difficult to reconcile with predictions of a frictionless baseline model where financial structure is irrelevant for the same reasons that render a firm's optimal capital structure or a government's optimal timing of tax collections indeterminate (Modigliani and Miller, 1958; Barro, 1974). The government budget constraint and rational expectations of bond holders associate a sequence of primary surpluses with an equilibrium level of net government debt; gross financial positions, the maturity structure, and the amount of debt rolled over from one period to the next typically remain indeterminate.

In this paper, I ask whether lack of commitment on the part of sovereign borrowers breaks this neutrality result, giving rise to a role for the maturity structure of sovereign debt. Focusing on lack of commitment appears natural. As illustrated by the large literature concerned with sovereign lending subject to limited contract enforceability, lack of commitment is widely acknowledged to be pervasive and relevant. Nevertheless, its implications for the choice of maturity structure have received little attention.

[^0]I consider a government issuing short- and long-term debt in order to finance an exogenous, initial deficit. Successive selves of the government choose whether, and to what extent, to honor maturing debt; they also choose taxes and new debt issuance (dilution) to finance the debt repayment. This process repeats itself until eventually, all debt has been repaid or defaulted upon. Bondholders, taxpayers and the government form rational expectations. The prices of debt maturities therefore reflect the expected repayment rates, and under commitment the optimal financial policy would be indeterminate. Since the government cannot commit, however, and since it has incentives to default ex post, incentive compatibility constraints affect the optimal choice of maturity structure ex ante.

The incentive to default derives from the government's desire to redistribute - a default transfers wealth from bondholders to taxpayers. Alternatively, the incentive might derive from the government's desire to transfer funds from the private to the public sector, in order to avoid tax distortions. While the model allows for both interpretations, focusing on the former is attractive for two reasons. First, because conflict among interest groups indeed appears to affect governments' default decisions. ${ }^{1}$ Second, because abstracting from tax distortions allows to disregard a second source of time inconsistency that is not central for the analysis. ${ }^{2}$

The government's incentive to default interacts with an opposing incentive to avoid the costs of a default. I model these costs as forgone benefits (for citizens and thus, the government) of having good "institutions" in place. This captures the notion that trust in the government's support of property rights, contract enforcement etc. is conducive to trade and high productivity, while at the same time, this trust is undermined after a default. Trust may be lost completely or only partially, and it may take time to be regained. Accordingly, the costs of a default may have a fixed or variable character, and they may be temporary, persistent, or permanent.

In the main model considered, default triggers a discrete loss of institutional capital. When pondering wether to default, a government therefore trades off the benefit of default induced redistribution with the foregone benefits from high-quality institutions. Anticipating such ex-post considerations of its successors, the government at the debt issuance stage chooses a maturity structure that minimizes the expected costs arising due to opportunistic behavior ex post. Expost incentive compatibility constraints therefore give rise to an optimal maturity structure of debt, although - or because - no default induced redistribution occurs in equilibrium. This optimal structure matches the maturity of government assets (tax collections) and liabilities (debt redemption), and it avoids dilution as well as debt rollovers. Debt, maturity, and tax collections are determined simultaneously, unlike in the commitment case, and in spite of the absence of a tax-smoothing motive (taxes are non-distorting, and taxpayers have access to financial markets). Variable rather than fixed costs of a default give rise to premature debt redemption (the opposite of dilution); in this case, the optimal maturity structure varies with the level of the initial deficit.

Related Literature Lack of commitment is widely acknowledged in the sovereign debt literature. A common view holds that strategic default can only be prevented and thus, borrowing

[^1]sustained, because default would push a country into (partial) financial autarky; see Eaton and Gersovitz (1981), Bulow and Rogoff (1989), Grossman and Han (1999) and Kletzer and Wright (2000) for discussions of this hypothesis, and Alvarez and Jermann (2000) or Kehoe and Perri (2002), among many others, for applications.

To rationalize an optimal maturity structure, many authors have suggested that short-term debt renders a country vulnerable to rollover crises, and that long-term debt reduces such vulnerability (Calvo, 1988; Alesina, Prati and Tabellini, 1990; Giavazzi and Pagano, 1990; Cole and Kehoe, 2000); see also Phelan (2001). Angeletos (2002) argues that a sufficiently rich maturity structure of non state contingent bonds may serve as substitute for state contingent debt. Related points are made by Gale (1990) and Cochrane (2001). Calvo and Guidotti (1990) and Missale and Blanchard (1994) discuss the role of the maturity structure of nominal debt for the government's incentive to engineer surprise inflation. Diamond (1991) analyzes the choice of maturity structure in a corporate finance context. Short-term debt is advantageous if the borrower has private information about a likely improvement of future credit ratings while longterm debt helps avoid liquidity risk. Jeanne (2004) argues that long-term debt creates a "debt overhang" problem. Reducing the borrower's incentives to exert effort (assumed to pay off only in the long run), long-term debt shrinks the primary surpluses available for debt repayment. Broner, Lorenzoni and Schmukler (2004) assume that both primary government surpluses and the bond pricing kernel are endogenous to the government's choice of maturity structure.

Kydland and Prescott (1977) and Fischer (1980) discuss the government's ex-post incentive to default when taxes are distorting. Distributive conflicts affect a government's default decision in Tabellini (1991), Dixit and Londregan (2000), Kremer and Mehta (2000) or Niepelt (2004a). Rogers (1986), Bassetto (1999), Niepelt (2004b) and Armenter (2003) note that distributive implications of ex-post policy changes may counteract a government's incentive to renege on the ex-ante optimal policy. Assuming full commitment to debt, these latter papers focus on other sources of time-inconsistency. Persson, Persson and Svensson (1998) and Doepke and Schneider (2005) document the likely distributive effects of a surprise capital levy on nominal assets due to unanticipated inflation.

The presence of ex-post incentive compatibility constraints is at the heart of the failure of Modigliani and Miller's equivalence result in Lucas and Stokey (1983) or Persson, Persson and Svensson (1987) ${ }^{3}$ (maturity matters) and Tabellini (1991) (debt versus social security matters).

## 2 Model

### 2.1 Setup

Time is discrete and indexed by $n=0,1,2, \ldots$. In the initial period, $n=0$, the government must fund an exogenous funding requirement, $g$. To that end, the government issues debt of various maturities, $\left\{b_{0 m}\right\}$, where the first and second index denote the issuance and maturity dates, respectively. In later periods, $n>0$, the government decides whether to honor the debt maturing in the period, $b_{x n} \equiv \sum_{l=0}^{n-1} b_{l n}$. Denoting the repayment rate on debt maturing in period $n$ by $r_{n}$, the government's default decision thus amounts to choosing an $r_{n} \in[0,1]$. In addition to choosing the repayment rate $r_{n}$, the government also chooses taxes, $t_{n}$, and sales of new maturities, $\left\{b_{n m}\right\}$.

