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# Precautionary Balances and the Velocity of Circulation of Money

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**Abstract**

The observed low velocity of circulation of money implies that households hold more money than they normally spend. A natural explanation for this behavior is that households face uncertain expenditure needs, so they have a precautionary motive for holding money. We investigate the precautionary demand for money in a search model where households are subject to preference shocks. The model predicts that the velocity of circulation of money is not only low but also interest elastic. The model closely fits United States data on the velocity of circulation of money and interest rates that span the period 1892-2003. The empirical analysis reveals a dramatic reduction in precautionary balances towards the end of our sample, probably linked to innovations in the information technology. This drop in precautionary balances is crucial for important issues of monetary economics such as the elasticity of the demand for money and the welfare cost of inflation.

Keywords: Precautionary Balances, Velocity of Circulation of Money, Money Demand.
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# 1 Introduction

From 1892 to 2003, checkable deposits plus the currency in circulation inside the United States (M1*) was worth on average the GDP produced in 3 months.[1]  Since during this period almost all households received income at least once a month and quite often once in a fortnight or once a week, a large fraction of M1* must have remained unspent in the course of a pay period. Recently, the worth of M1* has fallen to be around 1 month of GDP. Therefore, the fraction of unspent M1* during a pay period, although probably still positive, must have dropped dramatically relative to historical levels. These observations raise the following two questions.

Why do households hold more money than they normally spend if other assets bear higher interest? A natural answer to this question is that households hold precautionary money balances in order to accommodate uncertain expenditure needs. For example, one never knows if the car will suddenly break down, or one will have to travel unexpectedly. Households typically hold precautionary balances in order to face these contingencies, even if holding money is costly.

Why have unspent money balances fallen so dramatically in recent years? A natural answer is that improvements in the information technology have made possible credit cards, telephone banking, internet banking, and low fees for rebalancing portfolios. Consequently, individuals are now able to face unexpected expenses without holding large precautionary balances.

Our paper shows how a simple search model with precautionary balances explains the evolution of the observed data on the demand for money in the United States and draws conclusions for substantive issues of monetary economics.

In this paper, we investigate the precautionary demand for money in a disaggregated

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[1]  M1* differs from the standard M1 by subtracting the currency in circulation outside the United States. See Section 2 for data sources.

model where households are subject to preference shocks. In earlier work, the precautionary demand for money was analyzed by Svensson (1985) in a representative agent model with a cash-in-advance constraint. As in our model, the agent experienced a preference shock after deciding the demand for money. In an influential paper, Hodrick, Kocherlakota, and Lucas (1991) dismissed the quantitative importance of this precautionary demand in the context of an aggregated model. In their numerical analysis, they found that once the purchases of the representative agent are calibrated to match the smooth aggregate consumption expenditures in the United States, the timing of the preference shock is quantitatively irrelevant. Yet, they acknowledged that "it is possible that a similar model that seriously treated the aggregation problem could produce reasonable velocity predictions." We address this issue by building a tractable yet fully disaggregated search model. This model, even in a steady state with constant aggregate consumption, generates large precautionary balances due to individual preference shocks.

We estimate our model using United States data from 1892-2003. Our model closely fits the velocity of circulation of M1* and its elasticity with respect to interest rates. Moreover, this empirical implementation reveals that the demand for precautionary balances has dramatically declined in recent years. This decline has profound consequences for most substantive issues of monetary economics. For example, it has reduced the elasticity of the demand for money, the seigniorage the Fed collects at a given rate of inflation, and the welfare cost of inflation. This last change is dramatic. For most of the past century, raising the nominal interest rate from 0 to 10 percent used to induce an equivalent reduction of consumption of around 1 percent. (This figure is similar to the estimates of Lucas, 2000). In contrast, the same increase today induces an equivalent reduction of consumption of only 0.15 percent.

Since the seminal work of Kiyotaki and Wright (1989), search models have become a common paradigm in the micro-foundations of money. In the earlier versions of these models, money was assumed indivisible to simplify the endogenous distribution of money holdings. Recent contributions built highly tractable models with divisible money by

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introducing devices which render the distribution of money holdings degenerate across individuals. Shi (1997) assumed that individuals belong to large symmetric households. More recently, Lagos and Wright (2005) assumed that individuals sometimes trade bilaterally and sometimes in a centralized market, and that the goods in the centralized market yield constant marginal utility. Our paper uses a framework related to these earlier contributions,[2] which proves useful to focus on precautionary money balances.

We assume that individuals belong to a large number of villages. Money is essential in facilitating trade across villages because the trading process is anonymous, enforcement is limited, and there is a double coincidence of wants problem. However, within a village, financial contracts  (credit and insurance) are viable because fellow villagers know each other. With this village construct we capture that individuals in modern societies sometimes deal with well know parties with whom financial contracts are viable, while other times they deal with relative strangers with whom future promises are impractical to enforce. The financial markets inside a village, by allowing individuals to rebalance their portfolios, render the distribution of money holdings tractable.[3]

Our model is in many ways different from previous search models of monetary exchange. One key difference is that we allow for preference shocks since, as we explain above, these shocks are the foundation for our precautionary demand for money. In addition, we abstract from the uncertainty of meeting a trading partner by assuming that matching is efficient (the short-side of the market is always served). Finally, we assume that the terms of trade are determined as a result of a competitive search process while most of the literature assumes Nash bargaining (see, however, Rocheteau and Wright (2005)).

The concept of competitive search has been widely used in labor economics since the work of Moen (1997) and Shimer (1996), and it is an attractive equilibrium concept for

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[2]   See Faig (2004) for a comparison of the three frameworks.

[3]   The role of financial markets inside a village replaces the role of the large household assumption in Shi (1997) and dispenses with the assumption that some goods yielding constant marginal utility are traded in a centralized market in Lagos and Wright (2005).

several reasons. First, competitive search is more tractable than Nash bargaining. Second, under competitive search the split of the trading surplus between a buyer and a seller is determined endogenously as a result of competition instead of being determined by an arbitrary bargaining weight. Finally, as shown by Rocheteau and Wright (2005), competitive search leads to efficiency under the Friedman rule, while in a search environment this is not generically true neither with Nash bargaining nor with Walrasian pricing.

The paper is organized as follows. Section 2 reviews historical data on the velocity of circulation of money in the United States and documents the facts the paper seeks to explain. Section 3 presents the theoretical model. Section 4 estimates the model using United States data. Section 5 concludes.

## 2  The  Data

Figure 1 displays the annual time series of a short term nominal interest rate and the velocity of circulation of money (ratio of nominal GDP over the quantity of money) in the United States from 1892 to 2003.[4] We plot the velocity for two measures of money: M1 and M1*. M1 is the standard aggregate reported by the Federal Reserve that includes currency and checkable deposits. M1* is M1 minus the currency reported by the Federal Reserve as being abroad. Measures of currency abroad are first reported for December 1964, when its importance was minimal. However, since that date currency abroad has grown to be 25 percent of M1 in 2003. Our analysis concentrates on the velocity of M1* because it is the most meaningful of our two measures, but we plot the velocity of M1 to facilitate the comparison with prior studies.

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[4]  The data plotted in this figure are similar to those analyzed by Lucas (2000) extended backward from 1900 to 1892 and forward from 1994 to 2003. See the Appendix for the sources.

Figure 1

## Velocity of M1 and Interest Rates in the United States
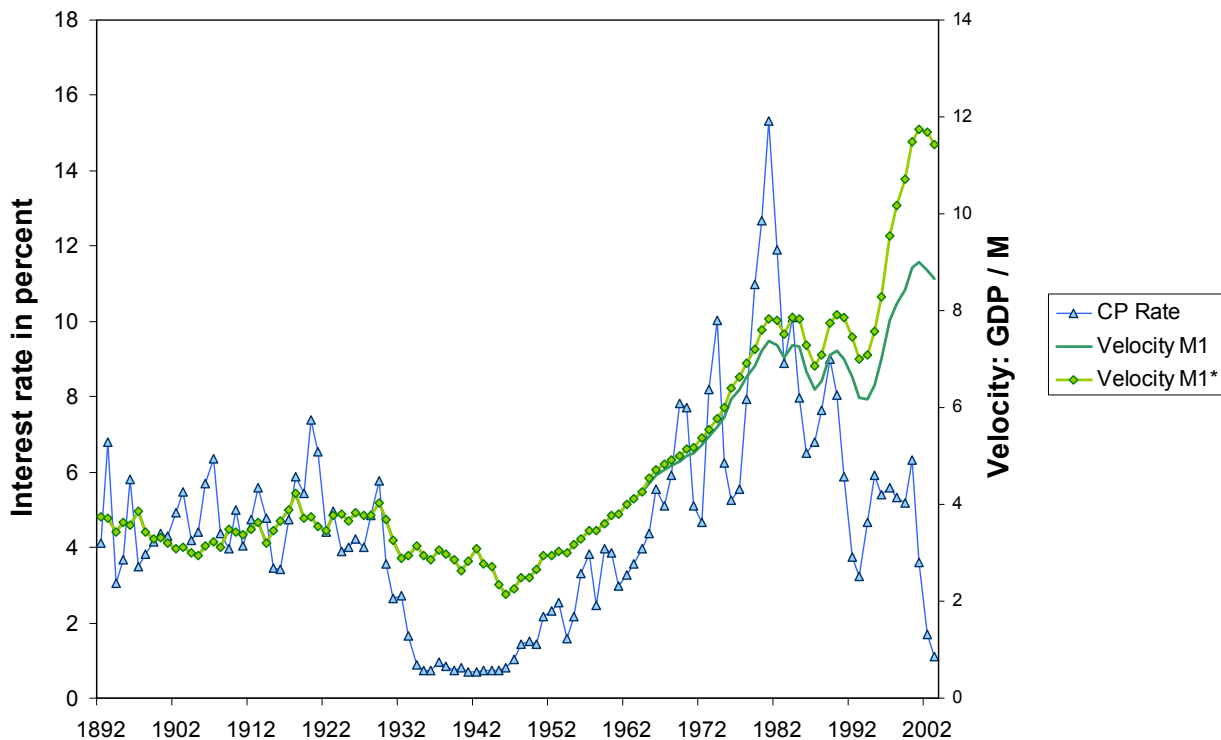


From 1892 to 2003, the velocity of circulation of money was on average low: M1*
circulated on average 4.8 times each year. Over time velocity has changed widely. Until
1946, velocity seldom reached 4, but since then it has experienced a marked upward trend.
This upward trend has accelerated in recent years, which is very informative about the
factors that may have driven the change. From 1946 to 1982, the upward trend in velocity
seems to be due to increases in the nominal rate of interest experienced during that period.
However, velocity has increased dramatically after 1982, at a time when nominal interest
rates have collapsed. Therefore, other elements must have played an important role in
the dynamics of velocity. In principle, an upward trend in velocity could be driven by
GDP growth if the transactions elasticity of the demand for money were lower than one.
However, this hypothesis does not fit the data well. GDP experienced long term growth

for the whole period of analysis, and actually it slowed down since the early seventies. In contrast, we observe in Figure 1 that the upward trend on velocity accelerated precisely at the time GDP growth experienced the slowdown. A much better explanation is that the upward trend in velocity is due to the revolution in the information technology, which in the last three decades has radically reduced the costs of communication and record keeping.

