Crime and the Labor Market*

Bryan Engelhardt
University of Iowa

Guillaume Rocheteau
Federal Reserve Bank of Cleveland
University of Pennsylvania

Peter Rupert
Federal Reserve Bank of Cleveland

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Abstract

The same policies or technological changes that affect the labor market can also affect the extent of criminal activities. For instance, while an increase in unemployment benefits can raise unemployment duration it may also reduce crimes by unemployed. Or, a technological change in the home sector that affects participation in the market may also raise participation in criminal activities. To analyze these interactions we construct a search-theoretic model where labor market outcomes and crimes are determined jointly. The description of the labor market follows Pissarides' (2000) canonical model of unemployment extended to account for decisions to participate in the labor force. We introduce random arrivals of crime opportunities for all individuals irrespective of their labor market status. Furthermore, we allow for optimal employment contracts that internalize the effect of crime activities on job duration. We calibrate our model to US data focusing on females. We investigate whether the change in preferences for market activities that is necessary to account for female labor force participation can also account for the substantial increase in female crime over the last half century. We also look at the effects of unemployment insurance, workers' bargaining power, skill-biased technological progress and the availability of crime opportunities.

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1 Introduction

Typically, labor market policies are not designed to affect the extent of criminal activities. Similarly, technological change in the labor market are rarely characterized in terms of their effects on crime. However, since Becker (1968) it is well understood that participation in illegal activities are driven by similar economic incentives as the ones that motivate legitimate activities. As a result, changes in policy or in the economic environment that affect the labor market can also affect involvement in criminal activities. For instance, a reduction in unemployment benefits, aimed at reducing unemployment duration, may generate, as a by-product, an increase in crime. According to Machin and Marie (2004), this is precisely what happened in the U.K. following the 1996 reform of the unemployment benefit system.

Similarly, a technological change in the home sector that raises female participation in the labor force may well raise female involvement in criminal activities as well. Witt and Witte (2000) find that "...a one percentage point rise in the female labor force participation rate increases the crime rate by just over 5 percent." This last observation is particularly relevant given that the female labor force participation rate has risen substantially over the last 50 years, from about 40 percent in 1960 to more than 60 percent in 2003.

Over the same period of time, the fraction of offenses perpetrated by women has increased by a factor of three. In 1948 about 12 percent of larcenies were committed by women; in 2002, about 40 percent of larcenies were perpetrated by women. Women accounted for only about 2 percent of motor vehicle thefts in 1948; they now account for about 15 percent. While the property crime rate in the U.S. has fallen from about 70 crimes per 1000 persons in 1975 to 50 crimes per 1000 persons in 2003, crime rates for women have *risen* substantially over the last 50 years.

Understanding how labor market outcomes and crimes are determined jointly calls for a theoretical model in which unemployment, participation in the labor force, wages and crime activities are all endogenous. While several frameworks are available, we propose an alternative one with the following key ingredients. First, we believe that there is no need to come up with a new description of the labor market. Macroeconomists have settled down on the canonical description proposed by Pissarides (2000).

Unemployment is generated by search-matching frictions and wages are determined via bargaining between employees and employers. We adopt this standard description and add a home sector to account for labor force participation. One advantage of using the canonical model of equilibrium unemployment is that the various extensions and studies of labor market policies can be used directly to think about the implications for crime.

Second, it is well documented that crimes are committed by individuals who are employed, unemployed or out of the labor force. For instance, more than 50 percent of females convicted of criminal activities were previously employed. In our model, we assume that all individuals receive random opportunities to commit crimes. Crime opportunities have different values and individuals choose which ones to undertake and which ones to leave aside. As in the standard textbook model of sequential search, all individuals will follow an optimal stopping rule summarized by a reservation value for crimes above which they undertake the crime opportunity. In equilibrium, all categories of individuals can commit crimes; however, the frequency with which they do so may vary with their labor force status. The fact that employed workers can also commit crimes implies a negative externality on firms by reducing average job duration. This type of externality is well understood in models with search-on-the-job and can lead to inefficient separations and a distribution of wages.[1] We will assume that employees and employers can write optimal employment contracts that take care of these externalities and so generate an efficient turnover from the point of view of a worker and the employer.

We prove that equilibrium exists and we provide simple conditions for uniqueness – despite strategic complementarities between firms' decisions to open vacancies and workers' decisions to commit crimes.[2] Like in Mortensen and Pissarides (1994) the determination of equilibrium can be represented in a simple two-dimensional figure with workers' crime decisions on one axis and firms' hiring decisions on the other axis. We can determine how a change in some relevant parameters (workers' bargaining power, productivity, separation rates, and so on) affect both the labor market and crime.

We calibrate our model to the U.S. data focusing on females. The crimes we consider are against property. With the calibrated model in hand, we quantify the effects of several experiments. We

[1] See the model by Burdett and Mortensen (1998), the extensions by Burdett and Coles (2003) and Stevens (2004).

[2] One can consider various extensions of the model that generate multiplicity of steady-state equilibria. We believe, however, that it is interesting that a benchmark version of the model predicts a unique equilibrium.

investigate whether a change in preferences toward market activities that has led to a massive increase in female participation in the labor force can explain the higher involvement in criminal activities. We show that the same force that explains the rise in participation for women can also lead to an increase in the crime rate by roughly 50 percent. We also consider skill-biased technological progress that is thought to have increased wage inequality, especially among women, changes in workers' bargaining power that could reflect changes in discriminatory behavior, as well as several other changes in various policies.

The closest paper to ours is that of Burdett, Lagos and Wright (2003) – BLW hereafter. While BLW adopt the wage posting framework of Burdett-Mortensen (1998), we use the Pissarides model where wages are determined via bilateral bargaining. Furthermore, our description of the employment contract is fundamentally different. In BLW the employment contract is restricted to a constant wage. Such contracts happen to be inefficient when workers can engage in criminal activities. This restriction can lead to a wage distribution and multiple equilibria. In contrast, we will consider optimal employment contracts in which the firm can charge a hiring fee. Another difference is the fact that the job finding rate is exogenous in BLW while it is endogenous in our model through a free-entry condition of vacancies. So changes in policies or technological changes in the BLW model do not affect the availability of jobs which could matter for unemployed workers' incentives to commit crimes. Also, we endogenize the decision to participate in the labor force by introducing a home sector. Finally, the value of crime opportunities are random draws from a distribution; this allows us to obtain endogenous crime rates for individuals in any state, i.e., employed, unemployed or out of the labor force.

Huang, Laing, and Wang (2004) is also related to our model in that they employ a search-theoretic framework with bilateral bargaining. Their description of criminal activities is, however, very different. In their model individuals specialize in criminal activities. As a consequence of this assumption, employed workers never commit crimes which is at odds with the evidence. Also, they do not formalize participation decisions but they include an endogenous human capital choice.

2 Model

The environment is similar to Pissarides (2000, ch. 7) extended to allow for criminal activities. Time, denoted t, is continuous and goes on forever. The economy is composed of a unit-measure set $\mathcal{K}$ of infinitely-lived individuals indexed by κ and a large measure of firms. There is one final good produced in the market by firms and workers. Each individual is endowed with one indivisible unit of time. This unit of time has three alternative, mutually exclusive uses. It can be used to search for a job ($\ell_u = 1$), to work for a firm ($\ell_e = 1$), or to stay out of the labor force and work or enjoy leisure at home ($\ell_o = 1$).

The utility function of an individual at time 0 is given by:

$$\mathbb{E}_0 \int_0^\infty c(t) e^{-rt} dt,$$

where $c(t)$ is the consumption flow of the final good and $r > 0$ is the rate of time preference. Individuals are not liquidity constrained and have access to a competitive market for private loans.

An unemployed worker who is looking for a job enjoys a utility flow b. One can interpret b as the utility from not working or as unemployment benefits paid by the government. When an unemployed worker and a vacant job meet they negotiate the terms of the employment relationship. We will see that the decision of an employed worker to engage in criminal activities affects the tenure of the job. We will therefore consider optimal employment contract, composed of a hiring fee ϕ and a constant wage w, that internalize this effect.

Individuals out of the labor force enjoy the utility flow κp where $\kappa \geq 0$ is individual-specific and $p \geq 0$ is common across all individuals. Individuals are heterogeneous in terms of their utilities at home. The distribution of the κs across individuals is $H(\kappa)$. The common component p can capture how society perceives women work at home and in the market, as well as the productivity of the technology in the home sector.[3] We assume that κ does not affect the utility of unemployed workers because of some indivisibilities in the use of time.

Firms are composed of a single job which is either filled or vacant. Vacant firms are free to enter the

[3] According to Fortin (2005), "female attitudes towards working women are developed in youth, influenced by parental education and religious affiliation." Or, following Fernandez, Fogli and Olivetti (2004), p could reflect women's spouses attitudes toward working women. Following Greenwood, Seshadri and Yorukoglu (2004) we can also interpret p as representing the productivity in the home sector. See the Appendix for such an interpretation.

labor market. There is a flow cost γ to advertise a vacancy. The production flow of a filled job is $y > b$. Firms are risk-neutral and discount future utility at rate $r > 0$.