[^2]
### 2.2 Private Sector

Two groups of households inhabit the economy. First, debt-holders or investors who hold all maturities of the debt issued by the government. Second, taxpayers who bear the burden of all taxes levied by the government. Our assumption that the two groups of taxpayers and investors are distinct is made for simplicity. Assuming instead that the tax burden is shared between groups, or that both groups hold government debt, would not significantly alter the analysis. ${ }^{4}$

Taxpayers and investors have time- and state-additive preferences over consumption and discount the future according to the discount factor $\beta \in(0,1)$. Households also benefit from the quality of institutions in place, to be discussed in more detail below. This quality is exogenous to the households and does not affect their marginal utility of consumption. For the time being, I therefore abstract from it. Households have access to a large international capital market on which they can trade state contingent one-period claims whose returns depend on the state of nature in the following period. I denote a realization of the exogenous state in period $n$ by $s_{n} .{ }^{5}$ A claim traded in period $n$ and paying one unit of the good after history $s^{n+1} \equiv\left\{s_{1}, s_{2}, \ldots, s_{n+1}\right\}$ and zero otherwise is denoted by $a_{n}\left(s^{n+1}\right)$, and the price of this claim is denoted by $p_{n}\left(s^{n+1}\right)$.

Let $U_{n}\left(a_{n}^{1} ; \mathbf{z}_{n}\right)$ and $V_{n}\left(a_{n}^{2}, \mathbf{b}_{n} ; \mathbf{z}_{n}\right)$ denote the value functions of taxpayers and investors, respectively, in period $n$, after the realization of $s_{n}$, and net of the utility derived from the quality of institutions. Here, $a_{n}^{1}$ and $a_{n}^{2}$ denote the maturing claims held by taxpayers and investors, respectively; $\mathbf{b}_{n} \equiv\left\{\sum_{l=0}^{n-1} b_{l i}\right\}_{i \geq n}$ denotes the vector of government debt maturities held by investors at the beginning of period $n$; and the vector $\mathbf{z}_{n}$ collects the exogenous variables in the households' programs. These exogenous variables are given by current and anticipated state-contingent exogenous incomes of taxpayers and investors, $\mathbf{y}_{n}^{1} \equiv\left\{y_{i}^{1}\left(s^{i}\right)\right\}_{i \geq n}$ and $\mathbf{y}_{n}^{2} \equiv$ $\left\{y_{i}^{2}\left(s^{i}\right)\right\}_{i \geq n}$, respectively; current and anticipated prices of the state-contingent claims, $\mathbf{p}_{n} \equiv$ $\left\{p_{i}\left(s^{i+1}\right)\right\}_{i \geq n}$; current and anticipated state-contingent prices of government debt maturing in future periods, $\mathbf{q}_{n} \equiv\left\{q_{j i}\left(s^{j}\right)\right\}_{j \geq n, i>j}$; current and anticipated state-contingent taxes, $\mathbf{t}_{n} \equiv$ $\left\{t_{i}\left(s^{i}\right)\right\}_{i \geq n}$; as well as current and anticipated state-contingent repayment rates, $\mathbf{r}_{n} \equiv\left\{r_{i}\left(s^{i}\right)\right\}_{i \geq n}$. The Bellman equations for the two groups are given by

$$
\begin{gathered}
U_{n}\left(a_{n}^{1} ; \mathbf{z}_{n}\right)=\max _{a_{n+1}^{1}} u\left(a_{n}^{1}+y_{n}^{1}-t_{n}-\sum_{s^{n+1}} p_{n}(\cdot) a_{n+1}^{1}(\cdot)\right)+\beta \mathrm{E} U_{n+1}\left(a_{n+1}^{1}\left(s^{n+1}\right) ; \mathbf{z}_{n+1}\right), \\
V_{n}\left(a_{n}^{2}, \mathbf{b}_{n} ; \mathbf{z}_{n}\right)=\max _{a_{n+1}^{2},\left\{b_{n i}\right\}_{i \geq n+1}} v\left(a_{n}^{2}+y_{n}^{2}+b_{x n} r_{n}-\sum_{s^{n+1}} p_{n}(\cdot) a_{n+1}^{2}(\cdot)-\sum_{i \geq n+1} q_{n i} b_{n i}\right) \\
+\beta \mathrm{E} V_{n+1}\left(a_{n+1}^{2}\left(s^{n+1}\right), \mathbf{b}_{n+1} ; \mathbf{z}_{n+1}\right) .
\end{gathered}
$$

The utility functions $u(\cdot)$ and $v(\cdot)$ are strictly increasing and concave.
The optimality conditions of these programs define asset demand functions as well as implied consumption functions for the households. I denote these functions by $a_{n+1}^{1 \star}\left(s^{n+1}\right)\left(a_{n}^{1} ; \mathbf{z}_{n}\right)$ and $c_{n}^{1 \star}\left(a_{n}^{1} ; \mathbf{z}_{n}\right)$ for taxpayers and by $a_{n+1}^{2 \star}\left(s^{n+1}\right)\left(a_{n}^{2}, \mathbf{b}_{n} ; \mathbf{z}_{n}\right), \mathbf{b}_{n+1}^{\star}\left(a_{n}^{2}, \mathbf{b}_{n} ; \mathbf{z}_{n}\right)$, and $c_{n}^{2 \star}\left(a_{n}^{2}, \mathbf{b}_{n} ; \mathbf{z}_{n}\right)$

[^3]for investors. These demand functions satisfy the usual implementability constraints and in particular, asset pricing conditions. Omitting (some) arguments of functions, we have
\[

$$
\begin{aligned}
& \frac{\partial U_{i}}{\partial t_{i}}=-u^{\prime}\left(c_{i}^{1 \star}\right), \frac{\partial V_{i}}{\partial r_{i}}=v^{\prime}\left(c_{i}^{2 \star}\right) b_{x i}, p_{i}\left(s^{i+1}\right)=\beta \frac{v^{\prime}\left(c_{i+1}^{2 \star}\left(s^{i+1}\right)\right)}{v^{\prime}\left(c_{i}^{2 \star}\right)} \\
& q_{i j}=\beta^{j-i} \mathrm{E}\left[\frac{v^{\prime}\left(c_{j}^{2 \star}\left(s^{j}\right)\right) r_{j}\left(s^{j}\right)}{v^{\prime}\left(c_{i}^{2 \star}\right)}\right]=\mathrm{E}\left[p_{i}\left(s^{i+1}\right) \cdots p_{j-1}\left(s^{j}\right) r_{j}\left(s^{j}\right)\right]
\end{aligned}
$$
\]

for all $i \geq n$. According to the last condition, the equilibrium price of debt reflects the exogenous asset pricing kernel as well as the anticipated repayment rate at maturity. This is a direct consequence of the assumption that investors have access to financial markets with exogenously given prices.

Conditional on $\left(a_{n}^{1}, a_{n}^{2}, \mathbf{b}_{n}, \mathbf{y}_{n}^{1}, \mathbf{y}_{n}^{2}, \mathbf{p}_{n}\right)$, an equilibrium as of period $n$ consists of policies $\left(\mathbf{t}_{n}, \mathbf{r}_{n},\left\{\mathbf{b}_{i}\right\}_{i \geq n+1}\right)$, prices $\mathbf{q}_{n}$, and savings choices $\left(\left\{a_{i}^{1}(\cdot), a_{i}^{2}(\cdot)\right\}_{i \geq n+1},\left\{\tilde{\mathbf{b}}_{i}\right\}_{i \geq n+1}\right)$, such that (i) savings choices solve the households' programs, (ii) bond markets clear, $\left\{\mathbf{b}_{i}\right\}_{i \geq n+1}=\left\{\tilde{\mathbf{b}}_{i}\right\}_{i \geq n+1}$, and (iii) the dynamic government budget constraints are satisfied,

$$
t_{i}+\sum_{j \geq i+1} q_{i j} b_{i j}=b_{x i} r_{i} \text { for all } i \geq n
$$

As noted before, investors' implementability constraints and the exogenous asset pricing kernel fully determine the market clearing bond prices. We can therefore alternatively define an equilibrium as of period $n$ as a set of policies and savings choices $\left\{a_{i}^{1}(\cdot), a_{i}^{2}(\cdot)\right\}_{i \geq n+1}$ such that
i. savings choices solve the households' programs, and
ii. the dynamic government budget constraints are satisfied
iii. subject to $q_{i j}=\mathrm{E}\left[p_{i}\left(s^{i+1}\right) \cdots p_{j-1}\left(s^{j}\right) r_{j}\left(s^{j}\right)\right]$.