Besides an upward trend in velocity, Figure 1 also shows a positive correlation between velocity and interest rates. Since Lucas and Stokey (1987), a common interpretation of this correlation is that cash must be held for a while prior to the purchase of certain goods (cash goods), while this cash-in-advance constraint does not apply for other goods (credit goods). Variations in nominal interest rates affect the relative cost of these two kinds of goods, thus affecting their demands and the demand for money. While this approach allows to successfully fit the demand for M1 in the United States (see Lagos and Wright, 2005, for an empirical implementation in a search-theoretic context), it unfortunately implies counterfactual predictions such as an excessively long pay period, very infrequent consumption purchases, or a drastic reduction on the demand for cash goods due to inflation. Our model, without relying on the distinction between cash and credit goods, explains the low velocity of money and its correlation with interest rates with a reasonable pay period during which consumers always make purchases.

## 3 The Model

The economy consists of a measure one of individuals. Individuals live in a large number of symmetric villages. The members of each village are ex ante identical. They all produce a perishable good specific to their own village and consume the goods produced in all other villages. Consequently, individuals must trade outside their village to consume.

Time is a discrete, infinite sequence of days. Each morning an individual must choose to be either a buyer or a seller in the goods market that convenes later in the day. Within a village some individuals will be buyers and others will be sellers each day. However, over

time individuals will alternate between these two roles.

Individuals seek to maximize their expected lifetime utility:

$$E \sum_{t=0}^{\infty} \beta^t \mathcal{U} \left( \varepsilon, q_t^b, q_t^s \right), \tag{1}$$

where

$$\mathcal{U}(\varepsilon, q^b, q^s) = \varepsilon U(q^b) - C(q^s) \tag{2}$$

is the one-period utility function and $\beta \in (0,1)$ is the discount factor. The one-period utility depends on the quantity consumed $q^b$ if the individual chooses to be a buyer during the period, and on the quantity produced $q^s$ if the choice is to be a seller. It also depends on an idiosyncratic preference shock $\varepsilon$ which affects the utility of consumption $\varepsilon U(q^b)$, but does not affect the disutility of production $C(q^s)$. The preference shock is distributed in the interval $[1, \bar{\varepsilon}]$ with a cumulative distribution $F(\varepsilon)$, and drawn in such a way that the Law of Large Numbers holds across individuals. Both $U$ and $C$ are continuously differentiable and increasing. Also, $U$ is strictly concave and $C$ is convex, with $U(0) = C(0) = 0$, and $U'(0) = \infty$. Finally, there is a maximum quantity $q^{\max}$ that the individual can produce each day which satisfies $\bar{\varepsilon} U(q^{\max}) \leq C(q^{\max})$.

Money is an intrinsically useless, perfectly divisible, and storable asset. Units of money are called dollars. The supply of money grows at a constant factor $\gamma > \beta$, so

$$M_{+1} = \gamma M, \tag{3}$$

where $M$ is the quantity of money per individual.[5] Each day new money is injected via a lump-sum transfer $\tau$ common to all individuals. For money to grow at the rate $\gamma$, this transfer must satisfy:

$$\tau = (\gamma - 1) M. \tag{4}$$

Each day, goods are traded in a decentralized market where buyers and sellers from different villages meet bilaterally. In this market, buyers and sellers search for trading

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[5] The subscript $t$ is omitted in most expressions, i.e. $M$ stands for $M_t$ and $M_{+1}$ stands for $M_{t+1}$.

opportunities and the terms of trade are determined by a competitive search process (as in Moen (1997) and Shimer (1996)). Before the market opens, each seller simultaneously posts a trading offer, which is a contract detailing the terms at which the seller commits to trade. Once the market opens, buyers direct their search towards the sellers posting the most attractive offer for them (possibly randomizing over offers for which they are indifferent). The set of sellers posting the same offer and the set of buyers directing their search towards them form a submarket. In each submarket, buyers and sellers from different villages are then matched in bilateral pairs.

We specialize the matching process to focus on the precautionary demand for money as follows. We assume that individuals experience only one match with an individual from another village and the short-side of the market is always served. That is, the probability that a buyer meets a trading partner in a submarket is

$$\pi^b(\alpha) = \min(1, \alpha), \tag{5}$$

where $\alpha$ is the ratio of sellers over buyers in that submarket. Similarly, the probability that a seller meets a trading partner is

$$\pi^s(\alpha) = \min(1, \alpha^{-1}). \tag{6}$$

As we shall show, this matching technology implies that in equilibrium $\alpha = \pi^b = \pi^s = 1$ in all active submarkets. Therefore, individuals know that they will find a trading partner each day. However, they do not know who this trading partner is going to be. When a buyer and a seller meet in a submarket they trade according to the posted offer that characterizes the submarket.

Outside their village, individuals are anonymous. This anonymity combined with the absence a double coincidence of wants (implied by the ex-ante choice of trading roles) makes money essential in the goods market. In other words, financial contracts engaging individuals from different villages are not enforceable. However, inside a village financial contracts are enforceable. In particular, in each village there is a centralized credit market

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where a one-period risk-free bond is traded. There is also a centralized insurance market where individuals can buy insurance against their idiosyncratic risks. As it will become apparent, these two centralized markets exhaust the gains from trade inside a village.

A typical day proceeds as follows (see Table 1). In the morning, centralized financial markets are open in each village. During this time, financial contracts from the previous day are settled. The government hands our monetary transfers that increase the money supply. Individuals decide whether to be buyers or sellers. Sellers post their trading offers. Then, all individuals adjust their holdings of bonds and money, and purchase insurance if they wish. At noon, once financial markets have closed, buyers experience an idiosyncratic preference shock that determines their willingness to pay for goods and is publicly observed. In the afternoon, the goods market is open. Buyers direct their search to the sellers posting the most attractive offer for them. This organizes traders in submarkets where the short side of the market is always served. When a buyer and a seller meet in a submarket they trade according to the specified offer. As a result of trade, sellers produce, buyers consume, and money changes hands from buyers to sellers.

Table 1

| MORNING | | | NOON | AFTERNOON | | |
|---------|---|---|------|-----------|---|---|
| Financial markets are open | | | | Goods market is open | | |
| Financial claims are settled. | Choice buyer-seller. Post trading offers. | Choice of bonds, money, insurance. | Realization preference shocks. | Buyers pick among trading offers. | Buyers-sellers are matched. | Trade takes place. |

Our equilibrium concept combines perfect competition in all centralized financial markets with competitive search in the decentralized goods market. In equilibrium, individuals make optimal choices in the environment where they live. This environment includes a sequence of nominal interest rates and insurance premia, and a sequence of conditions in the goods market to be detailed below (essentially the reservation surpluses of other traders).

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Individuals have rational expectations about the future conditions of this environment. We focus on symmetric and stationary equilibria where all individuals follow identical strategies and real allocations are constant over time.

To characterize an equilibrium, we adopt the following strategy. First, we describe the buyer-seller choice and the financial decisions of a representative individual given the equilibrium nominal interest rates and insurance premia, as well as some conjectures about the conditions in the afternoon goods market. Then, we characterize the conditions in the goods market in a competitive search equilibrium. Finally, we show that these conditions satisfy our former conjecture, and we provide a formal definition of an equilibrium.

## 3.1  The Buyer-Seller Choice and the Financial Decisions

Consider an individual facing the following environment:

In the credit market, the equilibrium nominal interest rate is:

$$i = \frac{\gamma - \beta}{\beta}, \tag{7}$$

where $\gamma$ is the growth factor of the money supply and $\beta$ is the subjective discount factor. Since good prices are proportional to $M$, which grows at the factor $\gamma$, the real interest rate is then equal to the subjective discount rate: $\beta^{-1} - 1$.

In the insurance market, the equilibrium insurance premia are actuarially fair. More specifically, an individual that decides to be a buyer can purchase an insurance contract which delivers $\mu_\varepsilon^b$ dollars next day contingent on experiencing a shock $\varepsilon$ at noon. The premium $\tilde{\mu}^b$ of such a contract is $\tilde{\mu}^b = \int_1^{\bar{\varepsilon}} \mu_\varepsilon^b dF(\varepsilon)$. In our environment, there is no need for insuring risks on trading opportunities because such risks vanish in equilibrium (all individuals that search find a trading partner with probability one).