The labor market is subject to search-matching frictions. The flow of hirings is given by the aggregate matching function $m(U, V)$ where U is the measure of unemployed workers actively looking for jobs and V is the measure of vacant jobs. The matching function $m(\cdot, \cdot)$ is strictly increasing and strictly concave with respect to each of its arguments and it exhibits constant returns to scale. Furthermore, $m(0, \cdot) = m(\cdot, 0) = 0$ and $m(\infty, \cdot) = m(\cdot, \infty) = \infty$. Following Pissarides' terminology, we define the ratio $\theta \equiv V/U$ as labor market tightness. Each vacancy is filled according to a Poisson process with arrival rate $\frac{m(U,V)}{V} \equiv q(\theta)$. Similarly, each unemployed worker finds a job according to a Poisson process with arrival rate $\frac{m(U,V)}{U} = \theta q(\theta)$. Filled jobs receive negative idiosyncratic productivity shocks that make matches unprofitable with a Poisson rate s.[4]

We now turn to the crime sector. Each individual, irrespective of their labor market status, in the economy receives an opportunity to commit a crime according to a Poisson process with arrival rate λ_i, where i indicates the individual's state: $i = u$ if unemployed, $i = e$ if employed and $i = o$ if out of the labor force.[5] The value of a crime is εm, where ε is a random draw from a distribution $G(\varepsilon)$ with support $[0, \bar{\varepsilon}]$ and m is an aggregate component. A worker who commits a crime is caught with probability π and is sent to jail. For simplicity, a criminal not caught instantly is never caught. Being in jail means that the individual cannot make any productive use of time. Workers in jail receive a flow of utility x that can be negative. A prisoner exits jail according to a Poisson process with arrival rate δ. We assume that the average time spent in jail is independent of the value ε of the crime.[6]

We capture the impact of criminal activities on individuals' payoffs by a lump-sum tax τ_i, where $i \in \{e, u, o, p\}$ is the state of the individual. Note that individuals in jail can be victimized which simply means that they have some properties that can be stolen just like other individuals. The model offers

[4] One could adopt a more explicit description of the idiosyncratic shocks received by firms and endogenize s. See Mortensen and Pissarides (1994).

[5] The fact that individuals can have different arrival rates for crime opportunities depending on their status in the labor force can be justified in various ways. For instance, one may think that opportunties to steal are more frequent for individuals participating in the market. Also, assuming different arrival rates for crime opportunities can be seen as an indirect way to relax the assumption that time is indivisible.

[6] In many cases, the punishment for a crime depends on the nature of the crime more than the gain enjoyed by the perpetrator of the crime. For instance, the punishment for a murder does not depend so much on whether the murder was profitable or not.

few guidelines on what those τ_i should be and how they should be related to the amount stolen in the economy. Therefore, in the following we will consider the case where $\tau_i = \tau$ for all i. Also, we assume that firms are not victimized. Those assumptions would be easy to relax. We will impose a "balanced budget" requirement according to which the aggregate amount stolen by criminals is equal to the aggregate cost of being victimized.

3 Bellman equations

We focus on steady state equilibria where the distribution of individuals across states is constant over time.

3.1 Individuals

An individual is in one of the following four states: Out-of-the-labor-force (o), unemployed (u), employed (e), or in prison (p). The value of being an individual in state $i \in \{o, u, e, p\}$ is denoted $\mathcal{V}_i$. The flow Bellman equations for individuals' value functions are

$$r\mathcal{V}_u = b - \tau + \theta q(\theta)\left(\mathcal{V}_e - \mathcal{V}_u - \phi\right) + \lambda_u \int \left[\varepsilon m + \pi(\mathcal{V}_p - \mathcal{V}_u)\right]^+ dG(\varepsilon), \tag{1}$$

$$r\mathcal{V}_e = w - \tau + s\left(\mathcal{V}_u - \mathcal{V}_e\right) + \lambda_e \int \left[\varepsilon m + \pi\left(\mathcal{V}_p - \mathcal{V}_e\right)\right]^+ dG(\varepsilon), \tag{2}$$

$$r\mathcal{V}_o = \kappa p - \tau + \lambda_o \int \left[\varepsilon m + \pi\left(\mathcal{V}_p - \mathcal{V}_o\right)\right]^+ dG(\varepsilon), \tag{3}$$

$$r\mathcal{V}_p = x - \tau + \delta\left[\max\left(\mathcal{V}_u, \mathcal{V}_o\right) - \mathcal{V}_p\right]. \tag{4}$$

where $[x]^+ = \max(x, 0)$. Note that when there is no ambiguity we omit the dependence of the value functions on κ. Equation (1) has the following interpretation. An unemployed worker enjoys a flow revenue of $b - \tau$ where b is the income of unemployed workers and τ is the cost of being victimized. A job is found with an instantaneous probability $\theta q(\theta)$. Upon taking a job an individual pays a hiring fee, ϕ (or receives a up-front payment if $\phi < 0$), and enjoys the capital gain $\mathcal{V}_e - \mathcal{V}_u$. When unemployed the individual receives an opportunity to commit a crime with an instantaneous probability λ_u. The value of the crime opportunity is drawn from the cumulative distribution $G(\varepsilon)$. If a worker chooses to commit a crime she enjoys utility εm but is at risk of being caught and sent to jail with probability π, in which case there is a capital loss, $\mathcal{V}_p - \mathcal{V}_u$. From (2), an employed worker receives a wage w, loses the job with an instantaneous probability s and has the opportunity to commit a crime with an instantaneous probability λ_e. As can be seen in (3), an individual out of the labor-force enjoys the utility κp and receives the opportunity to commit a crime with an instantaneous probability λ_o.

According to (4), an imprisoned worker receives a consumption flow x, suffer the loss τ, and a prisoner exits jail with an instantaneous probability δ . After release a decision has to be made whether to participate in the labor force as an unemployed worker or to be out of the labor force. In steady-state a prisoner who was previously in the labor force returns to the labor force upon release from jail. Similarly, a prisoner who was previously out of the labor-force returns to home production activities after exiting jail.

From (1), (2) and (3), an individual κ in state i chooses to commit a crime whenever $\varepsilon \geq \varepsilon_i$ where

$$\varepsilon_u = \frac{\pi(\mathcal{V}_u - \mathcal{V}_p)}{m}, \tag{5}$$

$$\varepsilon_e = \frac{\pi\left(\mathcal{V}_e - \mathcal{V}_p\right)}{m}, \tag{6}$$

$$\varepsilon_o(\kappa) = \frac{\pi\left[\mathcal{V}_o(\kappa) - \mathcal{V}_p(\kappa)\right]}{m}. \tag{7}$$

From (5)-(7) the value of the marginal crime that makes an individual indifferent between committing a crime and not committing it, $\varepsilon_i m$, is the expected cost of punishment, $\pi(\mathcal{V}_i - \mathcal{V}_p)$.

An individual chooses to stay at home if $\mathcal{V}_o(\kappa) \geq \mathcal{V}_u$. From (3) the utility from staying at home is increasing with κ. Hence, there exists a threshold κ_u such that an individual chooses not to participate in the labor force if $\kappa \geq \kappa_u$. This threshold satisfies $\mathcal{V}_o(\kappa_u) = \mathcal{V}_u$. From (5) and (7), $\varepsilon_o(\kappa_u) = \varepsilon_u$. Therefore, from (1) and (3), and using the fact that $\int_{\varepsilon_i}^{\bar{\varepsilon}} (\varepsilon - \varepsilon_i) dG(\varepsilon) = \int_{\varepsilon_i}^{\bar{\varepsilon}} [1 - G(\varepsilon)] d\varepsilon$ from integration by parts,

$$\kappa_u p = b + \theta q(\theta)\left(\mathcal{V}_e - \mathcal{V}_u - \phi\right) + (\lambda_u - \lambda_o) m \int_{\varepsilon_u}^{\bar{\varepsilon}} [1 - G(\varepsilon)] d\varepsilon. \tag{8}$$

According to (8), the reservation utility κ_u below which individuals choose to participate in the labor market is such that the instantaneous surplus from staying at home, the left-hand side of (8), is equal to the the sum of the income flow received by an unemployed worker, the expected surplus from finding a job and the difference of the returns from criminal activities for unemployed individuals and individuals out-of-the-labor-force, the right-hand side of (8). Other things being equal, as the labor market becomes tighter individuals have higher incentives to participate in the market. Also, if there are more opportunities to commit crimes when unemployed ($\lambda_u > \lambda_o$) then individuals tend to participate more. If $\lambda_u = \lambda_o$ then crime opportunities do not affect participation decisions.

3.2 Firms

Firms participating in the market can be in either of two states: they can hold a vacant job (v) or a filled job (f). Firms' flow Bellman equations are

$$r\mathcal{V}_v = -\gamma + q(\theta)\left(\phi + \mathcal{V}_f - \mathcal{V}_v\right), \tag{9}$$

$$r\mathcal{V}_f = y - w - s\left(\mathcal{V}_f - \mathcal{V}_v\right) - \lambda_e \pi \left[1 - G(\varepsilon_e)\right]\mathcal{V}_f. \tag{10}$$

According to (9), a vacancy incurs an advertising cost γ; finds an unemployed worker with an instantaneous probability q in which case it enjoys the capital gain $\phi + \mathcal{V}_f - \mathcal{V}_v$. According to (10) a filled job enjoys an instantaneous profit $y - w$ and it is destroyed if a negative shock occurs with an instantaneous probability s or if the worker commits a crime and is caught, an event occurring with an instantaneous probability $\lambda_e \pi[1 - G(\varepsilon_e)]$. Free-entry of firms implies $\mathcal{V}_v = 0$ and therefore, from (9),

$$\mathcal{V}_f + \phi = \frac{\gamma}{q(\theta)}. \tag{11}$$

From (11), the value of the filled job plus the up-front payment is equal to the average recruiting cost incurred by the firm.