### 2.3 Government

The government's objective is to maximize a weighted average of the welfare of taxpayers and investors. I denote the weight the government attaches to the welfare of taxpayers by $\theta^{1}$, and the weight attached to the welfare of investors by $\theta^{2}$. These weights can be interpreted, for example, as reflecting relative political influence of the two groups. Crucially, and realistically, I assume that policies are chosen sequentially. This implies that current political decision makers cannot commit their successors (or future selves) to implement the ex-ante preferred sequence of policy choices. In particular, the government cannot commit its successors (or future selves) to honor maturing debt.

A large literature on sovereign debt has discussed the restrictions that lack of commitment imposes on a government's ability to issue debt. This literature emphasizes various costs of defaulting that induce the government to honor its obligations rather than renege on them. ${ }^{6}$ I take a similar approach here, assuming that default has persistent negative repercussions. In particular, I assume that the government benefits from high-quality institutions, interpreted as an environment characterized by good faith, well protected property rights, a large stock of social capital etc; that it takes time to build high-quality institutions; and that the quality of

[^4]institutions suffers after a default. Formally, I model the quality of institutions in period $n$ as the product of two indicator functions, $\mathbf{1}_{\left[r_{n}=1\right]} \cdot \mathbf{1}_{\left[r_{n-1}=1\right]}$ : The quality is high (equal to unity) if the government has honored its debt obligations in the current and the previous period; it is low (equal to zero) if the government has defaulted at least once over the last two periods. The benefit to the private sector and thus, the government of having good institutions in place in period $n$ is given by $B_{n} \geq 0 ; B_{n}$ is the realization of a random variable, identically and independently distributed over time, with cumulative density function $F(\cdot)$ and associated probability density function $f(\cdot)$. For simplicity, I assume that the support of $B_{n}$ is unbounded above and that $f(\cdot)$ is strictly positive for all $B_{n}$ above the non-negative lower bound of the support of $B_{n}$. The government is assumed to learn about the realization of $B_{n}$ at the beginning of period $n$, before it chooses the policy instruments.

Lack of commitment imposes additional constraints on the equilibrium since policy choices have to be ex-post optimal. A time-consistent equilibrium as of period $n$ is a set of policies $\left(\mathbf{t}_{n}, \mathbf{r}_{n},\left\{\mathbf{b}_{i}\right\}_{i \geq n+1}\right)$ and savings choices $\left\{a_{i}^{1}(\cdot), a_{i}^{2}(\cdot)\right\}_{i \geq n+1}$ such that
i. savings choices solve the households' programs, and
ii. the dynamic government budget constraints are satisfied
iii. subject to $q_{i j}=\mathrm{E}\left[p_{i}\left(s^{i+1}\right) \cdots p_{j-1}\left(s^{j}\right) r_{j}\left(s^{j}\right)\right]$,
iv. where anticipated policies coincide with the ex-post optimal policies that successive governments (solving parallel problems) choose to implement.

The objective of the government in period $n$ to select the "best" time-consistent equilibrium can now be stated as follows:

$$
\begin{aligned}
\max _{t_{n}, r_{n} \in[0,1],\left\{b_{n i}\right\}_{i \geq n+1}} & \theta^{1} U_{n}\left(a_{n}^{1} ; \mathbf{z}_{n}\right)+\theta^{2} V_{n}\left(a_{n}^{2}, \mathbf{b}_{n} ; \mathbf{z}_{n}\right)+\sum_{j \geq 0} \beta^{j} \mathrm{E}\left[\mathbf{1}_{\left[r_{n+j}=1\right]} \mathbf{1}_{\left[r_{n+j-1}=1\right]} B_{n+j}\right] \\
\text { s.t. } & \left(t_{i}, r_{i} \in[0,1],\left\{b_{i j}\right\}_{j \geq i+1}\right) \text { is optimal, conditional on }\left(a_{i}^{1}, a_{i}^{2}, \mathbf{b}_{i}, r_{i-1}\right), \text { for all } i>n, \\
& q_{n i}=\mathrm{E}\left[p_{n} \cdots p_{i-1} r_{i}\right] \text { for all } i>n, \\
& \left\{\begin{array}{ll}
\sum_{i \geq n+1} q_{n i} b_{n i}+t_{n}=b_{x n} r_{n} & \text { if } n>0 \\
\sum_{i \geq 1} q_{0 i} b_{0 i}=g & \text { if } n=0
\end{array},\right.
\end{aligned}
$$

where the value functions of the households are evaluated at the privately optimal savings choices (and the rationally anticipated policy choices).

## 3 Discussion

Consider the effect of a marginal reduction in the repayment rate, $d r_{n}$, holding the deficit constant. Taxes can then be reduced by $b_{x n} d r_{n}$. From the implementability constraints derived earlier, we know that the derivatives of the value functions with respect to taxes and repayment rates are related to households' marginal utilities. Abstracting from the induced effects on the quality of institutions, the effect of a marginal reduction in the repayment rate on the government's objective therefore equals $\left(\theta^{1} u^{\prime}\left(c_{n}^{1 \star}\right)-\theta^{2} v^{\prime}\left(c_{n}^{2 \star}\right)\right) b_{x n} d r_{n}$ : Default amounts to a lump-sum transfer from investors to taxpayers. Such a lump-sum transfer is attractive for the government when the social marginal utility (as perceived by the government) of taxpayers exceeds the one of investors. In that case, a government only refrains from exploiting the default option if the foregone benefits from high-quality institutions outweigh the gains from redistribution. Whatever the outcome of this cost-benefit comparison, With rational expectations, the government's
optimal behavior (either to default or to refrain from defaulting) is anticipated, debt when issued is priced accordingly, and no default-induced redistribution occurs in equilibrium. Nevertheless, the threat of opportunistic behavior ex-post imposes constraints at the debt-issuance stage.

The strength of the ex-post incentive to default depends on the difference between taxpayers' and investors' marginal utilities. With strictly concave utility functions, this difference depends on the equilibrium consumption levels and thus, wealth of the two groups of households. The government's optimal policy choice therefore generally varies with asset holdings of taxpayers and investors. The contemporaneous choice of $t_{n}$ or $r_{n}$ in turn induces changes in the endogenous state variables and thereby affects successive policy choices. This interdependence renders the characterization of equilibrium policies and allocations difficult.