The afternoon goods market consists of a continuum of submarkets, one for each buyer type $\varepsilon$. In each submarket, the terms of trade are identical in all bilateral meetings. The submarket of type $\varepsilon$ is characterized by a triple $(q_\varepsilon, d_\varepsilon, \alpha_\varepsilon)$ where $q_\varepsilon$ is the quantity traded, $d_\varepsilon$ is the buyer's total payment in dollars, and $\alpha_\varepsilon$ is the ratio of buyers over sellers. This

description allows for the possibility that some submarkets are inactive with $q_\varepsilon = d_\varepsilon = 0$, so the corresponding buyer types do not trade. Since the payments $d_\varepsilon$ change over time as the money supply grows, it is convenient to also characterize the submarket by a triple $(q_\varepsilon, z_\varepsilon, \alpha_\varepsilon)$ where $z_\varepsilon$ obeys:

$$z_\varepsilon = \frac{\beta d_\varepsilon}{M_{+1}}. \tag{8}$$

In a stationary equilibrium the triples $(q_\varepsilon, z_\varepsilon, \alpha_\varepsilon)$ are time invariant. The variable $z_\varepsilon$ can be interpreted as real payments in utils. The money supply is used to deflate nominal quantities. This deflator is appropriate in the environment the individual faces because goods prices increase proportionately with $M$ (see (3) and (4)). The discount factor $\beta$ and $M_{+1}$ appears in (8) because the payment $d_\varepsilon$ cannot be spent until next period.

Prior to all financial choices, each morning the individual chooses the trading role that yields maximal utility. The value function $V$ of the individual at the beginning of a day then obeys:

$$V\left(\frac{A}{M}\right) = \max\left\{V^b\left(\frac{A}{M}\right), V^s\left(\frac{A}{M}\right)\right\}; \tag{9}$$

where $A$ is the initial wealth in dollars, and $V^b$ and $V^s$ are the value functions conditional on being a buyer or a seller during the day, respectively. The ratio $A/M$ can be interpreted as initial real wealth and is denoted by $a$.

While financial markets are open, the individual reallocates wealth and may also purchase insurance. Conditional on being a buyer the individual chooses the demands for money, $m^b$, bonds, $b^b$, and the insurance coverages, $\left\{\mu_\varepsilon^b\right\}_{\varepsilon \in [1,\bar{\varepsilon}]}$, to solve:

$$V^b(a) = \max \int_1^{\bar{\varepsilon}} \left\{\pi^b(\alpha_\varepsilon)\left[\varepsilon U(q_\varepsilon) + \beta V\left(a_{+1}^{b\varepsilon}\right)\right] + \left[1 - \pi^b(\alpha_\varepsilon)\right]\beta V\left(a_{+1}^{b0}\right)\right\} dF(\varepsilon) \tag{10}$$

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subject to

$$a_{+1}^{b\varepsilon} = \frac{m^b + b^b(1+i) + \mu_\varepsilon^b - \tilde{\mu}^b + \tau - d_\varepsilon}{M_{+1}}, \tag{11}$$

$$a_{+1}^{b0} = \frac{m^b + b^b(1+i) - \tilde{\mu}^b + \tau}{M_{+1}} \tag{12}$$

$$a = \frac{m^b + b^b}{M}, \text{ and} \tag{13}$$

$$m^b \geq d_\varepsilon \text{ for all } \varepsilon \in [1, \bar{\varepsilon}]. \tag{14}$$

Contingent on the realization of the shock $\varepsilon$, the buyer either meets a seller and buys $q_\varepsilon$ for $d_\varepsilon$ dollars or does not meet a seller and purchases nothing. The probabilities of these two events are $\pi^b(\alpha_\varepsilon)$ and $1 - \pi^b(\alpha_\varepsilon)$, respectively. If the buyer meets a seller, next period's real wealth $a_{+1}^{b\varepsilon}$ is given by (11). If the buyer does not meet a seller, next period's real wealth $a_{+1}^{b0}$ is given by (12). The choice of how to allocate wealth between money $m^b$ and bonds $b^b$ must satisfy the budget constraint (13). The buyer must also carry enough money to make each possible contingent payment, so $m^b$ must satisfy (14).

Analogously, conditional on being a seller the individual chooses the demands for money $m^s$ and bonds $b^s$ to solve:

$$V^s(a) = \max \pi^s(\alpha_\varepsilon)\left[\beta V\left(a_{+1}^{s\varepsilon}\right) - C(q_\varepsilon)\right] + [1 - \pi^s(\alpha_\varepsilon)]\beta V\left(a_{+1}^{s0}\right) \tag{15}$$

subject to

$$a_{+1}^{s\varepsilon} = \frac{m^s + b^s(1+i) + \tau + d_\varepsilon}{M_{+1}}, \tag{16}$$

$$a_{+1}^{s0} = \frac{m^s + b^s(1+i) + \tau}{M_{+1}}, \tag{17}$$

$$a = \frac{m^s + b^s}{M}, \text{ and} \tag{18}$$

$$m^s \geq 0. \tag{19}$$

for all $(q_\varepsilon, d_\varepsilon, \alpha_\varepsilon)$ where $q_\varepsilon > 0$. Since the seller gets the same utility in all active submarkets, $V^s$ is not indexed by $\varepsilon$. The seller does not need to carry money to make payments, but money holdings cannot be negative as stated in (19).

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In addition to all constraints specified above, the individual faces an endogenous lower bound on next period's real wealth because he or she must be able to repay the amounts borrowed with probability one without reliance to unbounded borrowing (No-Ponzi game condition):

$$a_{+1} \geq a_{\min} \text{ with probability one.} \tag{20}$$

We denote by $a_{+1}$ the stochastic real wealth for next period, which depends on the choice of being a buyer or a seller, the realization of $\varepsilon$, and the trading match. The endogenous lower bound $a_{\min}$ is equal to minus the present discounted value of the maximum guaranteed income the individual can obtain as a seller.

The optimization program described in equations (9) to (20) is easily solved once the value function $V$ is known. The value function $V$ is a well defined function of $a$ that can be characterized using standard recursive methods. Also, $V$ is concave with a linear segment as stated in the following proposition and proved in the Appendix.

**Proposition 1:** *There is an interval $[\underline{a}, \overline{a}] \subset [a_{\min}, \infty)$ where the equilibrium value function $V$ takes the linear form*

$$V(a) = v_0 + a. \tag{21}$$

*where $v_0$ is a term independent of $a$. Outside this interval, $V$ is strictly concave and continuously differentiable. Finally, the interval $[\underline{a}, \overline{a}]$ is absorbing, that is $a \in [\underline{a}, \overline{a}]$ implies $a_{+1} \in [\underline{a}, \overline{a}]$ with probability one.*

The linear segment of $V$ is due to the endogenous choice of the trading role individuals make each day. Intuitively, if an individual is not rich enough to be a buyer forever and not so poor to have to be a seller at perpetuity, then the individual will alternate between being a buyer and a seller. As the individual does so, wealth does not affect the quantities consumed or produced, instead it affects how often and how early the individual consumes or produces. Since utility is linear on the times and the timing an individual consumes and produces, the value function is linear.

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The property that the interval $[\underline{a}, \overline{a}]$ is absorbing simplifies the model dramatically. As long as all individuals have initial wealth in the interval $[\underline{a}, \overline{a}]$, as we assume from now on, the behavior of all buyers and all sellers is independent from their wealth. Therefore, there is no incentive to create submarkets that cater to individuals of different wealth and the distributions of money holdings are easily characterized.

The optimal demands for money follow from the fact that money earns not interest but bonds earn $i > 0$. This implies that it is not optimal to carry money balances that are never used. Therefore, $m^b$ is equal to the highest contingent payment: $m^b = \max\{d_\varepsilon\}_{\varepsilon \in [1, \bar{\varepsilon}]}$ and $m^s = 0$. Using these optimal demands for money, (21), and $a_{+1} \in [\underline{a}, \overline{a}]$ with probability one, the value functions of the buyer (10) and the seller (15) simplify into:

$$V^b(a) = S^b + \beta\left(v_0 + \frac{\gamma - 1}{\gamma}\right) + a, \text{ and} \tag{22}$$

$$V^s(a) = S^s + \beta\left(v_0 + \frac{\gamma - 1}{\gamma}\right) + a. \tag{23}$$

These value functions differ only on the expected trading surpluses of buyers and sellers in the goods market. Namely,

$$S^b = \int_1^{\bar{\varepsilon}} \pi^b(\alpha_\varepsilon)\left[\varepsilon U(q_\varepsilon) - z_\varepsilon\right] dF(\varepsilon) - im, \text{ and} \tag{24}$$

$$S^s = \pi^s(\alpha_\varepsilon)\left[z_\varepsilon - C(q_\varepsilon)\right]. \tag{25}$$

In (24), we define $m$ to be the real money in next day utils: $m \equiv \beta m^b / M_{+1}$. Since buyers carry only enough money to make the highest contingent payment, we have

$$m = \max\{z_\varepsilon\}_{\varepsilon \in [1, \bar{\varepsilon}]}. \tag{26}$$

Note that the insurance coverages are missing from (24). As long as $a_{+1} \in [\underline{a}, \overline{a}]$ with probability one, an individual is risk neutral and thus indifferent between purchasing insurance or not. The only role played by insurance in this model is to ensure that wealth

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does not drift out of the interval $[\underline{a}, \overline{a}]$. This role is only important if individuals fail to trade along the equilibrium path with positive probability. With efficient matching, insurance is redundant if the interval of preference shocks is sufficiently narrow, so buyers purchase positive amounts for all realizations of $\varepsilon$. In this case, the individual prevents $a_{+1}$ from drifting below $\underline{a}$ by choosing to be a seller and prevents it from drifting above $\overline{a}$ by choosing to be a buyer.

## 3.2 Competitive Search Equilibrium

In this section we characterize the equilibrium conditions for the goods market and define a symmetric monetary stationary equilibrium. We assume that all individuals have initial wealth in the interval $[\underline{a}, \overline{a}]$, so the trading surpluses of buyers and sellers are given by (24) and (25).