3.3 Matches

Define $\mathcal{S} \equiv \mathcal{V}_e - \mathcal{V}_u + \mathcal{V}_f$ as the total surplus of a match (Recall that $\mathcal{V}_v = 0$). From (2) and (10),

$$r\mathcal{S} = y - \tau - r\mathcal{V}_u - s\mathcal{S} + \lambda_e \int_{\varepsilon_e}^{\bar{\varepsilon}} \left[\varepsilon m - \pi\mathcal{S} - \pi(\mathcal{V}_u - \mathcal{V}_p)\right] dG(\varepsilon). \tag{12}$$

Equation (12) has the following interpretation. A match generates a flow surplus $y - \tau - r\mathcal{V}_u$ composed of the output of the job plus home output minus taxes (or the loss due to victimization) and the permanent income of an unemployed person, $r\mathcal{V}_u$. The match is destroyed if an exogenous shock occurs, with an instantaneous probability s, or if the worker commits a crime and is caught. In this case, the value $\mathcal{S}$ of the match is lost and the worker goes to jail which generates an additional capital loss $\mathcal{V}_u - \mathcal{V}_p$.

Suppose a worker and a firm could *jointly* determine the crime opportunities undertaken by the worker. It can be seen from (12), that the surplus of the match is maximized if $\varepsilon_e = \varepsilon_e^*$ where

$$\varepsilon_e^* = \frac{\pi(\mathcal{S} + \mathcal{V}_u - \mathcal{V}_p)}{m} = \frac{\pi\left(\mathcal{V}_e + \mathcal{V}_f - \mathcal{V}_p\right)}{m}. \tag{13}$$

Comparison of (6) and (13) reveals that if $\mathcal{V}_f > 0$ then $\varepsilon_e < \varepsilon_e^*$, implying that employed workers commit too much crime and the total surplus of the match is not maximized. Indeed, if the value of a filled job is strictly positive then workers do not internalize the negative externality they impose on the firm if they commit a crime and are sent to jail.

If the employment contract is limited to a constant wage, w, then a firm faces a trade-off between the wage paid to the worker and the duration of the job.[7] In the following, we will allow the firm to charge an up-front fee, ϕ, so that the worker and the firm reach an efficient contract while keeping the expected profit of the firm positive.[8] The employment contract (ϕ, w) is determined by the generalized Nash solution where the worker's bargaining power is $\beta \in [0, 1]$.[9] It satisfies

$$(\phi, w) = \arg\max\left(\mathcal{V}_e - \mathcal{V}_u - \phi\right)^{\beta}\left(\mathcal{V}_f + \phi\right)^{1-\beta}. \tag{14}$$

[7]In the wage-posting model of Burdett, Lagos and Wright (2003, 2004), firms are restricted to post contracts promising a constant wage. This restriction generates an inefficient turnover of workers and, for some parameter values, a nondegenerate distribution of wages.Turnover is jointly efficient iff $w = y$, in which case no profit is left for firms. Firms, however, will choose $w < y$ and let workers commit more crimes than is optimal.

[8]In a different but somewhat related model, Burdett and Coles (2003) describe the optimal contract when workers are liquidity-constrained, i.e., they cannot afford a hiring fee, and risk-averse. They show that wages increase with tenure and that there is a distribution of initial wage offers. As the coefficient of risk aversion goes to 0 then the unique contract converges to a contract that is payoff equivalent to the one with an hiring fee, turnover is pairwise efficient and there is no distribution of wages.

[9]Contracts with an upward-sloping wage profile may be more realistic but are inefficient, unless liquidity constraints are imposed. Furthermore, they generate analytical complications arising from crime decisions which are not stationary. If the employment contract is restricted to a constant wage, the bargaining set may not be convex in which case the Nash solution cannot be used. For an elaboration of this point in a related context, see Shimer (2005).

Lemma 1 *The employment contract solution to (14) is such that*

$$w = y, \tag{15}$$

$$\phi = (1-\beta)\left(\mathcal{V}_e - \mathcal{V}_u\right). \tag{16}$$

Proof. See Appendix. ■

According to Lemma 1, the wage is set to be equal to the worker's productivity. Since the worker gets the entire output generated by the match this wage setting guarantees that the worker internalizes the effect of their crime decision on the total surplus of the match. The up-front payment is used to split the surplus of the match according to each agent's bargaining power.

From (11), $\mathcal{V}_f = 0$ implies $\phi = \gamma/q(\theta)$. The gain from filling a vacancy is equal to the up-front payment ϕ which equals the average recruiting cost incurred by the firm to fill a vacancy. Similarly, the expected surplus received by an unemployed worker who finds a job is $-\phi + \mathcal{V}_e - \mathcal{V}_u = \beta\gamma/\left[(1-\beta)q(\theta)\right]$.

4 Equilibrium

In order to characterize the equilibrium of the economy, we determine first the steady-state distribution of individuals across states. Denote $n_i(\kappa)$ the density measure of individuals in state $i \in \{e, u, h, p\}$. More precisely, $\int_E n_i(\kappa)d\kappa$ is the measure of individuals in state i whose utility at home is $\kappa \in E \subseteq \mathcal{K}$. Consider first individuals who do not participating in the labor force, $\kappa \geq \kappa_u$. The condition that the flows in and out each state are equal imply

$$n_o(\kappa)\lambda_o\pi[1 - G(\varepsilon_o(\kappa))] = \delta n_p(\kappa), \tag{17}$$

$$n_o(\kappa) + n_p(\kappa) = g(\kappa). \tag{18}$$

Equation (17) has the following interpretation. The flow of individuals from out-of-the-labor-force to jail, $n_o(\kappa)\lambda_o\pi[1-G(\varepsilon_o(\kappa))]$, has to be equal to the flow of individuals from jail to out-of-the-labor-force, $\delta n_p(\kappa)$. Equation (18) is an identity that specifies that individuals whose home utility is lower than κ_u are either out-of-the-labor-force or in jail.

Consider next workers who participate in the labor market ($\kappa < \kappa_u$). These individuals are either unemployed, employed or in jail. The distribution $[n_u(\kappa), n_e(\kappa), n_p(\kappa)]$ is determined by the following steady-state conditions:

$$sn_e(\kappa) + n_p(\kappa)\delta = \{\theta q(\theta) + \lambda_u \pi[1 - G(\varepsilon_u)]\} \, n_u(\kappa), \tag{19}$$

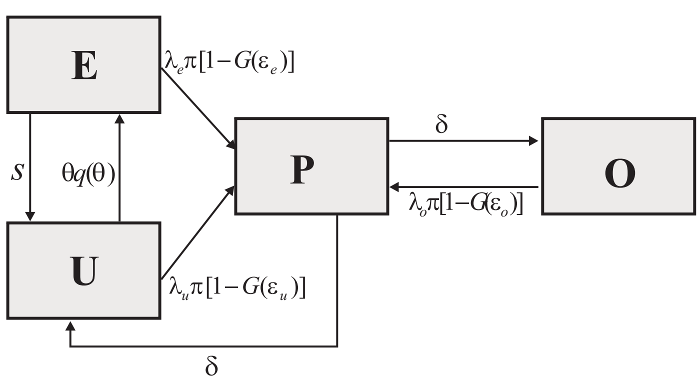
$$\theta q(\theta) n_u(\kappa) = \{s + \lambda_e \pi[1 - G(\varepsilon_e)]\} \, n_e(\kappa), \tag{20}$$

$$n_e(\kappa) + n_u(\kappa) + n_p(\kappa) = g(\kappa). \tag{21}$$

According to (19) the flows in and out of unemployment must be equal. The measure of individuals with home-productivity κ entering unemployment is equal to the sum of the employed workers who lose their jobs, $sn_e(\kappa)$, and the individuals who exit jail, $n_p(\kappa)\delta$. The flow of individuals exiting unemployment corresponds to individuals finding jobs, $\theta q(\theta) n_u(\kappa)$, or unemployed individuals committing crimes and sent to jail, $\lambda_u \pi[1 - G(\varepsilon_u)] n_u(\kappa)$. Similarly, (20) prescribes that the flows in and out of employment must be equal in steady state. Note that in steady state there are no flows between out of the labor force and into the labor force.

Figure 1 diagrams the above-mentioned flows in the labor market. We indicate above each arrow the rates at which transitions occur.

Figure 1: Flows



The equilibrium unemployment rate u is defined as the fraction of individuals in the labor force who are unemployed,

$$u = \frac{\int_0^{\kappa_u} n_u(\kappa) d\kappa}{\int_0^{\kappa_u} n_e(\kappa) d\kappa + \int_0^{\kappa_u} n_u(\kappa) d\kappa}. \tag{22}$$

From (20), it satisfies

$$u = \frac{s + \lambda_e \pi \left[1 - G(\varepsilon_e)\right]}{\theta q(\theta) + s + \lambda_e \pi \left[1 - G(\varepsilon_e)\right]}. \tag{23}$$

Note that the unemployment rate is for the non-institutional population–similar to what is commonly reported in the data. The unemployment rate decreases with market tightness and increases when employed workers commit crimes at a higher frequency, that is, ε_e decreases. As in Mortensen and Pissarides (1994) the rate at which jobs are destroyed is endogenous. In our model it depends on employed workers' decision to commit crimes.