Nevertheless, several conclusions can be drawn at this general level. First, although investors are "vulnerable" ex-post in the sense of being affected by the government's choice of repayment rate $\left(\partial V_{n}(\cdot) / \partial r_{n} \neq 0\right.$ if $b_{x n} \neq 0$ ), they are "insulated" against policy ex-ante: $\partial V_{0}(\cdot) / \partial r_{i}=0$ for all $i>0 .{ }^{7}$ This follows from two observations. On one hand, investors hold government debt only to the extent that its return characteristics render it a substitute to the state-contingent claims. With rational expectations, future repayment rates are therefore fully reflected in the prices that investors pay when buying government debt (see the implementability constraints). On the other hand, market completeness implies that government debt is a redundant asset from the investors' perspective. In equilibrium, all debt therefore has to be repaid in expected present discounted value terms, with the asset pricing kernel fixed at $\mathbf{p}_{0}$. For the same reasons, the expected present discounted value of tax payments must be equal to the level of government spending in the initial period, $g$. This implies, second, that the ex-ante welfare of taxpayers is independent of taxes and repayment rates as well (but not, of course, independent of the funding requirement) if taxpayers can access financial markets subject to the same pricing kernel $\mathbf{p}_{0}$. Without such access, the timing of taxes may matter since it influences the degree to which taxpayers' consumption is smoothed across time and states. The previous two conclusions imply, third, that the exante objective of the government reduces to maximizing $\sum_{j \geq 0} \beta^{j} \mathrm{E}\left[\mathbf{1}_{\left[r_{j}=1\right]} \mathbf{1}_{\left[r_{j-1}=1\right]} B_{j}\right]$ if both taxpayers and investors have access to complete financial markets. Summarizing, we have:

Lemma 1. Suppose taxpayers and investors have access to complete financial markets with an exogenous asset pricing kernel. Then, the ex-ante program of the government without commitment is given by

$$
\begin{aligned}
\max _{\left\{b_{0 i}\right\}_{i \geq 1}} & \sum_{j \geq 0} \beta^{j} \mathrm{E}\left[\mathbf{1}_{\left[r_{j}=1\right]} \mathbf{1}_{\left[r_{j-1}=1\right]} B_{j}\right] \\
\text { s.t. } & \left(t_{i}, r_{i} \in[0,1],\left\{b_{i j}\right\}_{j \geq i+1}\right) \text { is optimal, conditional on }\left(a_{i}^{1}, a_{i}^{2}, \mathbf{b}_{i}, r_{i-1}\right), \text { for all } i>0, \\
& q_{0 i}=\mathrm{E}\left[p_{0} \cdots p_{i-1} r_{i}\right] \text { for all } i>0, \\
& \sum_{i \geq 1} q_{0 i} b_{0 i}=g .
\end{aligned}
$$

The same ex-ante objective applies in an alternative setting with commitment on the part of the government. In such a setting with commitment, the constraints in the government's ex-ante program comprise all successive budget constraints, but no incentive compatibility constraints.

[^5]More specifically, the government's program under commitment reads

$$
\begin{aligned}
\max _{\left\{t_{n}, r_{n} \in[0,1]\right\}_{n \geq 1},\left\{b_{n i}\right\}_{n \geq 0, i>n}} & \sum_{j \geq 0} \beta^{j} \mathrm{E}\left[\mathbf{1}_{\left[r_{j}=1\right]} \mathbf{1}_{\left[r_{j-1}=1\right]} B_{j}\right] \\
\text { s.t. } & q_{n i}=\mathrm{E}\left[p_{n} \cdots p_{i-1} r_{i}\right] \text { for all } n \geq 0, i>n, \\
& \sum_{i \geq 1} q_{0 i} b_{0 i}=g, \sum_{i>n} q_{n i} b_{n i}+t_{n}=b_{x n} r_{n} \text { for all } n>0 .
\end{aligned}
$$

Due to the implementability constraints, the choice of repayment rates has no effect on the government's intertemporal budget constraint. In the solution to the program, all repayment rates therefore equal unity, $r_{n}=1, n \geq 1$, while the timing of taxes, the level of debt in all but the initial period, and the maturity structure are indeterminate. Determinacy of the optimal timing of taxes, net debt, and maturity structure therefore arises - if it arises - solely as a result of the government's lack of commitment. Summarizing:

Lemma 2. Suppose taxpayers and investors have access to complete financial markets with an exogenous asset pricing kernel. If the government can commit, it chooses never to default, $r_{n}=1$ for all $n \geq 1$. The timing of taxes and the maturity structure of debt are indeterminate.

## 4 Linear Utility

To solve the ex-ante program of the government, all ex-post programs have to be solved first since the solutions to the latter determine the equilibrium values of $\mathbf{1}_{\left[r_{j}=1\right]}, j \geq 1$. To simplify the analysis, I consider the special case of risk-neutral preferences. Under this assumption, marginal utilities are independent of wealth, ex-post optimal policy choices do not depend on the amount of claims held by households, and repayment rates are either zero or one.

Let $u^{\prime}$ and $v^{\prime}$ denote the constant marginal utilities of taxpayers and investors, respectively. In an interior equilibrium, the price of one-period state-contingent claims equals the discount factor, multiplied by the probability of those states. Consequently, the period $n$ price of debt maturing in period $i$ satisfies

$$
q_{n i}=\beta^{i-n} \mathrm{E}\left[r_{i}\right] \text { for all } i>n .
$$

Using these asset pricing relationships, we can express the value functions of taxpayers and investors in period $n$ (net of the utility derived from the quality of institutions) as

$$
\begin{align*}
U_{n}\left(a_{n}^{1} ; \mathbf{z}_{n}\right) & =u^{\prime} \cdot\left(a_{n}^{1}+\sum_{i \geq n} \beta^{i-n} \mathrm{E}\left[y_{i}^{1}-t_{i}\right]\right),  \tag{1}\\
V_{n}\left(a_{n}^{2}, \mathbf{b}_{n} ; \mathbf{z}_{n}\right) & =v^{\prime} \cdot\left(a_{n}^{2}+\sum_{i \geq n} \beta^{i-n} \mathrm{E}\left[y_{i}^{2}+\sum_{l=0}^{n-1} b_{l i} r_{i}\right]\right) . \tag{2}
\end{align*}
$$

According to these expressions, welfare of households is proportional to their wealth. For taxpayers, wealth consists of contingent claims at hand as well as the expected present discounted value of exogenous incomes net of taxes. Investor wealth is comprised of contingent claims at hand as well as the expected present discounted return on government bonds bought in the past. I denote the latter by $b_{x n i} \equiv \sum_{l=0}^{n-1} b_{l i}, i \geq n$, where $b_{x n n}=b_{x n}$.