In the morning, when individuals can still rebalance their portfolios, sellers post their trading offers in all submarkets where they wish to participate. A trading offer is a pair $(q_\varepsilon, z_\varepsilon)$ that specifies the output offered to and the payment demanded from a buyer of type $\varepsilon$.[6] All individuals have rational expectations regarding the number of buyers that will be attracted by each offer, and thus about the relative proportion of buyers and sellers that will trade in each submarket. In a competitive search equilibrium the offers posted by the sellers must be such that no seller has incentives to post a deviating offer.

Let $\Omega_\varepsilon$ be the set of vectors $(q_\varepsilon, z_\varepsilon, \alpha_\varepsilon)$ characterizing the submarkets where the buyers of type $\varepsilon$ choose to trade in equilibrium. If buyers of type $\varepsilon$ choose not to trade at all, then $\Omega_\varepsilon$ is a singleton with $q_\varepsilon = z_\varepsilon = 0$ (there is a single inactive submarket for this type). The set of all submarkets that are formed in equilibrium is then $\Omega = \Pi_{\varepsilon \in [1, \bar{\varepsilon}]} \Omega_\varepsilon$.

A competitive search equilibrium is a set $\{\Omega, \bar{S}^b, \bar{S}^s\}$ such that the following four

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[6]   We could allow for offers which are contingent both on the type $\varepsilon$ and the wealth $a$ of the buyer. Since the buyer's expected surplus (24) and money balances (26) are independent of $a$, from the sellers' view point all buyers of a given type $\varepsilon$ are identical even if their wealth is different. Hence, restricting to offers which are only contingent on $\varepsilon$ is without loss of generality.

conditions hold:

1. Sellers attain the same expected surplus $\bar{S}^s$ in all active submarkets.

2. Buyers attain the same expected surplus $\bar{S}^b$.

3. The expected surpluses of buyers and sellers are identical provided at least one submarket is active: $\bar{S}^b = \bar{S}^s$

4. Each $\omega \in \Omega$ solves the following program:

$$\bar{S}^b = \max_{\{(q_\varepsilon, z_\varepsilon, \alpha_\varepsilon)\}_{\varepsilon \in [1,\bar{\varepsilon}]}} \int_1^{\bar{\varepsilon}} \left\{ \pi^b(\alpha_\varepsilon)\left[\varepsilon U(q_\varepsilon) - z_\varepsilon\right] \right\} dF(\varepsilon) - im \tag{27}$$

subject to

$$m = \max \{z_\varepsilon\}_{\varepsilon \in [1,\bar{\varepsilon}]}, \tag{28}$$

$$\varepsilon U(q_\varepsilon) - z_\varepsilon \geq 0 \ \text{ for } \varepsilon \in [1,\bar{\varepsilon}], \text{ and} \tag{29}$$

$$\pi^s(\alpha_\varepsilon)[z_\varepsilon - C(q_\varepsilon)] = \bar{S}^s \text{ for all active } \varepsilon \in [1,\bar{\varepsilon}]. \tag{30}$$

The rationale for these conditions is the following. (1) Sellers are free to choose the submarket where they participate, so they must attain the same expected surplus in all active submarkets no matter the type of buyer they trade with. (2) Conditional on the realization of $\varepsilon$, buyers are also free to trade in any submarket in $\Omega_\varepsilon$, so they must attain the same conditional expected surplus in any of these submarkets. Since the distribution of $\varepsilon$ is identical for all buyers, all buyers must attain the same ex ante expected surplus. (3) An equilibrium with at least one active submarket must have both buyers and sellers present in that submarket, so individuals must be indifferent between being buyers or sellers. (4) Buyers choose among submarkets in order to maximize their expected surplus (27) subject to three constraints. Constraint (28) says that the buyer must be able to pay for the good in each submarket. Constraint (29) says that the buyer's utility must be non-negative in each submarket. Constraint (30) says that the buyer chooses among offers which give the seller a fixed surplus $\bar{S}^s$. Clearly, sellers never post deviating offers that imply a lower

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expected surplus than $\bar{S}^s$ because they can attain $\bar{S}^s$ an any active submarket.[7] If a seller tries to post an offer that attracts buyers and yields a higher expected surplus than $\bar{S}^s$, other sellers would profitably undercut this offer (e.g. by offering the same quantity for a slightly lower payment). Finally, a seller cannot create a deviating submarket that attracts several buyer types with cross-subsidies because other sellers would try to attract the type paying the subsidy with a more attractive offer.

The solution to program (27) to (30) must maximize the total expected surplus from a match subject to the cash constraint and the individual rationality constraints. Therefore, in any active submarket buyers and sellers must trade with probability one:

$$\alpha_\varepsilon = \pi^b (\alpha_\varepsilon) = \pi^s (\alpha_\varepsilon) = 1. \tag{31}$$

Some buyer types may not trade because there is no active submarket serving them. This is the case if the total surplus is lower than $\bar{S}^s$, for then no seller posts an offer targeting these types:

$$q_\varepsilon = z_\varepsilon = 0 \text{ if } \varepsilon U(q) - C(q) \leq \bar{S}^s \text{ for all feasible } q. \tag{32}$$

Using (31) and (32), solving for $z_\varepsilon$ in (30), and restating (28), program (27) to (30) simplifies to:

$$\bar{S}^b = \max_{m, \{q_\varepsilon\}_{\varepsilon \in [1,\bar{\varepsilon}]}} \int_1^{\bar{\varepsilon}} \max\{\varepsilon U(q_\varepsilon) - \bar{S}^s - C(q_\varepsilon), 0\} dF(\varepsilon) - im \tag{33}$$

subject to

$$\bar{S}^s + C(q_\varepsilon) \leq m \text{ for } \varepsilon \in [1, \bar{\varepsilon}], \text{ and} \tag{34}$$

$$\varepsilon U(q_\varepsilon) - C(q_\varepsilon) \geq \bar{S}^s \text{ if } q_\varepsilon > 0. \tag{35}$$

When (34) and (35) do not bind, the first order condition with respect to $q_\varepsilon$ is

$$\varepsilon U'(q_\varepsilon) = C'(q_\varepsilon). \tag{36}$$

---

[7]   Since individuals are infinitesimal in the market, they take as given the expected surplus of other individuals.

The output $q_\varepsilon$ that solves (36) is unique and increasing with $\varepsilon$ given the convexity of $C$ and concavity of $U$. Hence, either the cash constraint (34) is never binding or it binds in an interval $[\hat\varepsilon, \bar\varepsilon]$. Similarly, either the individual rationality constraint (35) is never binding or it binds for an interval $[1, \varepsilon_0]$. Therefore, the quantities of output that solve program (33) to (35) obey:

$$
\begin{cases}
q_\varepsilon = 0 & \text{for } \varepsilon \in [1, \varepsilon_0] \quad \text{if } \varepsilon_0 > 1, \\
\varepsilon U'(q_\varepsilon) = C'(q_\varepsilon) & \text{for } \varepsilon \in (\varepsilon_0, \hat\varepsilon], \qquad \text{and} \\
q_\varepsilon = q_{\hat\varepsilon} \equiv \hat q & \text{for } \varepsilon \in [\hat\varepsilon, \bar\varepsilon].
\end{cases}
\tag{37}
$$

Furthermore, the buyers' real money balances are given by

$$
m = \bar S^s + C(\hat q).
\tag{38}
$$

The break-point for zero output $\varepsilon_0$ is characterized by combining (35) with equality and (37). The break-point for a binding cash constraint $\hat\varepsilon$ is obtained from the first order condition of program (33) to (35) with respect to $m$:

$$
i = \int_1^{\bar\varepsilon} \delta_\varepsilon d\varepsilon,
\tag{39}
$$

where $\delta_\varepsilon$ is the Lagrange multiplier of (34). The Kuhn-Tucker Theorem implies

$$
\begin{cases}
\delta_\varepsilon = 0 & \text{for } \varepsilon \in [1, \hat\varepsilon], \quad \text{and} \\
\delta_\varepsilon C'(\hat q) = [\varepsilon U'(\hat q) - C'(\hat q)] \, dF(\varepsilon) & \text{for } \varepsilon \in [\hat\varepsilon, \bar\varepsilon].
\end{cases}
\tag{40}
$$

Using (37) and (40) to solve the integral in (39), we obtain the break-point $\hat\varepsilon$ as an implicit function of $i$:

$$
i = \int_{\hat\varepsilon}^{\bar\varepsilon} \left( \frac{\varepsilon}{\hat\varepsilon} - 1 \right) dF(\varepsilon).
\tag{41}
$$

To complete the characterization of a competitive search equilibrium, it remains is to determine $\bar S^s$. Since the expected surplus is the same for buyers and sellers, $\bar S^s$ is given by

$$
\int_{\varepsilon_0}^{\bar\varepsilon} \left[ \varepsilon U(q_\varepsilon) - \bar S^s - C(q_\varepsilon) \right] dF(\varepsilon) - i \left[ \bar S^s + C(\hat q) \right] = \bar S^s.
\tag{42}
$$

We are ready for a formal definition of equilibrium. A **symmetric monetary**

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**stationary equilibrium** is a vector of real numbers $\left(i, \varepsilon_0, \hat{\varepsilon}, m, \bar{S}^s\right)$ and a set of real functions $\{(\alpha_\varepsilon, q_\varepsilon, z_\varepsilon)\}_{\varepsilon \in [1, \bar{\varepsilon}]}$ that satisfy the system of equations: (7), (30), (31), (32), (37), (38), (41), and (42). This equilibrium is consistent with the environment conjectured in Subsection 3.1. The credit market clears because individuals have a perfectly elastic net demand for bonds at the interest rate (7). The insurance market clears by the way the fair premia have been defined. These financial markets exhaust the gains for trading financial securities in the morning because all individuals have identical marginal rates of substitution in their margins of choice. Finally, since the solution to program (27) to (30) is unique, there at most one active submarket for each type $\varepsilon$.