The participation rate is computed as the fraction of individuals who are not in jail who choose to participate in the labor market. It satisfies

$$\mathcal{P} = \frac{\int_0^{\kappa_u} n_e(\kappa) d\kappa + \int_0^{\kappa_u} n_u(\kappa) d\kappa}{1 - \int_0^{\bar{\kappa}} n_p(\kappa) d\kappa}. \tag{24}$$

The model has an interesting recursive structure. First, rearrange the conditions determining individuals' crime decisions. Using the Bellman equations (1)-(2)-(4), the crime decisions (5)-(7) can be rewritten as follows:

$$\left(\frac{r+\delta}{\pi}\right) \varepsilon_u m = b - x + \frac{\beta}{1-\beta} \theta \gamma + \lambda_u m \int_{\varepsilon_u}^{\bar{\varepsilon}} \left[1 - G(\varepsilon)\right] d\varepsilon, \tag{25}$$

$$\left(\frac{r+\delta}{\pi}\right) \varepsilon_e m = y - x + \frac{(\delta - s)\gamma}{q(\theta)(1-\beta)} + \lambda_e m \int_{\varepsilon_e}^{\bar{\varepsilon}} \left[1 - G(\varepsilon)\right] d\varepsilon. \tag{26}$$

$$\left(\frac{r+\delta}{\pi}\right) \varepsilon_o(\kappa) m = \kappa p - x + \lambda_o m \int_{\varepsilon_o(\kappa)}^{\bar{\varepsilon}} \left[1 - G(\varepsilon)\right] d\varepsilon. \tag{27}$$

Given θ and τ_i, (25)-(27) determine a unique list $[\varepsilon_u, \varepsilon_e, \varepsilon_o(\kappa)]$ of critical values for crime decisions. Notice that (25)-(27) correspond to standard optimal stopping rules where the left-hand side corresponds to the gain from stopping and the right-hand side is the gain from keeping searching for opportunities. From (25) as the labor market becomes tighter the probability that an unemployed worker commits a crime falls. Also, for given θ, an unemployed worker is less likely to commit a crime if: the probability to be caught is high; the time spent in jail is high; the income when unemployed is high; the worker's bargaining power is high. According to (26) an increase in θ raises ε_e if $\delta > s$ and it reduces ε_e if $\delta < s$. From (27) individuals out-of-the-labor-force are more likely to commit crime if the instantaneous utility from being at home, κp, is low.

Using (6) the crime decision of an employed worker can also be described by

$$\varepsilon_e = \varepsilon_u + \frac{\pi\gamma}{m(1-\beta)q(\theta)}. \tag{28}$$

So, for a given ε_u employed workers are also less likely to commit crimes as market tightness increases.

Let us turn to the permanent income of an unemployed worker, $\mathcal{V}_u$. Using that $\mathcal{V}_e - \mathcal{V}_u - \phi = \beta\gamma/\left[(1-\beta)q(\theta)\right]$ and integrating by parts the integral term in (1), $\mathcal{V}_u$ obeys

$$r\mathcal{V}_u = b - \tau + \frac{\beta}{1-\beta}\theta\gamma + \lambda_u m \int_{\varepsilon_u}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon. \tag{29}$$

As in Pissarides' model, the permanent income of an unemployed is proportional to labor market tightness. In our model, it also depends on the return of criminal activities, the last term on the right-hand side of (29). As unemployed workers commit more crimes, their permanent income increases.

In order to determine market tightness, we use the condition $\mathcal{V}_e - \mathcal{V}_u = \gamma/\left[(1-\beta)q(\theta)\right]$. From (2) and (29) we obtain

$$\frac{(r+s)\,\gamma}{(1-\beta)q(\theta)} = y - b - \frac{\beta}{(1-\beta)}\theta\gamma - \lambda_u m \int_{\varepsilon_u}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon + \lambda_e m \int_{\varepsilon_e}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon. \tag{30}$$

Given the thresholds ε_u and ε_e (30) determines a unique θ. Up to the last two terms on the right-hand side, (30) is identical to the equilibrium condition in the Pissarides model. If crime activities are more valuable for unemployed workers than for employed ones, i.e., the sum of the last two terms is negative, then the presence of crime opportunities tends to reduce market tightness. This will be the case if the arrival rates of crime opportunities are the same for employed and unemployed workers, $\lambda_e = \lambda_u$, since

$\varepsilon_e > \varepsilon_u$. Using (28), we obtain a relationship between ε_u and θ,

$$\begin{aligned}\frac{(r+s)\,\gamma}{(1-\beta)q(\theta)} &= y-b-\frac{\beta}{(1-\beta)}\theta\gamma-\lambda_u m\int_{\varepsilon_u}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon \\ &\quad +\lambda_e m\int_{\varepsilon_u+\frac{\pi\gamma}{m(1-\beta)q(\theta)}}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon. \end{aligned} \tag{31}$$

According to (31), if $\lambda_u[1-G(\varepsilon_u)] > \lambda_e[1-G(\varepsilon_e)]$ then θ increases with ε_u. This condition is satisfied, for instance, if $\lambda_u = \lambda_e$.

From (8), the reservation utility at home below which individuals participate in the labor force satisfies

$$\kappa_u p = b+\frac{\beta}{1-\beta}\theta\gamma+(\lambda_u-\lambda_o)m\int_{\varepsilon_u}^{\bar{\varepsilon}}[1-G(\varepsilon)]d\varepsilon. \tag{32}$$

The following defines an equilibrium.

Definition 2 *A steady-state equilibrium is a list* $\{\theta,\, \kappa_u,\, \varepsilon_u,\, \varepsilon_e,\, \varepsilon_o(\kappa),\, n_e(\kappa),\, n_u(\kappa),\, n_o(\kappa),\, n_p(\kappa)\}$ *such that:* θ *satisfies (31);* κ_u *satisfies (32);* $\{\varepsilon_u,\, \varepsilon_e,\, \varepsilon_o(\kappa)\}$ *satisfies (25), (28), (27);* $\{n_e(\kappa),\, n_u(\kappa),\, n_o(\kappa),\, n_p(\kappa)\}$ *satisfies (17)-(21).*

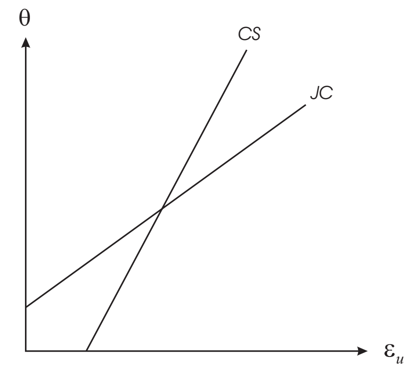
The model can now be solved recursively. First, the crime decisions of individuals out of the labor force are determined independently of other endogenous variables by (27). Second, the pair (θ, ε_u), are determined jointly from (25) and (31). Third, knowing (θ, ε_u), one can use (28) and (32) to find ε_e and κ_u. Finally, knowing $\{\theta, \kappa_u, \varepsilon_u, \varepsilon_e, \varepsilon_o(\kappa)\}$ the steady-state distribution $\{n_e(\kappa), n_u(\kappa), n_o(\kappa), n_p(\kappa)\}$ is obtained from (17)-(21).

Figure 2 represents the determination of the pair (θ, ε_u). We denote CS (crime schedule) the curve representing (25) and JC (job creation) the curve representing (31). Recall that CS always slopes upward while JC can slope upward or downward depending on the the values for λ_e and λ_u. In the case where $\lambda_u = \lambda_e$ the two curves are upward-sloping. Along CS, as the number of vacancies per unemployed increases workers are less likely to commit crimes. Along JC, as the frequency of crime falls the number of jobs in the market increases. The following Lemma establishes that if JC and CS intersect then they intersect once. This result is illustrated in Figure 2.

Lemma 3 *In the space* (ε_u, θ) *the curve* JC *intersects the curve* CS *by above.*

Proof. See Appendix. ■

Figure 2: Equilibrium



Interestingly, the determination of equilibrium is reminiscent to the one in the Mortensen-Pissarides (1994) model where labor market tightness and the job destruction rate are determined jointly through two conditions that can be represented graphically. The CS curve in our model is analogous to the job destruction curve in the Mortensen-Pissarides model in that workers' crime decisions affect the duration of a job. Denote ε_u^0 the value of ε_u that solves (25) when $\theta = 0$.

Proposition 4 *There exists a unique equilibrium such that $\theta > 0$ if*

$$y - b + (\lambda_e - \lambda_u)m \int_{\varepsilon_u^0}^{\bar{\varepsilon}} [1 - G(\varepsilon)]d\varepsilon > 0. \tag{33}$$

Proof. See the Appendix. ■

Proposition 4 shows that equilibrium exists and is unique. So despite the possibility of strategic complementarities between individuals' crime decisions and firms' entry decisions, there is no multiple steady-state equilibria in this model. The condition (33) for firms entering the market requires that the rate at which unemployed workers receive crime opportunities is not too high compared to the arrival rate of crime opportunities for employed workers; moreover, it is satisfied if $\lambda_e = \lambda_u$.

Proposition 5 *In any equilibrium where $\theta > 0$ then $\varepsilon_e > \varepsilon_u$ and $\varepsilon_o(\kappa) \geq \varepsilon_u$ for all $\kappa \geq \kappa_u$.*

Proof. The result according to which $\varepsilon_e > \varepsilon_u$ comes from (28). From (27), $\varepsilon_o(\kappa)$ is nondecreasing in κ. Since $\varepsilon_o(\kappa_u) = \varepsilon_u$ we have $\varepsilon_o(\kappa) \geq \varepsilon_u$ for all $\kappa \geq \kappa_u$. ■

Proposition 5 shows that unemployed workers tend to commit more crimes than employed workers and individuals out-of-the-labor-force. The intuition for this result is as follows. Employed workers are paid their marginal product which is larger than the income they receive when unemployed. Therefore, the opportunity cost of being caught and sent to jail is higher for employed workers. Also, individuals who choose not to participate in the labor force have a higher expected utility than individuals who are unemployed. Consequently, those individuals suffer a higher cost of being sent to jail than unemployed workers.

The following Proposition provides a condition under which the equilibrium is characterized by no criminal activities. Denote $\hat{\theta}$ the value of market tightness that solves

$$\frac{(r+s)\,\gamma}{q(\hat{\theta})} = (1-\beta)\,(y-b) - \beta\hat{\theta}\gamma. \tag{34}$$

This is the market tightness that would prevail in an economy without crime.