Let $\Delta \equiv \theta^{1} u^{\prime}-\theta^{2} v^{\prime}$ denote the marginal direct gain to the government of redistributing from investors to taxpayers. I will assume in the following that this marginal direct gain is strictly
positive - the government prefers to transfer resources from investors to taxpayers. Using (1) and (2), the government's objective in period $n$ to select the "best" time-consistent equilibrium can now be expressed as

$$
\begin{aligned}
\max _{r_{n} \in[0,1],\left\{b_{n i}\right\}_{i>n}} & \sum_{i \geq n} \beta^{i-n} \mathrm{E}\left[-\Delta b_{x n i} r_{i}+\mathbf{1}_{\left[r_{i}=1\right]} \mathbf{1}_{\left[r_{i-1}=1\right]} B_{i}\right]+\text { terms independent of policy } \\
\text { s.t. } & \left(r_{i} \in[0,1],\left\{b_{i j}\right\}_{j>i}\right) \text { is optimal, conditional on }\left(\mathbf{b}_{i}, r_{i-1}\right), \text { for all } i>n
\end{aligned}
$$

the government in the initial period, $n=0$, faces the additional constraint $\sum_{i \geq 1} \beta^{i} \mathrm{E}\left[r_{i}\right] b_{0 i}=g$.
The government aims at maximizing the expected present discounted benefits from highquality institutions on one hand and default-induced redistribution on the other, subject to the restrictions imposed by a time-consistent equilibrium. Optimality of contingent claims holdings does not constrain the government since these claims enter the objective function additively, due to risk neutrality (contingent claims wealth is subsumed under the "terms independent of policy"). The government's dynamic budget constraints have been imposed by expressing taxes in the value function of taxpayers as the difference between debt redemptions and deficits; and the equilibrium asset pricing relationships have been incorporated by expressing the prices of newly issued debt in terms of the discount factor and expected repayment rates. Ex-post incentive compatibility of policy choices constitutes the sole remaining condition to be satisfied in the time-consistent equilibrium.

The government has two sets of instruments at its disposal. On one hand, the repayment rate, $r_{n}$. By reducing that rate below unity, the government redistributes from investors to taxpayers, see the first term in the objective function. At the same time, it foregoes the benefit $B_{n}$ (if debt was honored in period $n-1$ ) as well as the benefit $B_{n+1}$ (to the extent that debt will be honored in period $n+1$ ), see the second term. On the other hand, debt issuance, $\left\{b_{n i}\right\}_{i>n}$. Due to rational expectations, newly issued maturities do not have direct revenue effects-they simply shift tax collections to the future, which is of no relevance to taxpayers if they are risk neutral or have access to complete financial markets. Issuing debt does have indirect effects, however, since it affects successive policy choices. By increasing the amount of debt maturing in later periods, current deficits increase the incentive to default in the future and dilute the outstanding long-term debt.

Rather than assuming that the government is tempted to default in order to redistribute between investors and taxpayers, I could alternatively have assumed that tax distortions drive a wedge between the shadow values of private and public funds and thus, that the government is tempted to default in order to redistribute from private to public coffers. In fact, subject to minor changes, the analysis to follow can alternatively be interpreted in this way. "Investors" then represent the homogenous private sector in the economy, "taxpayers" correspond to the public sector, and $\Delta$ measures the wedge between the shadow values of public and private funds.

## 5 Analysis

### 5.1 Fixed Cost, Short Horizon

To build intuition, I start by analyzing the optimal policy in the special case where all debt must, for unspecified reasons, be repaid or defaulted upon by the end of the second period. Formally, I impose the restriction $b_{n m}=0$ for all $m \geq 3$. With no debt maturing in periods $n=3$ and later, the optimal repayment rates in these periods are unity, $r_{n}=1$ for all $n \geq 3$. I consider a situation in which the government starts out with low-quality institutions, i.e., I assume " $r_{0}<1$ ".

I characterize the optimal policy by backward induction.

### 5.1.1 Temporary Loss of Social Capital

In this simplest case, the benefits from high-quality institutions, $B_{n}$, are realized if and only if the contemporaneous policy choice is to honor the debt, $r_{n}=1$. This renders the default decision a "static" one.

Period $n=2$ In period $n=2$, the government's program reads

$$
\max _{r_{2} \in[0,1]}-\Delta b_{x 2} r_{2}+\mathbf{1}_{\left[r_{2}=1\right]} B_{2} .
$$

We thus have

$$
r_{2}^{\star}= \begin{cases}1 & \text { if } B_{2} \geq \Delta b_{x 2}  \tag{3}\\ 0 & \text { otherwise }\end{cases}
$$

Period $n=1$ In period $n=1$, the government's program reads

$$
\max _{r_{1} \in[0,1], b_{12}}-\Delta b_{01} r_{1}-\beta \Delta b_{02} \mathrm{E}\left[r_{2}\right]+\mathbf{1}_{\left[r_{1}=1\right]} B_{1}+\beta \mathrm{E}\left[\mathbf{1}_{\left[r_{2}=1\right]} B_{2}\right] .
$$

Using (3), this can equivalently be expressed as

$$
\max _{r_{1} \in[0,1], b_{12}}-\Delta b_{01} r_{1}+\beta \int_{\Delta b_{x 2}}^{\infty}\left(B_{2}-\Delta b_{02}\right) f\left(B_{2}\right) d B_{2}+\mathbf{1}_{\left[r_{1}=1\right]} B_{1} .
$$

The second term in this objective function represents the value of an option that pays $B_{2}-\Delta b_{02}$ whenever $B_{2}$ is sufficiently high, and zero otherwise. Whether this option is "in the money" does not only depend on the realization of $B_{2}$, but also on $b_{x 2}$ and thus, the period-1 government's choice of debt rollover, $b_{12}$. The larger $b_{12}$, the higher the amount of debt maturing in period $n=2$ and the probability that the period-2 government defaults (see condition (3)). The associated gains from dilution are accompanied by a reduced probability of realizing the benefits from high-quality institutions.

The marginal value of an increase in $b_{12}$ is given by

$$
\beta \frac{\partial \int_{\Delta b_{x 2}}^{\infty}\left(B_{2}-\Delta b_{02}\right) f\left(B_{2}\right) d B_{2}}{\partial b_{12}}=-\beta b_{12} \Delta^{2} f\left(\Delta b_{x 2}\right) .
$$

If the smallest possible realization of $B_{2}$ exceeds $\Delta b_{x 2}$, then $f\left(\Delta b_{x 2}\right)=0$ and the marginal value of an increase in $b_{12}$ equals zero. Intuitively, there is no risk of an upcoming default in this case, and marginally increasing $b_{12}$ keeps the default probability unchanged. Newly issued debt therefore does not dilute outstanding debt, nor does it affect the expected benefits from high-quality institutions (nor does it generate net revenue). The choice of $b_{12}$ therefore is of no consequence, and without loss of generality we can set $b_{12}^{\star}=0$. If the smallest possible realization of $B_{2}$ falls short of $\Delta b_{x 2}$, in contrast, then issuing additional debt does affect the probability of an upcoming default. ${ }^{8}$ An increase (decrease) of $b_{12}$ leads the period-2 government to repay in fewer (more) instances, giving rise to a benefit (loss) as far as dilution is concerned and a loss (benefit) as far as the gains from high-quality institutions are concerned. At $b_{12}=0$, the two effects cancel, due to the envelope theorem, while for $b_{12} \neq 0$, the respective losses outweigh the

[^6]gains. Any deviation of $b_{12}$ from zero therefore reduces the value of the option, see Figure 1 . In conclusion,
\[

r_{1}^{\star}=\left\{$$
\begin{array}{ll}
1 & \text { if } B_{1} \geq \Delta b_{01}  \tag{4}\\
0 & \text { otherwise }
\end{array}
$$ \quad, b_{12}^{\star}=0\right. always.
\]



Figure 1: Option payoffs for different choices of $b_{12}$.