An interesting property of the equilibrium is that it can be implemented if sellers post a trading offer which consists of a simple price schedule:

$$Z(q) = \bar{S}^s + C(q), \tag{43}$$

together with the promise to sell any quantity $q$ for $Z(q)$ utils $\left(Z(q)\beta^{-1}M_{+1}\right.$ dollars). The price schedule (43) has two tiers. The first tier is a flat amount that covers the seller's expected surplus. The second tier is a variable amount that covers the production cost of output. One can easily check that a buyer facing (43) chooses a quantity of output consistent with (37) and a quantity of money consistent with (41).

Interestingly, the price schedule in (43) is independent of the buyer's type. This is important because it means that all buyers face the same prices. Hence, the equilibrium can be implemented even if types can not be publicly observed because buyers have no incentives to lie about their type.

The welfare effects of inflation are captured by equations (32), (37), (38) and (41), together with the equation that determines the equilibrium nominal interest rate (7). At the Friedman rule $(\gamma \downarrow \beta)$, the opportunity cost of holding money vanishes since $i \downarrow 0$. Buyers then hold enough money to avoid being liquidity constrained in all contingencies: $\hat{\varepsilon} = \bar{\varepsilon}$. In this instance, in any active submarket $q_\varepsilon$ is efficient since the marginal utility of

consumption is equal to the marginal disutility of production.[8] Therefore, the equilibrium is efficient at the Friedman rule. For $\gamma > \beta$, the opportunity cost of holding money is positive since $i > 0$. Buyers then react by reducing their money balances relative to the Friedman rule and they are liquidity constrained for high realizations of the preference shock: $\hat{\varepsilon} < \bar{\varepsilon}$. When this happens, the output traded is below the efficient level of output, and the marginal utility of consumption for liquidity constrained individuals is above the marginal cost of production. This is the source of the welfare cost of inflation in this model.

The interest elasticities of the demand for money and the velocity of circulation of money are implicitly determined by equations (41) and (38). As $i$ increases, $\hat{\varepsilon}$ falls so buyers reduce their real money balances and spend on average a larger fraction of them. As a result, money circulates faster. There are two effects of $i$ on $m$. Buyers carry enough money to pay for $\bar{S}^s + C(\hat{q})$. If $i$ increases, $\hat{q}$ falls and so $m$ falls. Furthermore, the total expected surplus decreases as a result of inflation, so the seller's surplus $\bar{S}^s$ falls implying a further reduction in $m$.

## 3.3 Extension

As seen in Section 2, the time series of velocity in the United States displays an upward trend which we argued was likely due to advances in the information technology. We do not view these advances as having eliminated M1* as the main media of exchange, but as allowing conversions in and out of M1* assets more easily and speedily. Thus individuals can now face unexpected expenses without holding large precautionary balances. To incorporate these technological advances in our model, we assume that a fraction of individuals in each village are able to communicate with other fellow villagers early in the afternoon, so they can rebalance their portfolio after they know their preference shock but prior meeting a seller.[9] Consequently, there is a fraction $\theta$ of individuals that experience the preference

---

[8]  As noted above, a market has no active submarket if the total expected surplus is lower than the opportunity cost of sellers $\bar{S}^s$. This is also an efficiency condition.

[9]  See Berentsen, Camera, and Waller (2004) for a model that views banks as institutions that facilitate

shock after deciding the demand for money, while the rest experience the preference shock prior to this decision. For tractability, we assume that all individuals have the same ex ante probability of a particular timing. However, over time, as technology advances, the fraction $\theta$ of individuals that need precautionary balances declines.

The analysis of this extended model is analogous to the one in the previous sections. A competitive search equilibrium must still satisfy conditions 1 to 3. Condition 4 is now that each $\omega \in \Omega$ solves the following program:

$$
\begin{aligned}
\bar{S}^b &= \max_{\{(\alpha_\varepsilon, q_\varepsilon, z_\varepsilon), (\alpha_\varepsilon^*, q_\varepsilon^*, z_\varepsilon^*, m_\varepsilon^*)\}_{\varepsilon \in [1, \bar{\varepsilon}]}} \theta \left( \int_1^{\bar{\varepsilon}} \left\{ \pi^b (\alpha_\varepsilon) \left[ \varepsilon U(q_\varepsilon) - z_\varepsilon \right] \right\} dF(\varepsilon) - im \right) \\
&\quad + (1 - \theta) \left( \int_1^{\bar{\varepsilon}} \left\{ \pi^b (\alpha_\varepsilon^*) \left[ \varepsilon U(q_\varepsilon^*) - z_\varepsilon^* \right] \right\} dF(\varepsilon) - im_\varepsilon^* \right)
\end{aligned}
\tag{44}
$$

subject to

$$
m = \max \{z_\varepsilon\}_{\varepsilon \in [1, \bar{\varepsilon}]} \quad \text{and} \quad m_\varepsilon^* = z_\varepsilon \text{ for all } \varepsilon \in [1, \bar{\varepsilon}],
\tag{45}
$$

$$
\varepsilon U(q_\varepsilon) - z_\varepsilon \geq 0 \quad \text{and} \quad \varepsilon U(q_\varepsilon^*) - z_\varepsilon^* \geq 0 \text{ for } \varepsilon \in [1, \bar{\varepsilon}], \quad \text{and}
\tag{46}
$$

$$
\pi^s (\alpha_\varepsilon) \left[ z_\varepsilon - C(q_\varepsilon) \right] = \bar{S}^s \quad \text{and} \quad \pi^s (\alpha_\varepsilon^*) \left[ z_\varepsilon^* - C(q_\varepsilon^*) \right] = \bar{S}^s \quad \text{for all active } \varepsilon \in [1, \bar{\varepsilon}].
\tag{47}
$$

For the individuals that carry precautionary balances, the conditions for the optimality of $\{(\alpha_\varepsilon, q_\varepsilon, z_\varepsilon)\}_{\varepsilon \in [1, \bar{\varepsilon}]}$ are the same as in the previous subsection. For the individuals that carry only the money they know they are going go spend, the conditions of optimality of $\{(\alpha_\varepsilon^*, q_\varepsilon^*, z_\varepsilon^*, , m_\varepsilon^*)\}_{\varepsilon \in [1, \bar{\varepsilon}]}$ are:

$$
m_\varepsilon^* = z_\varepsilon^* = \bar{S}^s + C(q_\varepsilon^*), \quad \text{and}
\tag{48}
$$

$$
\begin{cases}
\alpha_\varepsilon^* = 0 & q_\varepsilon^* = 0 & \text{if } \varepsilon U(q) - C(q)(1+i) \leq \bar{S}^s(1+i) \text{ for all } q \\
\alpha_\varepsilon^* = 1 & \varepsilon U'(q_\varepsilon^*) = C'(q_\varepsilon^*)(1+i) & \text{otherwise.}
\end{cases}
\tag{49}
$$

---

this type of arrangements.

Buyers that know their preference shock before deciding the demand for money carry only the money they know they are going to spend. The real value of this money is equal to the cost of producing the goods to be purchased plus the cost of selling them. Efficient matching implies the ratio of buyers over sellers is one in all active submarkets. A submarket is active if the trading surplus in this submarket can cover the cost of selling goods. Finally, the marginal utility of consumption is equal to the marginal cost of acquiring goods, which includes the cost of carrying money, in all active submarkets.

In this extended model, the equality between the expected trading surpluses of buyers and seller is given by

$$
\begin{aligned}
\bar{S}^s \ = \ & \theta \int_{\varepsilon_0}^{\bar{\varepsilon}} \left[ \varepsilon U\left(q_\varepsilon\right) - \bar{S}^s - C\left(q_\varepsilon\right) \right] dF(\varepsilon) - i \left[ \bar{S}^s + C\left(\hat{q}\right) \right] + \\
& (1-\theta) \int_1^{\bar{\varepsilon}} \left\{ \varepsilon U\left(q_\varepsilon^*\right) - \left[ \bar{S}^s + C\left(q_\varepsilon^*\right) \right] (1+i) \right\} dF(\varepsilon).
\end{aligned}
\tag{50}
$$

A **symmetric monetary stationary equilibrium** is a real vector $\left(i, \varepsilon_0, \hat{\varepsilon}, m, \bar{S}^s\right)$, and a set of real functions $\left\{ \left(\alpha_\varepsilon, q_\varepsilon, z_\varepsilon\right), \left(\alpha_\varepsilon^*, q_\varepsilon^*, z_\varepsilon^*, m_\varepsilon^*\right) \right\}_{\varepsilon \in [1, \bar{\varepsilon}]}$ that satisfy the system of equations: (7), (31), (32), (37), (38), (41), (47), (48), (49), and (50).

# 4 The Estimation of the Model

This section estimates the extended model advanced in Subsection 3.3 using primarily the United States data described in Section 2. The estimated model is then used to address several important issues in monetary economics.

In the empirical implementation, we adopt specific functional forms for $U$, $C$, and $F$. The functional forms for the utility of consuming and the disutility of producing are assumed to be respectively isoelastic and linear:

$$
U(q_\varepsilon) = \frac{q_\varepsilon^{1-\sigma}}{1-\sigma}, \ \sigma \in (0,1), \ \text{and}
\tag{51}
$$

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$$C(q_\varepsilon) = q_\varepsilon. \tag{52}$$

These functional forms are the most commonly used in the literature. With these functional forms, an interesting property that will be used below is that the average commercial margin is increasing with the curvature parameter $\sigma$. In particular, the average commercial margin is $\sigma/(2 - \sigma)$ as long as buyers purchase goods for all $\varepsilon$ and $i = 0$.[10] Intuitively, if $U$ has a large curvature parameter $\sigma$, individuals seek to consume small quantities often because marginal utility is strongly decreasing in $q_\varepsilon$. As a result, individuals require a large remuneration, in the form of a large commercial margin, to sacrifice their time to be sellers.