Proposition 6 *If*

$$\frac{(r+\delta)}{\pi}\bar{\varepsilon}m \leq b - x + \frac{\beta}{1-\beta}\hat{\theta}\gamma \tag{35}$$

then the equilibrium is such that $\theta = \hat{\theta}$ and no crime occurs.

Proof. From Proposition 5, no crime occurs in equilibrium iff $\varepsilon_u \geq \bar{\varepsilon}$. From (30) if $\varepsilon_u \geq \bar{\varepsilon}$ then $\theta = \hat{\theta}$. From (25) the condition $\varepsilon_u \geq \bar{\varepsilon}$ requires (35). ■

According to Proposition 6, there is no crime in equilibrium provided that the probability of being caught is sufficiently high and the time spent in jail is sufficiently long.

5 Calibration

In this section and the following we use the model to quantitatively analyze the relationship between the labor market and criminal activity. Since female involvement in criminal activities has been widely understudied, we focus on females as an illustration of our model. Recall that there is no element in our model that is gender specific. However, we can assume the parameter values for males and females differ in terms of their utility at home, the arrival rates of jobs, the productivity of those jobs, the arrival rate of crimes, and the probability of being caught committing those crimes. We take these and other differences into account in our calibration. In addition, they may differ along other dimensions. We explore a few of them including differences with respect to the bargaining power and the unemployment flow.

The model is calibrated to the U.S. labor market along the lines of Shimer (2005). The unit of time corresponds to a year and the rate of time preference is set to $r = 0.048$. The output from a match is normalized to $y = 1$. The flow of utility when unemployed is $b = 0.4$. The matching function, $m(U, V) = AU^{\eta}V^{1-\eta}$, is assumed to be Cobb-Douglas with constant returns to scale and set $\eta = 0.72$.[10] To ensure search externalities are internalized through the wage mechanism, we impose the Hosios (1990) condition, i.e., $\beta = \eta = 0.72$.[11] The distribution of returns in the home sector is exponential, $g(\kappa) = e^{-\kappa}$. We calibrate p so the models participation rate matches the female participation rate in 2003, which was 59%. The calibration implies a value of $p = 1.04$.

To calibrate the job finding rate, we use total female unemployment as well as female short term unemployment, i.e., those unemployed less than 5 weeks.[12] Let u_t^s denote the number of workers unemployed for less than one month in month t, and u_t be the total number of unemployed women in month t. The job finding rate is defined as

$$f_t = 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t}. \tag{36}$$

For the years 1976-2004 (the only years this data is available for females) $f_t = 0.434$ per month, implying

[10] For a survey of the literature on the aggregate matching function, see Petrongolo and Pissarides (2001).

[11] The Hosios condition emerges endogenously if wages are posted and search is directed. See Moen (1997).

[12] The female data series which began in 1976 can be found at http://www.bls.gov/cps. Implicitly, we assume that the labor market is segmented by gender. This may reflect the fact that male and female are to a large extent specialized by occupations.

the annualized expected number of job offers, $\theta q(\theta)$, is 5.208. We infer the job separation rate for females using the two unemployment series given above. In the data, when a worker is separated from her job, she has on average half a month to find a new job before she is recorded as unemployed. Therefore, letting e_t be the number of women employed in month t we calculate the separation rate as

$$s_t = \frac{u^s_{t+1}}{e_t(1 - \frac{1}{2}f_t)}, \tag{37}$$

which can be calculated for females from 1976-2004. The result is the average monthly separation rate for females is 0.038 which implies an annualized rate of 0.456, i.e., jobs last, on average, about 2 years. The parameters A and γ are chosen to match the average job finding rate and the average $v-u$ ratio. In the model the $v-u$ ratio, or θ, is arbitrary and normalized to one.[13] Therefore, we set $A = 5.208$ and $\gamma = 0.2056$.

We now turn to the parameters of the crime sector. To begin, we consider only Type I property crimes as defined by the FBI, which includes larceny, burglary, and motor-vehicle theft. We exclude violent crimes because they do not necessarily respond to economic motivations. We do not include drug related crimes because of the difficulty in capturing addictive behaviors which are inherent in all types of drug crimes.[14] Finally, the FBI defines Forgery, Fraud, and Embezzlement as a Type II offense and does not collect the number of these types of crimes. We have taken the data that exists, arrest data, and assumed the probability of being caught is the same for Type I and Type II offenses in order to extrapolate out the number of Type II offenses. The result for our analysis is a level shift that does not effect our comparative statics. Therefore, we follow the crime literature and exclude Type II offenses in our analysis.

The female property crime rate is the product of the total number of property crimes, as defined by

[13] The reason $\theta = v/u$ is arbitrary when choosing γ is because θ and γ appear as a product in the equilibrium conditions. To see this, rewrite the equilibrium conditions as

$$\begin{aligned}
\left(\frac{r+\delta}{\pi}\right)\varepsilon_u m &= b - x - \tau + \frac{\beta}{1-\beta}\theta\gamma + \lambda_u m \int_{\varepsilon_u}^{\bar{\varepsilon}} [1 - F(\varepsilon)]\, d\varepsilon \\
\frac{(r+s)\gamma\theta}{(1-\beta)f} &= y - b - \frac{\beta}{(1-\beta)}\theta\gamma - \lambda_u m \int_{\varepsilon_u}^{\bar{\varepsilon}} [1 - F(\varepsilon)]\, d\varepsilon + \lambda_e m \int_{\varepsilon_u + \frac{\pi\gamma\theta}{m(1-\beta)f}}^{\bar{\varepsilon}} [1 - F(\varepsilon)]\, d\varepsilon.
\end{aligned}$$

where $f = \theta q(\theta)$ denotes the job finding rate given by the data.

[14] Refer to Cozzi(2005) for an analysis on the link between drugs and crime. The study is empirically driven and builds on the framework of Imrohoroglu, Merlo, and Rupert(2004).

the FBI, and the percent of female arrests. The probability of being caught is derived from the number of females sent to prison divided by the number of female crimes, implying $\pi = 1.5\%$. We exclude those sentenced to probation when calculating the probability of being caught because individuals on probation or parole are not forced out of employment and/or home production.[15] The annual number of females sent to prison is the product of the total number of convictions, the percentage of the total who were female, and the percent of those females who were incarcerated.[16] The length of incarceration for women was 17 months in 2002, so that $\delta = 0.706$. The average per capita loss from crime, τ, is set to 0.001.[17] Since we do not have much information on the utility or disutility from being in jail, we let $x = 0$.[18]

We assume that the distribution of the value of crime opportunities $G(\cdot)$ is exponential, $g(\varepsilon) = e^{-\varepsilon}$. While there is no hard evidence to support one particular distribution for the values of crime opportunities, the exponential distribution captures the idea that it is much less likely to receive large value crime opportunities than low value ones. In contrast, under a uniform distribution it is as likely to get a high crime opportunity than a low value one. As a consequence, to match the data the arrival rate of crime opportunities has to be low when using the uniform distribution, roughly fifty times lower than under the exponential (the annual values are: $\lambda_o = 0.078$, $\lambda_u = 0.30$ and $\lambda_e = 0.045$). As a result, since the mean of the exponential distribution is normalized to one, the average value of a crime opportunity is m. The average amount stolen in the data is calculated as the ratio of the dollar value stolen and not recovered divided by the number of crimes. This gives $m = 0.00525$. [19]

The values of λ_e, λ_u, and λ_o are calibrated to target the probability an individual commits a crime

[15] Fines, mandatory volunteer service, and other types of punishments that do not involve jail have not been modeled given they are constructed to allow individuals to continue to participate in the labor market.

[16] The values are taken from the most recent survey "State Court Sentencing of Convicted Felons, 2002," Tables 1.1, 2.1, and 2.4, respectively. These numbers represents both state and federal convictions. The Bureau of Criminal Justice Statistics collects data at the state level and then estimates that another six percent are convicted at the federal level.

[17] The total number of property crimes is reported in the Uniform Crime Reports, 2003, Table 1. The percent of female arrests is located in Table 42 of the Uniform Crime Reports, 2003. The percent of females convicted to jail or prison can be found in "State Court Sentencing of Convicted Felons, 2002," Table 2.4 and the average time spent in jail or prison is located in Table 2.6. The total dollar amount lost and not recovered from crime is published in the Uniform Crime Reports 2003, Table 24. The population is non-institutional as defined and calculated by the Bureau of Labor Statistics.

[18] In BLW's benchmark calibration, $x = 0.25$ whereas Imrohoroglu, Merlo, and Rupert (2004) target $x = 0.15$. We have tested different values for x and we have verified that the calibration is basically unaffected. The threshold values ε_i fall as x rises, which decreases our target for m. The effects on the arrival rates of crime are found to be quite small.

[19] Note that the value used to calculate the amount stolen in the data is not necessarily its worth to the criminal. Even the most liquid of stolen goods, arguably currency which makes up 10% of all stolen goods, is subject to laundering costs. These are assumed to be negligible and if taken into account would be compensated by lowering the value of m.

given they are in a particular state. The crime rates for employed, unemployed, and non-participants, (which correspond to $\lambda_i[1 - G(\varepsilon_i)]$ in the model) are 2.5%, 17.7%, and 2% respectively. The crime rate of an individual in a particular state is computed as the product of the number of female crimes and the percent of females incarcerated when in the particular state, divided by the number of females in that state.[20] The implied values of the λ's are: $\lambda_e = 1.25$, $\lambda_u = 5.94$, and $\lambda_o = 3.25$. Our interpretation of why they differ here is due to our restrictive assumption on time use–the full unit of time is used in either working, search while unemployed, or producing at home. If we would impose $\lambda_e = \lambda_u = \lambda_o$ then $\lambda_i = 2.03$. Even though $\lambda_e \neq \lambda_u \neq \lambda_o$, the theoretical result that unemployed agents commit more crimes continues to hold in our calibration.