Period $n=0$ Using Lemma 1, the government's program in the initial period reads

$$
\begin{aligned}
\max _{b_{01}, b_{02}} & \beta \mathrm{E}\left[\mathbf{1}_{\left[r_{1}=1\right]} B_{1}\right]+\beta^{2} \mathrm{E}\left[\mathbf{1}_{\left[r_{2}=1\right]} B_{2}\right] \\
\text { s.t. } & (3),(4), \beta \mathrm{E}\left[r_{1}\right] b_{01}+\beta^{2} \mathrm{E}\left[r_{2}\right] b_{02}=g
\end{aligned}
$$

Substituting the optimal repayment rates, the program can be expressed as

$$
\begin{aligned}
\max _{b_{01}, b_{02}} & \beta \int_{\Delta b_{01}}^{\infty} B_{1} f\left(B_{1}\right) d B_{1}+\beta^{2} \int_{\Delta b_{02}}^{\infty} B_{2} f\left(B_{2}\right) d B_{2} \\
\text { s.t. } & \beta\left(b_{01} \int_{\Delta b_{01}}^{\infty} f\left(B_{1}\right) d B_{1}+\beta b_{02} \int_{\Delta b_{02}}^{\infty} f\left(B_{2}\right) d B_{2}\right)=g .
\end{aligned}
$$

Letting $\lambda$ denote the multiplier on the funding constraint, the first-order conditions with respect to $b_{01}$ and $b_{02}$, respectively, are given by

$$
\begin{aligned}
& -\Delta^{2} b_{01} f\left(\Delta b_{01}\right)+\lambda\left(1-F\left(\Delta b_{01}\right)-\Delta b_{01} f\left(\Delta b_{01}\right)\right)=0 \\
& -\Delta^{2} b_{02} f\left(\Delta b_{02}\right)+\lambda\left(1-F\left(\Delta b_{02}\right)-\Delta b_{02} f\left(\Delta b_{02}\right)\right)=0
\end{aligned}
$$

If the hazard function $f(B) /(1-F(B))$ is weakly increasing then these conditions yield a unique optimal maturity structure, $b_{01}=b_{02} .{ }^{9}$ Intuitively, with an increasing hazard function, the expected loss of benefits per unit of revenue raised by a certain debt maturity increases in the level of that maturity. The ex-ante optimal policy therefore amounts to "smoothing" the expected losses due to opportunistic behavior ex post across the available maturities, in parallel to the tax-smoothing prescription developed in Barro (1979).

The maximal revenue to be raised by a particular debt maturity is attained at the top of the Laffer curve, at $b^{\text {max }}$, where this debt level is defined as the solution to the equation $1-F(\Delta b)=\Delta b f(\Delta b) .{ }^{10}$ The following Proposition summarizes these results.

[^7]Proposition 1. Consider the case of a temporary loss of social capital in the wake of a default. Suppose the hazard function of $B$ is weakly increasing. Then, the optimal maturity structure minimizes the probability of strategic default. For any feasible funding requirement, $g$, the optimal maturity structure is balanced. The maximal feasible funding requirement is given by $g^{\max }=\left(\beta+\beta^{2}\right) b^{\max }\left(1-F\left(\Delta b^{\max }\right)\right)$. Debt is not diluted, and no debt is rolled over, $b_{12}=0$.

### 5.1.2 Persistent Loss of Social Capital

In this case, the benefit from high-quality institutions is realized only if debt was honored in the current and previous period.

Period $n=2$ In period $n=2$, the government's program reads

$$
\max _{r_{2} \in[0,1]}-\Delta b_{x 2} r_{2}+\mathbf{1}_{\left[r_{1}=1\right]} \mathbf{1}_{\left[r_{2}=1\right]} B_{2}+\beta \mathbf{1}_{\left[r_{2}=1\right]} \mathrm{E}\left[B_{3}\right],
$$

where the last term follows from the observation that $r_{3}=1$.
We thus have

$$
r_{2}^{\star}=\left\{\begin{array}{ll}
1 & \text { if } \mathbf{1}_{\left[r_{1}=1\right]} B_{2}+\beta \mathrm{E}\left[B_{3}\right] \geq \Delta b_{x 2}  \tag{5}\\
0 & \text { otherwise }
\end{array} .\right.
$$

Period $n=1$ In period $n=1$, the government's program reads

$$
\begin{aligned}
\max _{r_{1} \in[0,1], b_{12}} & -\Delta b_{01} r_{1}-\beta \Delta b_{02}\left\{\mathbf{1}_{\left[r_{1}=1\right]} \mathrm{E}\left[r_{2} \mid r_{1}=1\right]+\left(1-\mathbf{1}_{\left[r_{1}=1\right]}\right) \mathrm{E}\left[r_{2} \mid r_{1}=0\right]\right\} \\
& +\mathbf{1}_{\left[r_{1}=1\right]} \mathrm{E}\left[\mathbf{1}_{\left[r_{2}=1\right]}\left(\beta B_{2}+\beta^{2} \mathrm{E}\left[B_{3}\right]\right) \mid r_{1}=1\right]+\left(1-\mathbf{1}_{\left[r_{1}=1\right]}\right) \mathrm{E}\left[\mathbf{1}_{\left[r_{2}=1\right]} \beta^{2} \mathrm{E}\left[B_{3}\right] \mid r_{1}=0\right] .
\end{aligned}
$$

The terms in the first line capture the direct distributive effects of a default in period $n=1$ or $n=2$. The first term in the second line represents the expected benefit from high-quality institutions if the government chooses not to default in the current period. In this case, benefits accrue in periods $n=2$ and $n=3$ if the government in period $n=2$ honors its obligations as well. The second term in the second line represents the expected benefit from high-quality institutions if the government chooses to default. In this case, no benefits accrue in period $n=2$. Benefits do accrue in period $n=3$, however, if the government in period $n=2$ does not default.

Using (5), the program can equivalently be expressed as

$$
\begin{aligned}
\max _{r_{1} \in[0,1], b_{12}} & -\Delta b_{01} r_{1}+\beta \mathbf{1}_{\left[r_{1}=1\right]} \int_{\Delta b_{x 2}-\beta \mathrm{E}\left[B_{3}\right]}^{\infty}\left(B_{2}+\beta \mathrm{E}\left[B_{3}\right]-\Delta b_{02}\right) f\left(B_{2}\right) d B_{2} \\
& +\beta\left(1-\mathbf{1}_{\left[r_{1}=1\right]}\right) \mathbf{1}_{\left[\beta \mathrm{E}\left[B_{3}\right] \geq \Delta b_{x 2}\right]}\left(\beta \mathrm{E}\left[B_{3}\right]-\Delta b_{02}\right) .
\end{aligned}
$$

The second term in this objective function represents the value of an option. Conditional on $r_{1}=1$, this option pays $B_{2}+\beta \mathrm{E}\left[B_{3}\right]-\Delta b_{02}$ whenever $B_{2}$ is sufficiently high, and zero otherwise. Whether this option is "in the money" does not only depend on the realization of $B_{2}$, but also on $b_{x 2}$ and thus, the period-1 government's choice of debt rollover, $b_{12}$. The larger $b_{12}$, the higher the amount of debt maturing in period $n=2$ and the probability that the period-2 government defaults (see condition (5)). The associated gains from dilution are accompanied by a reduced probability of realizing the benefits from high-quality institutions. According to the third term in the objective function, the value of $B_{2}$ does not affect the expected benefits from high-quality institutions if the period- 1 government defaults. These benefits are still realized in period $n=3$ if the government in period $n=2$ does not default, i.e., if the maturing debt in period $n=2$ is not too high.