Previous literature offers little guidance about the distribution of preference shocks. After some experimentation, we discovered that to generate a realistic elasticity of the demand for money, large preference shocks must be rare relative to low preference shocks. A convenient way to capture this is to assume that the distribution of shocks is uniform on the interval $(1, \bar{\varepsilon}]$ but has mass probability at 1. This distribution has the following convenient interpretation. With probability $p$, buyers have a "normal" desire to consume in which case $\varepsilon$ is normalized to 1. With probability $1 - p$, buyers experience a larger than normal desire to consume. When this happens $\varepsilon$ is uniformly distributed on the interval $(1, \bar{\varepsilon}]$. Algebraically, the distribution function is

$$F(\varepsilon) = \begin{cases} p & \text{at } \varepsilon = 1, \text{ and} \\ \frac{1-p}{\bar{\varepsilon}-1}(\varepsilon - 1) & \text{for } \varepsilon \in (1, \bar{\varepsilon}]. \end{cases} \tag{53}$$

Thus, the density for $\varepsilon \in (1, \bar{\varepsilon}]$ is constant and equal to

$$\varphi = \frac{1 - p}{\bar{\varepsilon} - 1}. \tag{54}$$

With the distribution function (53), condition (41) that determines the critical shock $\hat{\varepsilon}$ at

---

[10]  The derivation of this formula is lengthy and unrelated to the main issues of the paper, so it is not provided here.

which individuals start being liquidity constrained simplifies into

$$\frac{i}{\varphi} = \frac{(\bar{\varepsilon} - \hat{\varepsilon})^2}{2\hat{\varepsilon}}. \tag{55}$$

Equation (55) determines the key properties of the demand for money for individuals that choose their money balances before knowing their preference shock. At $i \downarrow 0$, the liquidity constraint never binds ($\hat{\varepsilon} = \bar{\varepsilon}$), so the demand for money depends on the maximum realization of the preference shock $\bar{\varepsilon}$. Large values of $\bar{\varepsilon}$ imply a large demand for money. At positive interest rates, the cost of carrying money balances induces individuals to accept a positive probability of being liquidity constrained ($\hat{\varepsilon} < \bar{\varepsilon}$). The size of this effect falls with the density of the preference shocks $\varphi$. Intuitively, if large preference shocks are rare ($p$ close to one and $\varphi$ close to zero), the losses from carrying less money are small because individuals are seldom liquidity constrained. Hence, individuals are willing to cut money balances substantially in response to a rise in $i$. As a result, the demand for money is highly elastic.

We estimate the parameters of the model $(\beta, \sigma, \bar{\varepsilon}, p, \theta)$ using primarily the time series of the velocity of circulation of M1* and the nominal rate of interest examined in Section 2. The velocity of circulation of money in our model, the theoretical counterpart of the velocity of M1*, is equal to:

$$\text{Velocity} \equiv \frac{\text{GDP}}{M} = \frac{\theta \int_{\varepsilon_0}^{\bar{\varepsilon}} \left[ \bar{S}^s + C\left(q_\varepsilon\right) \right] \varphi d\varepsilon + (1-\theta) \int_1^{\bar{\varepsilon}} \left[ \bar{S}^s + C\left(q_\varepsilon^*\right) \right] \varphi d\varepsilon}{\theta \left[ \bar{S}^s + C\left(\hat{q}\right) \right] + (1-\theta) \int_1^{\bar{\varepsilon}} \left[ \bar{S}^s + C\left(q_\varepsilon^*\right) \right] \varphi d\varepsilon}. \tag{56}$$

If $\theta = 0$, velocity is one because all individuals carry exactly the money they know they are going to spend. In contrast, if $\theta > 0$, some individuals hold precautionary balances. As a result, velocity is below one and it is interest elastic.

Assuming that variations in nominal interest rates are driven by exogenous shifts in monetary policy, the time series examined in Section 2 provide information not only on the level of velocity and its time trend, but also on the response of velocity to changes in nominal interest rates. Therefore, we are able to identify $\bar{\varepsilon}$, $p$, and the time profile of $\theta$. The parameters $\bar{\varepsilon}$ and $p$ are assumed constant, so preferences are time invariant. The parameter

$\theta$ is assumed to be the following polynomial of time: $\theta = \theta_0 + \theta_1 T + \theta_2 T^2 + \theta_3 T^3$, where $T$ measures time from $T = -1$ at the beginning of the sample to $T = 1$ at the end of the sample,[11] and $\theta_0$ is normalized so the maximum value of $\theta$ is one.

To properly identify the rest of parameters in our model, we complement the data on velocity and nominal interest rates with additional information. We calibrate $\beta$ so the real rate of interest is a realistic 3 percent. Similarly, we calibrate $\sigma$ to match the average commercial margin reported by the Bureau of the Census (www.census.gov/svsd/www/artstbl.htlm) (around 28 percent) [12]. Finally, we choose the length of the period to be 2 weeks. This captures the fact that most households receive income and make regular purchases at a fairly high frequency.

We estimated the model using nonlinear least squares and treating velocity as the independent variable. More precisely, we converted the velocities and interest rates in our data from annual to biweekly[13], and we searched for the vector of parameter values $(\sigma, \bar{\varepsilon}, p, \theta_0, \theta_1, \theta_2, \theta_3)$ that minimizes the sum of squared residuals (difference between actual and predicted velocities) subject to the constraints $\sigma = 0.435$ and $\max(\theta) = 1$ ($\beta$ is not needed to calculate velocity for a given nominal interest rate). The parameter estimates are reported in Table 2.

---

[11]   That is, $T = (\text{Year} - 1947.5)/55.5$.

[12]   Because preferences are additive $\sigma$ is also the inverse of the intertemporal elasticity of substitution of consumption. However, we do not think that calibrating $\sigma$ to match the intertemporal elasticity of substitution is a reasonable choice here. Faig and Jerez (2004) present a model with multiple purchases each period that distinguishes between the two roles that $\sigma$ plays here.

[13]   The biweekly velocity is the annual velocity divided by 26. One plus the biweekly interest rate is equal to one plus the annual interest rate to the power of 1/26.

Table 2

ESTIMATION OF THE MODEL

Sample: Annual time series United States 1892-2003

Dependent variable: Velocity (GDP/M1*)

Independent variables: Commercial paper rate and time

Method: Non-linear least squares

Period length: 2 weeks

$$\sigma = 0.435$$

$$\theta_0 = 0.859$$

| Parameter | $\bar{\varepsilon}$ | $p$ | $\theta_1$ | $\theta_2$ | $\theta_3$ |
|---|---|---|---|---|---|
| Estimate | 4.000 | 0.966 | $-0.275$ | $-0.290$ | $-0.156$ |
| Std. dev. estimates[14] | 0.080 | 0.007 | 0.059 | 0.052 | 0.090 |

Sum of squared residuals $= 0.029$

$R^2 = 0.994$

As we can see in Table 2, the five parameters $(\bar{\varepsilon}, p, \theta_1, \theta_2, \theta_3)$ are precisely estimated and have reasonable values. The first four parameters are significantly different from zero at all reasonable confidence intervals. The last parameter is marginally significant. The high $R^2$ signals a very close fit between the model and the data. To better judge this close fit we plot in Figure 2 the annual velocity predicted by the model (dark plain line) and the actual velocity (line with circles). The two lines move very closely together. To ascertain how much this close fit is due to the correct estimation of the trend and how much it is due to the correct predictions of the theoretical model, Figure 4 displays the deviations of the actual and the predicted velocities from trend velocity (trend velocity is defined as the predicted velocity for a constant interest rate at the average level of 4.56 percent). Figure 3 shows that our estimated model predicts very accurately the deviations of velocity from

---

[14]    The standard deviations of the estimates were calculated using the formula in Doan (2002) pp. 218-9 with $k = 5$. This formula is robust to hetereostadicity and autocorrelations of the error term up to five years apart.

trend. For example, the model correctly predicts the fall of velocity during the low interest rate years of the Great Depression. It also predicts the rise in velocity, well above its trend, during the period of high interest rates that went from the mid-sixties to the mid-eighties. Finally, it predicts with a slight lead, the dramatic ups and downs of detrended velocity in the last portion of the sample. The largest persistent residuals between the actual and the predicted velocities correspond to the Second World War and its aftermath. However, this deviation is not too surprising because this was a period of massive price and financial controls. The time profile of trend velocity, displayed in Figure 2 with a thin line, is interesting in itself. Trend velocity was almost constant in the first half of the sample. Whereas it increased at an accelerating rate in the second half. As examined below, this sharp rise in trend velocity has major implications for a variety of issues.