Table 1 provides a summary of the parameters used in the calibration.

Table 1: Parameters

r	0.048	real interest rate
b	0.400	unemployed utility flow
β	0.720	bargaining power of workers
η	0.720	elasticity of matching function
γ	0.206	recruiting cost
s	0.456	job destruction rate
A	5.21	efficiency of matching technology
p	1.04	price of intermediate good
τ	0.001	loss from crime
x	0.00	payment when in jail
π	0.015	apprehension probability
δ	0.706	rate of exit from jail
λ_e	1.25	flow of crime opportunities when employed
λ_u	5.94	flow of crime opportunities when unemployed
λ_o	3.25	flow of crime opportunities when NILF
m	0.005	average value of a crime oppurtunity
θ	1.00	vacancy-unemployment ratio

6 Comparative statics

In this section we examine both qualitatively and quantitatively how changes in some relevant variables affect female crime and female outcomes in the labor market. We distinguish changes in terms of policies and changes in terms of technology or preferences. Policies related to welfare programs and

[20]The percent of females incarcerated when in $i = \{e, u, o\}$ is taken from the most recent "Survey of Federal and State Correctional Facilities," 1997. The number of females in $i = \{e, u, o\}$ is taken from the Bureau of Labor Statistics for 2003.

unemployment insurance can have important implications for the labor market and crime. We will consider the case of unemployment benefits since a noticeable difference between unemployed men and women is in their eligibility to receive unemployment insurance benefits. Policies aimed at reducing discrimination against women can also affect wages and job finding rates, and therefore incentives to engage in criminal activities. Finally, crime policies can directly affect criminal behavior, and indirectly the labor market. In terms of technological change, we consider a technological improvement (or an alteration of preferences in favor of market activities) in the home sector, and a skill-biased technological shock in the market. These two types of shocks have been well documented in the literature. We will also consider a change in the availability of crime opportunities.

With a slight abuse of language, an increase in participation in what follows means an increase in κ_u, the reservation home productivity above which individuals choose to work at home. Proofs for the following propositions can be found in the Appendix.

6.1 Benefits

Unemployment insurance is often blamed for high unemployment duration. Several countries have reduced the generosity of their unemployment systems in order to increase the incentives of the unemployed to accept jobs and to reduce the pressure on wages. A case in point is the United Kingdom which introduced the Job Seeker's Allowance in October 1996. According to Machin and Marie (2004), "...the duration of non means-tested contributory benefits was reduced from 12 to 6 months". They argue that this reform has led to an increase in crime. To illustrate the effects of such a policy in our model, consider the effects of an increase in the income flow b received by unemployed workers (See Panel (i) in Figure 3).[21]

Proposition 7 *An increase in b: reduces market tightness; raises market participation; reduces the crime rate of unemployed; increases the crime rate of employed workers if $\delta > s$ and decreases it if $\delta < s$.*

[21] Unemployment insurance in practise are different from a constant income flow when unemployed. Typically, the eligibility to unemployment insurance ends after a given duration of unemployment. See Atckeson and Micklewright (1991) and Holmlund (??).

For given θ, an increase in b provides unemployed workers with lower incentives to commit crimes: The curve CS shifts to the right. For given ε_u, an increase in b raises the threat point of workers when bargaining so that fewer firms enter the market: The curve JC shifts downward. So the overall effect seems ambiguous. However, Proposition 7 establishes that the measure of vacancies per unemployed falls as well as unemployed workers' incentives to commit crimes. Furthermore, a larger fraction of individuals participate in the labor force. The effect of an increase in the income flow of the unemployed on the crime rate of the employed depends on the relative sizes of δ and s. The intuition is as follows. As b increases the value of being unemployed increases. But both employed workers and workers in jail (provided they were in the labor force when sent to prison) will ultimately end up in the pool of unemployed. The transition from employment to unemployment occurs at rate s while the transition from jail to unemployment occurs at rate δ. If $\delta > s$ then the value of being in jail tends to increase relatively more, increasing the incentive to commit crimes, ε_e decreases.[22] In this case, the overall effect on crime is ambiguous. In contrast, if $\delta < s$ then the crime rate declines for all categories of individuals.

6.2 Discrimination

Among other things, the difference between male and female wages has been associated with discrimination against women. Over the last several decades policies have been put into place to promote equal opportunity and reduce discriminatory behavior. One way to capture the reduction in wage discrimination in our model is to assume that the bargaining power of women in the negotiation of their wages has increased. The next Proposition describes the effects of an increase in β on the labor market and crime.

Proposition 8 *An increase in β: reduces market tightness; raises participation if $\beta < \eta(\theta)$ and decreases it if $\beta > \eta(\theta)$; reduces the crime rate of the unemployed if $\beta < \eta(\theta)$ and increases it if $\beta > \eta(\theta)$; reduces the crime rate of the employed if $\delta > s$ and $\beta > \eta(\theta)$ or $\delta < s$ and $\beta < \eta(\theta)$, and increases it otherwise.*

An increase in β reduces firms' expected surplus from a match and therefore the supply of vacancies. The effect of an increase in β on the crime rate of unemployed workers depends on how β and $\eta(\theta)$, the elasticity of the matching function, are ordered. As it is well-known from Pissarides (2000), the value

[22] A related result can be found in Burdett, Lagos and Wright (2003).

of being unemployed, $\mathcal{V}_u$, is a nonmonotonic function of β that reaches a maximum when $\beta = \eta(\theta)$, called the Hosios Condition. If $\beta = \eta(\theta)$ then a change in β has only a second-order effect on crime. If $\beta < \eta(\theta)$ then an increase in β raises the value of being unemployed. As a consequence, unemployed workers reduce their crime rate and more agents participate in the labor force. If $\beta > \eta(\theta)$ then the opposite happens. If the average jail sentence is smaller than the average duration of a job then the crime rate of employed workers varies in the opposite direction as the crime rate of unemployed workers.

6.3 Crime policies

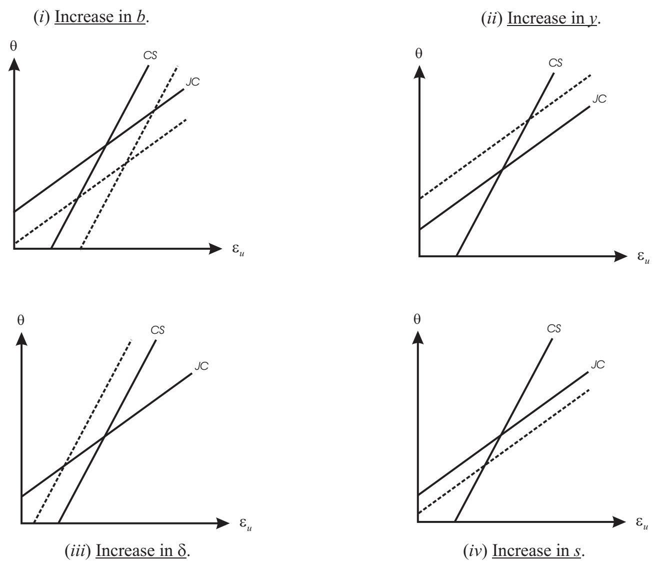
Policies aimed at deterring criminal activities have also been changing over time. The following Proposition considers the effects of a change in crime policies such as an increase in the length of jail sentences or an improvement in the technology to catch criminals. We focus here on the case where all individuals receive crime opportunities at the same rate, $\lambda_e = \lambda_u = \lambda_o$.

Proposition 9 *Assume $\lambda_e = \lambda_u = \lambda_o$. An increase in δ or a decrease in π: decreases market tightness; increases the crime rates of all individuals irrespective of their labor force status; decreases participation in the labor force.*

An increase in δ, the Poisson rate at which an individual exits jail, moves the CS curve to the left (See Panel (iii) in Figure 3). Since the punishment for committing crimes is weaker, both unemployed and employed workers commit more crimes and firms open fewer vacancies. If the arrival rate of crime opportunities is the same for all individuals then fewer individuals wish to participate in the market. The effects of an increase in π, the probability of being caught, are opposite to those of an increase in δ.

If individuals do not receive crime opportunities at the same rate then the previous results can be overturned. In particular, an increase in δ or a decrease in π can increase market tightness if $\lambda_e[1 - G(\varepsilon_e)] > \lambda_u[1 - G(\varepsilon_u)]$. If the time spent in jail or the probability of being caught fall then the return to criminal activities go up. If employed workers commit crimes at a higher frequency than unemployed workers then the relative value of being employed increases which gives firms incentives to open vacancies.

Figure 3: Comparative Statics



6.4 Changes in the home sector

Greenwood *et al.* (2005) have argued that technological progress in the household sector played a major role in liberating women from the home. Fernandez *et al.* (2004) and Fortin (2005) have emphasized changes in preferences towards market activities to explain this liberation. Criminologists have also claimed that gains in gender equality in the 60's and 70's, have reduced the gender gap in terms of involvement in criminal activities (Steffensmeier and Allan, 1996). We will determine here whether the technological improvements in the home sector that have generated the increased participation of women can also explain their higher involvement in criminal activities. In our model, changes in preferences towards market activities are represented by the parameter p.