To derive the optimal default decision, I start by characterizing the rollover policy, $b_{12}$, conditional on the choice of $r_{1}$. Consider first the case of no default, $r_{1}=1$. The marginal value of an increase in $b_{12}$ is then given by

$$
\beta \frac{\partial \int_{\Delta b_{x 2}-\beta \mathrm{E}\left[B_{3}\right]}^{\infty}\left(B_{2}+\beta \mathrm{E}\left[B_{3}\right]-\Delta b_{02}\right) f\left(B_{2}\right) d B_{2}}{\partial b_{12}}=-\beta b_{12} \Delta^{2} f\left(\Delta b_{x 2}-\beta \mathrm{E}\left[B_{3}\right]\right) .
$$

If the smallest possible realization of $B_{2}$ exceeds $\Delta b_{x 2}-\beta \mathrm{E}\left[B_{3}\right]$, then $f\left(\Delta b_{x 2}-\beta \mathrm{E}\left[B_{3}\right]\right)=0$ and the marginal value of an increase in $b_{12}$ equals zero. Intuitively, there is no risk of an upcoming default in this case, and marginally increasing $b_{12}$ keeps the default probability unchanged. Newly issued debt therefore does not dilute outstanding debt, nor does it affect the expected benefits from high-quality institutions. The choice of $b_{12}$ therefore is of no consequence, and without loss of generality we can set $b_{12}^{\star}=0$. If the smallest possible realization of $B_{2}$ falls short of $\Delta b_{x 2}-\beta \mathrm{E}\left[B_{3}\right]$, in contrast, then issuing additional debt does affect the probability of an upcoming default. An increase (decrease) of $b_{12}$ leads the period-2 government to repay in fewer (more) instances, giving rise to a benefit (loss) as far as dilution is concerned and a loss (benefit) as far as the gains from high-quality institutions are concerned. At $b_{12}=0$, the two effects cancel, due to envelope theorem, while for $b_{12} \neq 0$, the respective losses outweigh the gains. Any deviation of $b_{12}$ from zero therefore reduces the value of the option discussed earlier, see Figure 2. I conclude that the choice $r_{1}=1$ yields the value $-\Delta b_{01}+\beta \int_{\Delta b_{02}-\beta \mathrm{E}\left[B_{3}\right]}^{\infty}\left(B_{2}+\right.$ $\left.\beta \mathrm{E}\left[B_{3}\right]-\Delta b_{02}\right) f\left(B_{2}\right) d B_{2}$.




Figure 2: Option payoffs for different choices of $b_{12}\left(A \equiv \Delta b_{02}-\beta \mathrm{E}\left[B_{3}\right]\right)$.
Consider next the default case, $r_{1}<1$. The choice of $b_{12}$ then maximizes the expression $\beta 1_{\left[\beta \mathrm{E}\left[B_{3}\right] \geq \Delta b_{x 2}\right]}\left(\beta \mathrm{E}\left[B_{3}\right]-\Delta b_{02}\right)$, and $b_{12}^{\star}=0$ again constitutes the weakly optimal rollover choice. Consequently, the government's value equals $\beta \mathbf{1}_{\left[\beta \mathrm{E}\left[B_{3}\right] \geq \Delta b_{02}\right]}\left(\beta \mathrm{E}\left[B_{3}\right]-\Delta b_{02}\right)$. Summarizing,

$$
r_{1}^{\star}=\left\{\begin{array}{ll}
1 & \text { if }-\Delta b_{01}+\beta \int_{A}^{\infty}\left(B_{2}-A\right) f\left(B_{2}\right) d B_{2} \geq-\beta \mathbf{1}_{[A \leq 0]} A  \tag{6}\\
0 & \text { otherwise }
\end{array}, b_{12}^{\star}=0 \text { always },\right.
$$

where I have defined $A \equiv \Delta b_{02}-\beta \mathrm{E}\left[B_{3}\right]$.
The no-default condition in (6) defines a critical value for short-term debt above which it is optimal to default. To see this, suppose first that $A \leq 0$. Defaulting then yields the (positive) payoff $-\beta A$ while not defaulting yields $-\Delta b_{01}+\beta \mathrm{E}\left[B_{2}\right]-\beta A$. Default therefore is optimal whenever $\Delta b_{01}>\beta \mathrm{E}\left[B_{2}\right]$. Intuitively, if $b_{02}$ and therefore $b_{x 2}$ is small, then the period2 government honors maturing debt independently of the choice of $r_{1}$. The reward for not defaulting in period $n=1$ therefore is $\beta \mathrm{E}\left[B_{2}\right]$.

Suppose next that $A \geq 0$. Defaulting then yields the payoff zero, while not defaulting yields $-\Delta b_{01}+\beta \int_{A}^{\infty}\left(B_{2}-A\right) f\left(B_{2}\right) d B_{2}$. Totally differentiating the condition characterizing indifference between these two choices, $\Delta b_{01}=\beta \int_{A}^{\infty}\left(B_{2}-A\right) f\left(B_{2}\right) d B_{2}$, we find

$$
\frac{d b_{02}}{d b_{01}}=-\frac{1}{\beta\left(1-F\left(\Delta b_{02}-\beta \mathrm{E}\left[B_{3}\right]\right)\right)} \leq-\beta^{-1} .
$$

Figure 3 provides a schematic summary of the results.


Figure 3: Optimal default choices $\left(A \equiv \Delta b_{02}-\beta \mathrm{E}\left[B_{3}\right]\right)$.

Period $n=0$ Using Lemma 1, the government's program in the initial period reads

$$
\begin{aligned}
\max _{b_{01}, b_{02}} & \beta^{2} \mathrm{E}\left[\mathbf{1}_{\left[r_{1}=1\right]} \mathbf{1}_{\left[r_{2}=1\right]} B_{2}\right]+\beta^{3} \mathrm{E}\left[\mathbf{1}_{\left[r_{2}=1\right]} B_{3}\right] \\
\text { s.t. } & (5),(6), \beta \mathrm{E}\left[r_{1}\right] b_{01}+\beta^{2} \mathrm{E}\left[r_{2}\right] b_{02}=g .
\end{aligned}
$$

I start by focusing on the situation with $A>0$. The repayment rate in period $n=1$ then depends on $b_{01}$ and $b_{02}$, see condition (6), and $r_{2}$ equals unity only if $r_{1}=1$ and $B_{2}$ is sufficiently large, see condition (5). Substituting the optimal repayment rates, the program can therefore be expressed as

$$
\begin{aligned}
\max _{b_{01}, b_{02}} & \beta^{2} \mathbf{1}_{\left[r_{1}=1\right]} \int_{\Delta b_{02}-\beta \mathrm{E}\left[B_{3}\right]}^{\infty}\left(B_{2}+\beta \mathrm{E}\left[B_{3}\right]\right) f\left(B_{2}\right) d B_{2} \\
\text { s.t. } & \beta \mathbf{1}_{\left[r_{1}=1\right]}\left(b_{01}+b_{02} \beta \int_{\Delta b_{02}-\beta \mathrm{E}\left[B_{3}\right]}^{\infty} f\left(B_{2}\right) d B_{2}\right)=g .
\end{aligned}
$$