Figure 2

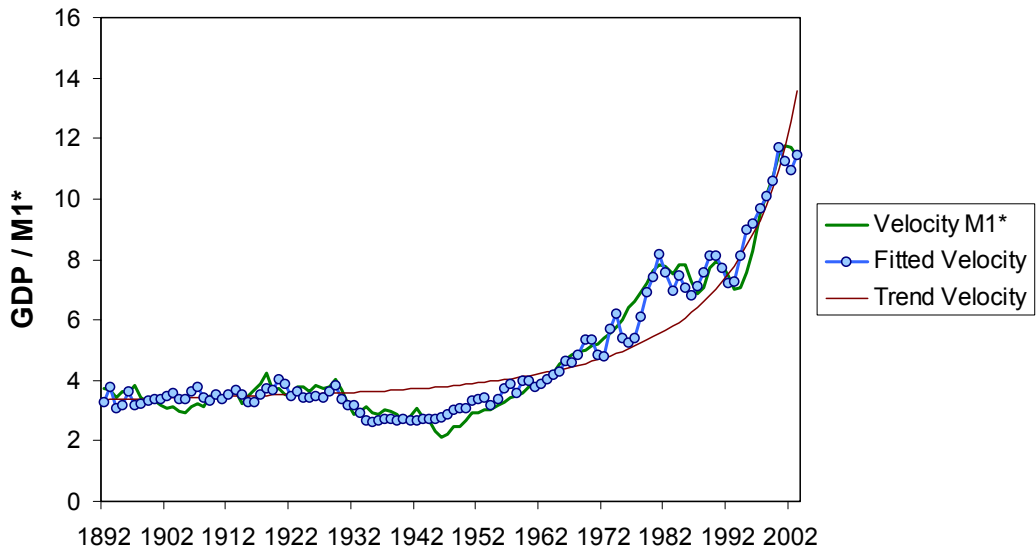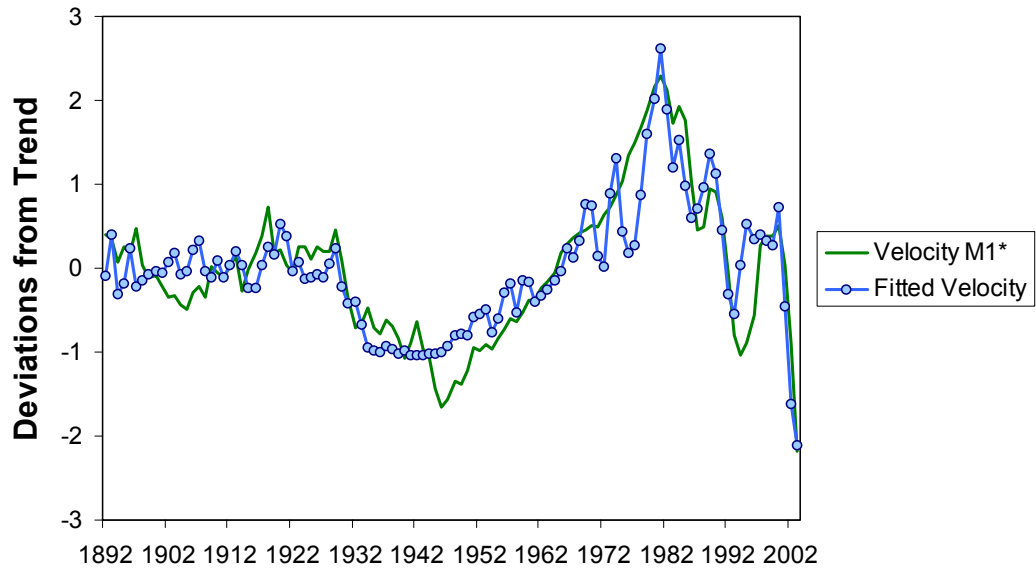## Fitted Versus Actual Velocity



Figure 3

## Fitted Versus Actual Deviations from Trend

One of the reasons behind the close fit of the model illustrated in Figures 2 and 3 is the correct response of velocity to changes in the nominal interest rate. The model accurately predicts that neither the interest elasticity nor the interest semi-elasticity of velocity are constant, but they change with the nominal interest rate. For the median value of $\theta$ (0.859), Figure 4 graphs the interest semi-elasticity and the interest elasticity of velocity as functions of the nominal interest rate. The elasticity is an increasing and concave function of the rate of interest. The semi-elasticity is a decreasing and convex function of the same rate. Curiously, the graph for this last function is approximately an hyperbola.
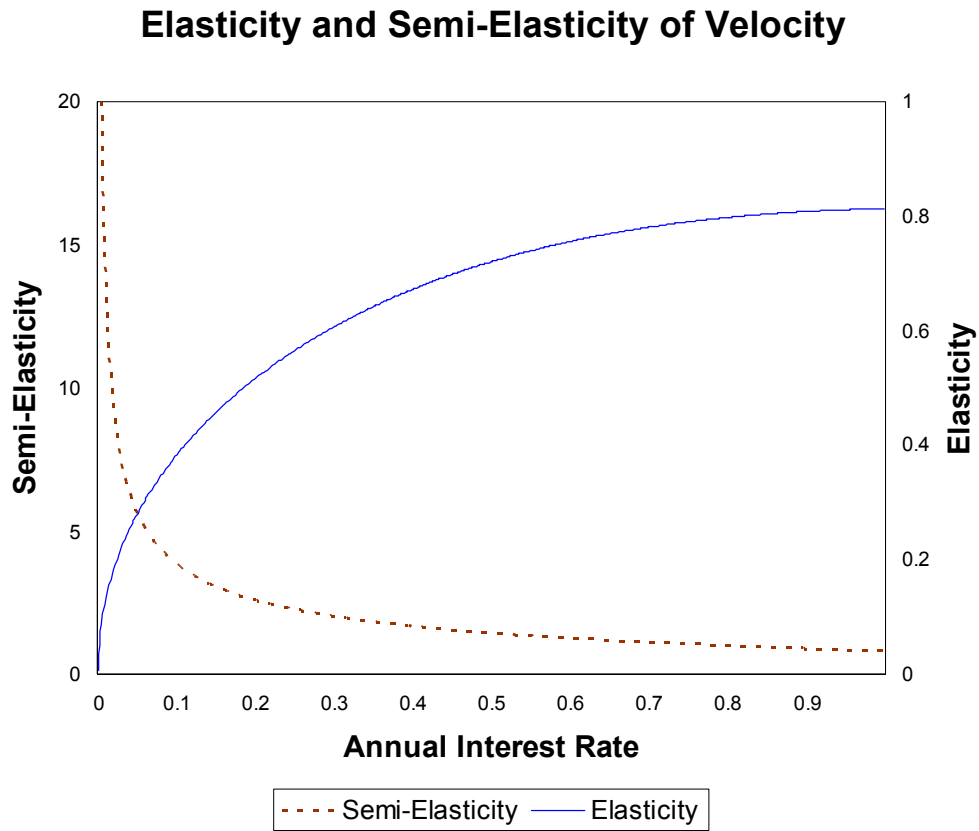
Figure 4

## Elasticity and Semi-Elasticity of Velocity

Table 3

IMPLICATIONS OF THE MODEL

| Year | 1892 | 1920 | 1948 | 1976 | 2003 |
|---|---|---|---|---|---|
| **Estimated value of $\theta$ (%)** | 100 | 94.3 | 85.6 | 62.0 | 13.9 |
| **Equilibrium at average** $i$ (4.56 % annual) | | | | | |
| Average commercial margin (%) | 28.1 | 28.1 | 28.1 | 28.0 | 27.9 |
| Precautionary balances over money supply (%) | 87.1 | 86.4 | 85.2 | 80.6 | 47.6 |
| Fraction of buyers liquidity constrained (%) | 1.11 | 1.03 | 0.94 | 0.68 | 0.15 |
| Seigniorage over GDP (%) | 0.45 | 0.43 | 0.39 | 0.30 | 0.11 |
| Interest semi-elasticity of velocity | 5.77 | 5.76 | 5.72 | 5.52 | 3.41 |
| Interest elasticity of velocity | 0.26 | 0.26 | 0.26 | 0.25 | 0.16 |
| **Effect of rising** $i$ **from 0 to 10% annual** | | | | | |
| Equivalent variation of consumption (%) | 1.06 | 1.00 | 0.91 | 0.66 | 0.15 |
| Reduction GDP (%) | 6.98 | 6.63 | 6.09 | 4.63 | 1.65 |
| Welfare cost over seigniorage (%) | 25.6 | 25.4 | 25.1 | 23.8 | 14.7 |

Table 3 calculates the implications of the model for five values of $\theta$ that correspond to our estimates for the years: 1892, 1920, 1948, 1976, and 2003. The upper part of Table 3 calculates implied equilibrium values if the nominal rate of interest had remained constant at its average level (4.56 percent) throughout the sample. The lower half calculates the implications of raising the nominal rate of interest from 0 to 10 percent. The time profile of $\theta$ mirrors the time profile of trend velocity: it is almost constant from 1892 to 1948, it falls significantly from 1948 to 1976, and it plummets from 1976 to 2003. This last drop in $\theta$

has had a major effect on the properties of the demand for money. As the table shows, the drop in $\theta$ from 1976 to 2003 has led to dramatic reductions in the following: precautionary balances, fraction of buyers that are liquidity constrained, ability to collect seigniorage, and interest semi-elasticity and elasticity of velocity. For example, at a 4.56 percent nominal rate of interest, the government could collect around 0.4 percent of GDP from 1892 to 1948, but it could only collect 0.11 percent of GDP in 2003. The semi-elasticity of velocity was between 5 and 6 between 1892 and 1976, but it dropped to 3.41 in 2003.

The sharp drop in the need for precautionary balances occurred in the last few decades has also major implications for the welfare cost of inflation. For most of our sample years, we find that the welfare cost of a 10 percent increase of inflation from the Friedman rule is equivalent to a reduction of consumption of about 1 percent. This is consistent with the estimates of Lucas (2000). However, we also find that the welfare cost of inflation has plummeted with $\theta$. Intuitively, the shift from a high and elastic demand for money to a low and inelastic demand has compressed the area below the demand for money and hence the welfare cost of inflation. The drop in $\theta$ has also reduced the effect of inflation on GDP and the ratio of the deadweight-loss of inflation over the seigniorage it generates.[15] In summary, the drop in $\theta$ has radically altered the answer to most substantive questions in monetary economics. The conventional wisdom about many dimensions of monetary economics needs to be reworked in the new environment created by the revolution of the information and communication technologies, which we believe are behind the drop in $\theta$ that we estimated in our analysis.

# 5 Conclusion

Precautionary balances, when carefully studied in a model with rigorous microeconomic foundations, are able to explain not only why the velocity of circulation of money has been historically low, but also why it correlates the way it does with the nominal rate

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[15]  Seigniorage is defined as $(\gamma - 1)\, m$. Deadweight-loss is defined as the change in $S^s$ (which is equal to $S^b$).

of interest. Our empirical implementation of the model discovers that precautionary balances have plummeted in the last three decades. We attribute the origin of this drop to the tremendous improvements in the information and communication technologies that occurred during this time. These improvements have allowed individuals to accommodate unexpected expenditure needs without holding large precautionary balances. This has radically transformed many important issues of monetary economics. The demand for money has not only fallen, but it is also less elastic. Moreover, both the seigniorage and the welfare cost of inflation are now a small fraction of what they were 30 years ago.