Proposition 10 *A decrease in p raises participation and the crime rates of individuals out of the labor force. It does not affect the crime rates of employed and unemployed workers.*

According to Proposition 10, as the utility for nonmarket activities falls, participation increases since the benefits of staying at home are smaller. Also, agents out of the labor force have higher incentives to commit crime.

To quantify the increase in the female labor force participation rate witnessed over the last 50 years, which went from 37% to 59%, we assume the average gains from staying at home have fallen relative to market participation. The average gain from staying at home could be falling for two reasons. Home production goods are cheaper, p, or individual preferences have shift, κ. Due to our characterization, changing p or the mean of $H(\kappa)$ are equivalent. Therefore without any loss of generality, we generate the rise in participation entirely through a drop in p. In order to generate the rise in the female participation rate that we see, the average gain from staying at home has to fall by roughly half as shown in Table 2.

Table 3 reports the steady-state outcome for crime corresponding to these different values of p. The technological progress in the home sector that is responsible for the increased participation in the labor market generates a 43% increase in crime. So, the liberating process of women from the home can explain a quantitatively significant increase in female crime.

In analyzing the mechanism for rising female crime, the change can be separated into two effects. The first effect arises from the fact that a woman's time is relatively less valuable at home, thanks to technological progress. Therefore, the cost of being caught committing a crime is smaller, which implies women who stay at home have a higher probability of committing crime. The first effect can be seen in the third row of Table 3. The second effect is due to the change in the composition of the labor force. More women are either employed or unemployed, and both types of women commit crimes at a higher frequency than those not in the labor force (as seen in the first three rows of Table 3). Therefore, as women enter the job market, the probability of those women committing a crime rises.

Table 2: Changes in Labor Force Participation

	1960	2005
p	2.00	1.04
Employed	34%	54%
Unemployed	3%	5%
NILF	63%	41%

Table 3: Changes in Crime

	1960	2005
Pr(Commit Crime | Employed)	0.025	0.025
Pr(Commit Crime | Unemployed)	0.177	0.177
Pr(Commit Crime | NILF)	0.011	0.02
Employed crime rate per 1000	4.4	6.99
Unemployed crime rate per 1000	2.75	4.38
NILF Crime rate per 1000	3.67	4.15
Total Crime rate per 1000	10.82	15.52

6.5 Productivity

Technological changes have also occurred in the market that have increased productivity. The next Proposition describes the effects of an increase in y.

Proposition 11 *An increase in y raises market tightness, labor force participation, and reduces unemployment and the crime rates of all workers.*

As workers become more productive a larger measure of firms enter the market. Graphically, the JC curve shifts upward. Consequently, both θ and ε_u increase. The fact that the labor market becomes tighter implies the cost to an unemployed worker of being caught committing a crime increases. As a consequence, unemployed workers commit fewer crimes. Similarly, the wage, which is equal to productivity, increases raising employed workers' cost of being caught committing a crime. So the crime rate of employed workers falls. Finally, since productivity in the market increases, participation increases as well. The effects of an increase in the separation rate, s, are analogous to those of a decrease in y (See Panel (iv) in Figure 3). Of course, if b, γ, m and x are proportional to y then an increase in productivity would be qualitatively equivalent to a decrease in p. In this case the unemployment rate is unchanged, participation increases and workers out of the labor force commit more crimes.

Technological improvements in the market, however, are often associated with increased inequality among workers. This has been labeled skill-biased technological progress. Hornstein *et al.* (2005) write that "the average and median wage have remained constant in real terms since the mid-1970's," while "the 90-10 weekly wage ratio rose by 35 percent for both males and females in the period 1965-

1995: from 1.20 to 1.55 for males, and from 1.05 to 1.40 for females. The increase in inequality took place everywhere in the wage distribution: both the 90-50 differential and the 50-10 differential rose by comparable amounts." Evidently, wage inequality for women has increased more than it has for men. We investigate the effect of such changes on the crime behavior of females.

In our model, workers are homogenous in terms of their productivity in the market. To study skill-biased technological progress we have to relax this assumption. Following Mortensen and Pissarides (1999), we assume that workers have different productivities, y, corresponding to different skill levels, and that labor markets are segmented by skills. Therefore, we solve for the equilibrium for each submarket separately. Each submarket is subject to the same matching frictions and the same separation rate. The experiment we perform allows for five groups representing the three quantiles (25^{th}, 50^{th} and 75^{th}) and the first and last deciles (10^{th} and 90^{th}).

To begin the illustration, we use information on income inequality in the U.S. for the years 1960 and 2000 taken from Katz and Autor (1998). Next, we choose wages for each group (five for each year) in order to simulate the inequality found in the data. Table 4 displays the choosen values for each group by year. Table 5 illustrates how the inequality found in the data matches the inequality produced from Table 4. Note that for 2000, the wages have been normalized so the median wage is equal to 1, and that the median in each year stays constant. The wage found in Table 4 shows, for example, that the wage of the lowest decile declined from 0.6 in 1960 to 0.5 in 2000. Wages at the 90^{th} percentile increased from 1.7 in 1960 to 2.0 in 2000.

Table 4: Simulated Wage Distribution

Distribution	1960	2000
10%	0.6	0.5
25%	0.8	0.65
50%	1.0	1.0
75%	1.2	1.35
90%	1.7	2.0

The goal is to illustrate how income ineqaulity is associated with higher levels of crime. It is possible to reformulate the parameters by skill group. As an illustration, we take a simplified approach by holding

Table 5: Female Inequality, Log Wage

Data			Model		
Inequality Measure	1960	2000	Inequality Measure	1960	2000
90-10 Ratio	1.04	1.38	90-10 Ratio	1.04	1.39
50-10 Ratio	0.50	0.68	50-10 Ratio	0.51	0.69
90-50 Ratio	0.54	0.70	90-50 Ratio	0.53	0.69
Standard Dev.	0.41	0.55	Standard Dev.	0.40	0.56

all the parameters constant except the arrival rates of crime and the welfare when unemployed. The arrival rates are chosen such that the overall crime rate in 2000 matches the benchmark. We normalize the λ_i's by a factor of 2.065, or $\lambda_e = .605$, $\lambda_u = 2.875$, and $\lambda_o = 1.574$. The unemployment benefit is kept at a constant fraction of the wage, or $b = 0.4y$.[23]

Once the ten equilibria are calculated, we find the overall crime rate, labor force participation rate, and employment status shown in Table 6 by taking the weighted averages (using equal weights of 0.2 for each group) of the five separate groups found in each year. The result is that crime in the model rises by roughly 35%. The mechanism through which crime increases is intuitive. Skilled workers tend to commit fewer crimes since their relative wage increases while unskilled workers commit more crimes due to a lower relative wage. Whether total crime increases or decreases depends on the distributions of the crime values. Under an exponential distribution, high value crimes are much less likely than low value ones. As a consequence, the decrease in the crime rate of skilled workers is less than the increase in the crime rate of unskilled workers. It should be noted that if one adopts a uniform distribution for crime values, the increase in the dispersion of workers' productivities has almost no effect on total crime.

Table 6: Effects of Changes in the Wage Distribution

	1960	2000
Total Crime	11.44	15.52
Employment	0.54	0.54
NILF	0.41	0.41
Unemployment	0.05	0.05

[23] We have tested the model using a constant $b = 0.4$. The results in Table 6, which are calculated for $b = 0.4w$, change only slightly. For instance, the change in the crime rate becomes 37% as opposed to 35% as found in Table 6.

6.6 Crime opportunities

Another reason that has been put forward for the increase in female crime is the fact that the opportunities to commit crime while in the market have become more frequent. We consider in the following Propositions how the availability of crime opportunities affect individuals' crime behavior and the labor market.

Proposition 12 *An increase in λ_e: raises market tightness and labor force participation; reduces the crime rate of unemployed workers.*

The effects of an increase in λ_e on the labor market are paradoxical. An increase in the arrival rate of crime opportunities for employed workers moves JC upward since the value of being employed increases. Consequently, both θ and ε_u increase: The market becomes tighter and unemployed workers commit fewer crimes. Since crime opportunities arrive at a higher frequency, employed workers become more choosy in terms of the crime opportunities they undertake. Therefore, the effect on the crime rate of employed workers and the overall effect on crime is ambiguous. Finally, more individuals participate in the market.

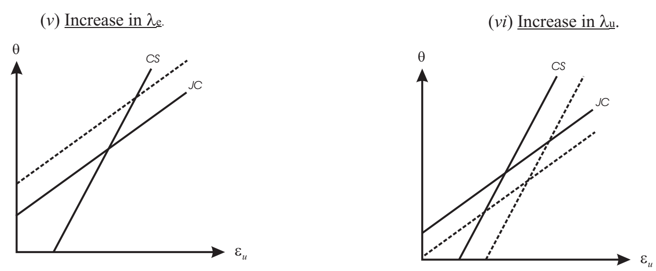
Consider next an increase in the arrival rate of crime opportunities for unemployed workers, shown in 4.

Proposition 13 *An increase in λ_u: reduces market tightness but increases labor force participation; raises ε_u and increases the crime rate of employed workers if $\delta - s > 0$ and decreases it if $\delta - s < 0$.*

Following an increase in λ_u, the curve CS moves to the right since unemployed workers can afford to be more selective in terms of their crime projects when crime opportunities are more readily available. The curve JC moves downward since the fact that workers can commit more crimes raises their threat points in the bargaining. Proposition 13 establishes that ε_u increases and θ falls. The effect on the crime rate of employed workers is ambiguous and depends on the sign of $\delta - s$. If the average jail sentence is lower than the average duration of a job then employed workers commit more crimes. Finally, since unemployed workers are made better-off a larger measure of individuals want to participate in the labor force.