Letting $\lambda$ denote the multiplier on the funding constraint (conditional on $r_{1}=1$ ),

$$
\beta\left(b_{01}+b_{02} \beta \int_{\Delta b_{02}-\beta \mathrm{E}\left[B_{3}\right]}^{\infty} f\left(B_{2}\right) d B_{2}\right)=g,
$$

and letting $\mu$ denote the multiplier associated with the ex-post incentive compatibility constraint,

$$
\Delta b_{01} \leq \beta \int_{A}^{\infty}\left(B_{2}-A\right) f\left(B_{2}\right) d B_{2}
$$

the first-order conditions with respect to $b_{01}$ and $b_{02}$, respectively, are given by

$$
\begin{aligned}
& \lambda \beta=\mu \Delta \\
& -\beta^{2} b_{02} \Delta^{2} f(A)-\lambda \beta^{2}\left\{\int_{A}^{\infty} f\left(B_{2}\right) d B_{2}-b_{02} \Delta f(A)\right\}+\mu \beta \Delta \int_{A}^{\infty} f\left(B_{2}\right) d B_{2}=0 .
\end{aligned}
$$

For $A>0$, the above conditions yield a contradiction. Intuitively, the slope of the constantrevenues curve (conditional on $r_{1}=1$ ) is always more negative (or even positive) than the slope of the conditional incentive compatibility constraint illustrated in Figure 3. For any level $b_{02}$ associated with a strictly positive $A$, the government can therefore improve its position by reducing $b_{02}$ and raising $b_{01}$. Issuing a marginal unit of short-term debt raises revenue $\beta$ and does not reduce the expected benefits from high-quality institutions. Issuing a marginal unit of long-term debt, in contrast, raises less revenue (since $A>0$ ) and reduces the expected benefits (for the same reason). Short-term debt therefore strictly dominates long-term debt. While the Modigliani-Miller neutrality result holds as far as the budgetary implications of the two maturities are concerned, the differential default risk of short- and long-term debt matters to the government since it affects the probability of being able to capture the benefits $B_{2}$ and $B_{3}$ ex post. Lack of commitment therefore renders the optimal maturity structure in period $n=0$ determinate (cf. Lemma 2).

Turning to the alternative case, $A \leq 0$, the repayment rate in period $n=1$ equals unity iff $\Delta b_{01} \leq \beta \mathrm{E}\left[B_{2}\right]$, while $r_{2}$ always equals one, see conditions (5) and (6). The government's program therefore reads

$$
\begin{aligned}
\max _{b_{01}, b_{02}} & \beta^{2} \mathbf{1}_{\left[\Delta b_{01} \leq \beta \mathrm{E}\left[B_{2}\right]\right]} \mathrm{E}\left[B_{2}\right]+\beta^{3} \mathrm{E}\left[B_{3}\right] \\
\text { s.t. } & \beta \mathbf{1}_{\left[\Delta b_{01} \leq \beta \mathrm{E}\left[B_{2}\right]\right]} b_{01}+\beta^{2} b_{02}=g .
\end{aligned}
$$

Independently of the level of $b_{02}$, it is then optimal to reduce $b_{01}$ below the default threshold $\beta \mathrm{E}\left[B_{2}\right] / \Delta$ in period $n=1$. Below that threshold, and for $b_{02} \leq \beta \mathrm{E}\left[B_{3}\right] / \Delta$, the optimal maturity structure is indeterminate. The following Proposition summarizes the results.

Proposition 2. Consider the case of a persistent loss of social capital in the wake of a default. Then, the optimal maturity structure avoids strategic default. For any feasible funding requirement, $g$, that is financed using risky long-term debt, there exists a superior financial strategy with more short-run and less long-run debt. The maximal feasible funding requirement is given by $g^{\max }=\beta^{2} \mathrm{E}\left[B_{2}\right] / \Delta+\beta^{3} \mathrm{E}\left[B_{3}\right] / \Delta$. Debt is not diluted, and no debt is rolled over, $b_{12}=0$.

### 5.1.3 Permanent Loss of Social Capital

Recursively, the government's program can be represented as

$$
\begin{aligned}
& G_{n}\left(B_{n}, \mathbf{1}_{\left[r_{n-1}=1\right]},\right.\left.\left\{b_{x n i}\right\}_{i \geq n}\right)= \\
& \max _{n} \in[0,1],\left\{b_{n i}\right\}_{i>n}
\end{aligned}-\Delta b_{x n} r_{n}+\mathbf{1}_{\left[r_{n-1}=1\right]} \mathbf{1}_{\left[r_{n}=1\right]} B_{n}+\beta \mathrm{E}\left[G_{n+1}\left(B_{n+1}, \mathbf{1}_{\left[r_{n}=1\right]},\left\{b_{x, n+1, j}\right\}_{j \geq n+1}\right)\right] .
$$

To be written.

### 5.2 Variable Cost, Short Horizon

To be written.

### 5.3 General Case

To be written.

## 6 Extensions

To be written.
How to rationalize rollover: news about new government spending. How to introduce default for "ability to pay" rather than "willingness to pay" reasons.

## 7 Conclusion

To be written.

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[^1]:    ${ }^{1}$ For example, Kohlscheen (2004) documents that parliamentary democracies rarely resort to rescheduling (despite shorter office terms of their executives), presumably because domestic constituencies opposed to default are more likely to be politically influential in representative democracies. MacDonald (2003) suggests that it is precisely in countries where a default does not generate clearly identifiable winners and losers (among politically influential groups) where sovereign defaults have been avoided.
    ${ }^{2}$ In the presence of tax distortions, the ex-ante optimal tax policy generally is not time consistent, due to pecuniary externalities generated by ex-post changes in tax rates (Lucas and Stokey, 1983). Krusell, Martin and Ríos-Rull (2004) characterize the time-consistent tax policy in Lucas and Stokey's (1983) model under the assumption that the government can commit to debt, but not tax rates.

[^2]:    ${ }^{3}$ See also Calvo and Obstfeld (1990), Alvarez, Kehoe and Neumeyer (2004) and Persson, Persson and Svensson (2005).

[^3]:    ${ }^{4}$ There is one caveat, however. In general, the government's default decision depends on the ownership structure of debt (relative to the distribution of tax burdens across the population), due to default-induced redistributive effects. Allowing for a mixed rather than concentrated ownership structure of debt therefore adds to the complexity of the model since it requires a theory of how the ownership structure is determined in equilibrium. No such (convincing) theory is currently available. Tabellini (1991) and Dixit and Londregan (2000) encounter similar problems. They address them by assuming that households can only save in the form of government debt (Tabellini, 1991), or that the return on the only alternative asset is household specific (Dixit and Londregan, 2000). Both assumptions are not applicable in the current context. See also Niepelt (2004a).
    ${ }^{5}$ The assumption that claims have a short maturity is not restrictive.

[^4]:    ${ }^{6}$ In particular, permanent (partial) financial autarky in the wake of a default has repeatedly been proposed as one potential cost, see Eaton and Gersovitz (1981), Bulow and Rogoff (1989), Grossman and Han (1999), Kletzer and Wright (2000), Alvarez and Jermann (2000) or Kehoe and Perri (2002), among many others.

[^5]:    ${ }^{7}$ Of course, we also have $\partial V_{0}(\cdot) / \partial t_{i}=0$ as investors pay no taxes.

[^6]:    ${ }^{8}$ Recall our assumption that $f(\cdot)>0$ to the right of the lower bound of the support of $B$.

[^7]:    ${ }^{9}$ The exponential distribution and some Weibull or Gamma distributions, among others, satisfy this regularity condition.
    ${ }^{10}$ Under the regularity condition stated before, this equation has a unique solution.