Our model abstracts from several features of reality that are likely to be important to the issues we study. For example, we abstract from the distinction between currency and checkable deposits, and from the fact that some checkable deposits earn interest. Also, we abstract from many ways that individuals can affect their demand for money, such as converting assets at a higher frequency. Finally, we abstract from many complexities in the production of goods such as the presence of physical capital. We list these features here not only to acknowledge the limitations of our work, but also to stimulate future research.

<div align="center">

**Appendix**

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**Data sources**

The interest rate is the short term commercial paper rate. For 1892-1971, it is from Friedman and Schwartz (1982), Table 4.8, Column 6. For 1972-2003, it is the DRI series FYCP90 (averaged).

Money is M1* = M1 - currency outside the country. M1 is the stock at the end of June of each year. For 1892-1928, the source of M1 is United States Bureau of the Census (1960), Series X267. For 1929-1958, it is the series constructed by St. Louis FED that extends backwards modern M1 http://research.stlouisfed.org/aggreg. For 1959-2003, it is the DRI series FM1. Currency in circulation abroad is from the FED *Flow of Funds* Table L-204 in the file ltab204d.prn downloaded from http://www.federalreserve.gov/releases/z1/current/data.htm.

For 1892-1928, GDP is calculated from the real GDP series in Kendrick (1961) and the implicit price deflator in Friedman and Schwartz (1982), Table 4.8, Column 4. For 1929-2003, it is from BEA NIPA Table 1.1.5 downloaded from www.bea.doc.gov/bea/dn/nipaweb in Dec. 2004.

**Proof of Proposition 1**

Consider the problem of an individual in the equilibrium of our basic model where all other individuals have value functions (21) and initial wealths in the interval $[\underline{a}, \overline{a}]$, except for a small positive measure of individuals that have identical wealth to that of the individual whose value function we are characterizing. The assumption that there is a positive measure of individuals whose problems are identical allows us to focus on deviations by a positive measure of individuals.[16] Throughout the appendix, we use without further proof the absence of uncertainty in trading opportunities because of efficient matching.

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[16] Deviations with a single individual are less interesting because the demand for money depends on the offers of all submarkets that buyers plan to visit. For instance, a single deviating seller who rises the quantity offered to the highest type $\bar{\varepsilon}$ to increase the expected payment fails to provide an incentive to any buyer to carry extra money (since the probability that $\bar{\varepsilon}$ is realized is zero). This would imply that $a_{\min}$ is

For all finite $a \geq a_{\min}$, the set of feasible time and state contingent policies is non empty. The feasible values of the quantities consumed and produced are bounded. Also, for all feasible policies the present discounted utility is well defined and finite because $\mathcal{U}$ is a continuous function. Consequently, we can use standard recursive methods to find the value function.

In competitive search, we can recursively characterize the individual optimization problem as follows. (This characterization uses a more general definition of competitive search than in Section 3.2 because it allows the individual to have wealth outside the interval $[\underline{a}, \overline{a}]$.) The individual chooses to be a buyer or a seller. As a seller, the individual chooses $(\varepsilon, q_\varepsilon^s, z_\varepsilon^s, m^s, b^s)$, where $\varepsilon$ is the buyer type the seller targets and $(q_\varepsilon^s, z_\varepsilon^s)$ are the corresponding posted offers. As a buyer the individual chooses $\left( \left\{ q_\varepsilon^b, z_\varepsilon^b, \mu_\varepsilon^b \right\}_{\varepsilon \in [1, \bar{\varepsilon}]}, m^b, b^b \right)$ where $\left\{ q_\varepsilon^b, z_\varepsilon^b, \mu_\varepsilon^b \right\}_{\varepsilon \in [1, \bar{\varepsilon}]}$ is the set of choices contingent on the realization of their preference shock. These choices are subject to the constraints (11)-(14), (16)-(19), and (20). Moreover, in the financial markets the individual takes as given the rate of interest and the insurance premia. In the goods market, the individual takes as given the reservation expected trade surpluses of other traders and has rational expectations about their actions. Therefore, as a seller the individual posts an offer $(q_\varepsilon^s, z_\varepsilon^s)$ to the buyers of type $\varepsilon$ which belong to a set of posted offers $\{(q_\varepsilon^s, z_\varepsilon^s)\}_{\varepsilon \in [1, \bar{\varepsilon}]}$ that satisfies the following two conditions: $\int_1^{\bar{\varepsilon}} [\varepsilon U(q_\varepsilon^s) - z_\varepsilon^s] dF(\varepsilon) - i \max \{z_\varepsilon^s\}_{\varepsilon \in [1, \bar{\varepsilon}]} \geq \bar{S}^b$ (buyers are guaranteed the reservation expected trade surplus), and $z_\varepsilon^s - C(q_\varepsilon^s) \leq \bar{S}^s$ for $\varepsilon \in [1, \bar{\varepsilon}]$ (the posted offers cannot be profitably undercut). As a buyer, the individual acts as if he/she were choosing $\left\{ q_\varepsilon^b, z_\varepsilon^b \right\}_{\varepsilon \in [1, \bar{\varepsilon}]}$ that satisfies $z_\varepsilon^b - C(q_\varepsilon^b) = \bar{S}^s$ for all $q_\varepsilon^b > 0$, because competition among sellers implies that all posted offers yield the same trade surplus $\bar{S}^s$ to the sellers.

Let $\mathcal{C}(a)$ be the space of bounded and continuous functions $f : [a_{\min}, \infty) \to R$, with the sup norm. Use the Bellman's equations (10) and (15) together with (9) to define the

equal to $\underline{a}$. In contrast, a positive measure of deviating sellers with similar offers to types in $[\hat{\varepsilon}, \bar{\varepsilon}]$ gives an incentive to a positive measure of buyers to carry extra money. Furthermore, the law of large numbers together with efficient matching implies that these buyers find a trading partner with probability one.

mapping $T$ of $\mathcal{C}(a)$ onto itself by substituting $f$ for $V$ in the right hand sides of (10) and (15) and denoting as $Tf(a)$ the left hand side of (9). The choice variables and constraints of these maximization programs are described in the previous paragraph. For a given $a$, the set of feasible policies is non-empty, compact-valued, and continuous. The utility function $\mathcal{U}$ is a bounded and continuous on the set of feasible policies, and $0 < \beta < 1$. Therefore, Theorem 4.6 in Stokey and Lucas with Prescott (1989) implies that there is a unique fixed point to the mapping $T$, which is the value function $V$.

Let $\mathcal{V}(a)$ be the set of functions $f : [a_{\min}, \infty) \to R$ that satisfy (21) where $v_0$, $\overline{a}$, and $\underline{a}$ are given by:[17]

$$
\begin{aligned}
v_0 &= \frac{\bar{S}^s}{1-\beta} + \frac{\beta}{1-\beta}\frac{\gamma-1}{\gamma}, \\
\overline{a} &= \frac{\int_1^{\bar{\varepsilon}} z_\varepsilon dF(\varepsilon) + im}{1-\beta} - \frac{\beta}{1-\beta}\frac{\gamma-1}{\gamma}, \text{ and} \\
\underline{a} &= -\frac{z_{\bar{\varepsilon}}}{1-\beta} - \frac{\beta}{1-\beta}\frac{\gamma-1}{\gamma};
\end{aligned}
\tag{57}
$$

and $i$, $m$, $\bar{S}^s$, and $z_\varepsilon$ satisfy the equilibrium system of equations described in 3.2. Consider the mapping $T$ defined in the previous paragraph. Since $V$ is concave, it is an optimal policy to fully insure preference shocks (full insurance is strictly optimal if there is a positive probability that $a_{+1} \notin [\underline{a}, \overline{a}]$). In consequence, $a_{+1}$ is not stochastic. Let $a_{+1}^b$ be next period real wealth for an optimal policy conditional on being a buyer. Similarly, let $a_{+1}^{s\varepsilon}$ be the optimal policy for a seller serving buyers of type $\varepsilon$. If $a_{+1}^b, a_{+1}^{s\varepsilon} \in [\underline{a}, \overline{a}]$ for all $\varepsilon$, $TV(a)$ is the maximum of $V^b(a)$ and $V^s(a)$ in equations (22) and (23), so $TV(a)$ is affine and the trade surpluses are those in (24) and (25). The optimal policies of the individual are the equilibrium ones characterized in Section 3.2. Therefore, the individual is indifferent between being a buyer or a seller, and as a seller he/she is indifferent to serve any type of buyer. This indifference is broken when one policy would lead to $a_{+1} \notin [\underline{a}, \overline{a}]$. In such a case, the strict concavity of $V$ outside the interval $[\underline{a}, \overline{a}]$ implies that it is suboptimal to

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[17] In the absence of insurance of preference shocks the integral in the expression for $\overline{a}$ should be replaced by $z_1$.

be a seller serving buyers of type $\varepsilon$ if $a_{+1}^{s\varepsilon} > \bar{a}$. Likewise, it is suboptimal to be a buyer if $a_{+1}^{b} < \underline{a}$. Consequently, the recursive budgets (11) to (13) and (16) to (18), together with (57), imply that $a_{+1} \in [\underline{a}, \bar{a}]$ if an only if $a \in [\underline{a}, \bar{a}]$. This implies that $TV(a)$ is affine in the interval $[\underline{a}, \bar{a}]$. Equation (23) implies that the constant term of this affine function is the value of $v_0$ in (57). If $a > \bar{a}$, the optimal policy is to be a buyer. Vice versa, if $a < \underline{a}$, an optimal policy is to be a seller serving the set of liquidity constrained buyers. In both cases, the strict concavity of $U$ and convexity of $C$ imply the strict concavity of $TV(a)$ for $a \notin [\underline{a}, \bar{a}]$. In summary, $T$ maps $\mathcal{V}(a)$ onto itself. Therefore, the value function $V$ satisfies (21). Finally, since $V$ is concave, $\mathcal{U}$ is continuously differentiable, and the solution is interior, $V$ is continuously differentiable.

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