Propositions 12 and 13 show that an increase in λ_e has very different effects on the labor market and crime compared to an increase in λ_u. If it becomes easier to commit crimes when employed the labor market tightness increases whereas labor market tightness decreases if unemployed workers have a greater access to crime opportunities. In both cases, participation to the labor force increases.

Figure 4: Comparative Statics



7 Conclusion

This paper has proposed a simple model of the labor market and crime. Our description of the labor market follows the canonical model of Pissarides (2000) extended to have a participation decision. Criminal activities are described as the result of rational decisions to undertake crime opportunities that occur randomly. The outcome of the labor market and the extent of criminal activities are determined jointly. We have been able to show how various parameters (unemployment benefits, workers' bargaining power, availability of crime opportunities) affect the equilibrium. The model generates crime rates that differ according to labor force status — the unemployed have the highest propensity to commit crime compared to being employed or out of the labor force —, a feature that is present in the data. The model has been calibrated to the US data and various explanations for the increase in female crime have been investigated. For instance, technological progress in the home sector can generate a rise in crime along with a rise in labor market participation. The model also generates an increase in crime through an increase in income inequality.

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8 Appendix 1: Home sector

In this appendix, we provide foundations for the utility that individuals get in the home sector. The home sector is similar in spirit to the one in Greenwood et al. (2005). There exists a technology that allows to produce the final good outside of the market from labor and an intermediate input. The home-technology is $F(\kappa\ell_o, k)$ where $\ell_o \in \{0,1\}$ is the time devoted to home production, κ is the specific productivity of an individual, and k is the quantity of the intermediate input. Individuals who do not spend any time at home ($\ell_o = 0$) can still use the home technology with intermediate goods only. The price of the intermediate good in terms of the final good is p.

Denote $z(\kappa\ell_o, p) = \max_k[-pk + F(\kappa\ell_o, k)]$ the net return of the home technology. We assume that the technology is such that the demand for intermediate goods is decreasing with ℓ_o. Consequently, $z(\kappa, p) - z(0, p)$ is increasing with p. As the price of the intermediate good rises individuals have higher incentives to use their time in the home technology. The above description of the home sector captures some of the features of the model in Greenwood et al. (2005). That is, on pg. 129 they argue

> "If the rental price of durables drops, then the household will demand more of them. When durables and housework are substitutes in the Edgeworth–Pareto sense, an increase in durables decreases the marginal product of housework denominated in utility terms (i.e. the marginal product of housework multiplied by marginal utility of home goods). Hence, housework falls. Thus, market work increases."

Consider the specification $F(\kappa\ell_o, k) = B\left[\kappa\ell_o + k\right]^a$ with $B > 0$, $a < 1$. Then, $z(\kappa\ell_h, p) = \max_k\left[-pk + B(\kappa\ell_h + k)^a\right]$. Assuming an interior solution, $k = \left(\frac{aB}{p}\right)^{\frac{1}{1-a}} - \kappa\ell_h$ and

$$z(\kappa\ell_h, p) = -p\left[\left(\frac{aB}{p}\right)^{\frac{1}{1-a}} - \kappa\ell_h\right] + B\left(\frac{aB}{p}\right)^{\frac{a}{1-a}}.$$

Consequently, the return from staying at home is $z(\kappa, p) - z(0, p) = p\kappa$ as in our current formulation. In order to guarantee that the choice for k be interior, $\left(\frac{aB}{p}\right)^{\frac{1}{1-a}}$ must be larger than κ.

9 Appendix 2

Proof of Lemma 1 According to Nash's axioms, (ϕ, w) must be pairwise Pareto-efficient. Since the up-front payment ϕ allows the worker and the firm to transfer utility perfectly, the wage, w, must be chosen to maximize the total surplus of the match. The comparison of (6) and (13) shows that the match surplus is maximized iff $\mathcal{V}_f = 0$. From (10), $\mathcal{V}_f = 0$ requires $w = y$. Finally, the first-order condition of (14) with respect to ϕ yields (16).

Proof of Lemma 3 The slope of CS in the (ε_u, θ) space is

$$\left.\frac{d\theta}{d\varepsilon_u}\right|_{CS} = (1-\beta)m\frac{r+\delta+\lambda_u\pi[1-G(\varepsilon_u)]}{\pi\beta\gamma}.$$

The slope of JC in the (ε_u, θ) space is

$$\left.\frac{d\theta}{d\varepsilon_u}\right|_{JC} = (1-\beta)m\frac{\lambda_u[1-G(\varepsilon_u)]-\lambda_e[1-G(\varepsilon_e)]}{\beta\gamma-\{(r+s)\,\gamma+\lambda_e\pi\gamma[1-G(\varepsilon_e)]\}\,\frac{q'(\theta)}{[q(\theta)]^2}}.$$

Observing that $\frac{r+\delta}{\pi}+\lambda_u[1-G(\varepsilon_u)] > \lambda_u[1-G(\varepsilon_u)]-\lambda_e[1-G(\varepsilon_e)]$ and $\beta\gamma \leq \{(r+s)\,\gamma+\lambda_e\pi\gamma[1-G(\varepsilon_e)]\}\,\frac{-q'(\theta)}{[q(\theta)]^2}+\beta\gamma$, it is easy to see that $\left.\frac{d\theta}{d\varepsilon_u}\right|_{JC} < \left.\frac{d\theta}{d\varepsilon_u}\right|_{CS}$.

Proof of Proposition 4 Summing (25) and (31) one obtains

$$\frac{(r+s)\,\gamma}{(1-\beta)q(\theta)}+\left(\frac{r+\delta}{\pi}\right)\varepsilon_u m = y-x+\lambda_e m\int_{\varepsilon_u+\frac{\pi\gamma}{m(1-\beta)q(\theta)}}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon. \tag{38}$$

From (38), it can be checked that θ is a strictly decreasing function of ε_u. So if a solution to (25) and (38) exists then it is unique. Denote $\varepsilon_u(\theta)$ the solution ε_u to the equation (25). Since $b-x-\tau>0$ then $\varepsilon_u(\theta)>0$. Furthermore, $\varepsilon_u(\theta)$ is non-decreasing in θ. Define $\Gamma(\theta)$ as

$$\Gamma(\theta) = y-x+\lambda_e m\int_{\varepsilon_u(\theta)+\frac{\pi\gamma}{m(1-\beta)q(\theta)}}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon-\frac{(r+s)\,\gamma}{(1-\beta)q(\theta)}-\left(\frac{r+\delta}{\pi}\right)\varepsilon_u(\theta)m.$$

An equilibrium is then a θ that solves $\Gamma(\theta) = 0$. Using the expression for $\left(\frac{r+\delta}{\pi}\right)\varepsilon_u(\theta)m$ given by (25), we have

$$\Gamma(0) = y - b + (\lambda_e - \lambda_u)\, m \int_{\varepsilon_u^0}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon.$$

So if (33) holds then $\Gamma(0) > 0$. Furthermore, $\Gamma(\infty) = -\infty$. Therefore, a solution exists and it is such that $\theta > 0$.

Proof of Proposition 7 The pair (ε_u, θ) is uniquely determined by (25) and (38). Differentiating these two equations, it is straightforward to show that $d\varepsilon_u/db > 0$ and $d\theta/db < 0$. From (26) the sign of $d\varepsilon_e/db$ is the same as $s - \delta$. To establish that $d\kappa_u/db > 0$ we first show that $d\mathcal{V}_u/db > 0$. To see this, consider an individual such that $\kappa < \kappa_u$. From (4)

$$(r + \delta)\mathcal{V}_p = x - \tau + \delta\mathcal{V}_u.$$

Rearranging (5) and using the previous equation,

$$m\left(\frac{r+\delta}{\pi}\right)\varepsilon_u = r\mathcal{V}_u - x + \tau.$$

Since $d\varepsilon_u/db > 0$ then $d\mathcal{V}_u/db > 0$. From (3) and (4),

$$r\mathcal{V}_o = \kappa p - \tau + \lambda_o \int \left[\varepsilon m + \pi\left(\frac{x - r\mathcal{V}_o}{r+\delta}\right)\right]^+ dG(\varepsilon),$$

and $d\mathcal{V}_o/db = 0$. Since $\mathcal{V}_u = \mathcal{V}_o(\kappa_u)$ then $d\kappa_u/db > 0$.

Proof of Proposition 8 The pair (ε_u, θ) is determined by (25) and (38). Differentiating these two equations one can establish that $d\theta/d\beta < 0$. In order to determine the effects on ε_u we adopt the following change of variable: $\tilde{\gamma} = \gamma/[(1-\beta)q(\theta)]$. Equations (25) and (38) can now be rewritten as

$$\left(\frac{r+\delta}{\pi}\right)\varepsilon_u m = b - x + \frac{\beta}{1-\beta}q^{-1}\left[\frac{\gamma}{(1-\beta)\tilde{\gamma}}\right]\gamma + \lambda_u m \int_{\varepsilon_u}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon, \tag{39}$$

$$(r+s)\,\tilde{\gamma} + \left(\frac{r+\delta}{\pi}\right)\varepsilon_u m = y - x + \lambda_e m \int_{\varepsilon_u + \frac{\pi\tilde{\gamma}}{m}}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon. \tag{40}$$

Equations (39) and (40) determine ε_u and $\tilde{\gamma}$. The term $\frac{\beta}{1-\beta}q^{-1}\left[\frac{\gamma}{(1-\beta)\tilde{\gamma}}\right]$ on the RHS of (39) increases in β if $\beta < \eta(\theta)$. Differentiating (39) and (40) one can show that $d\varepsilon_u/d\beta > 0$ if $\beta < \eta(\theta)$ and $d\varepsilon_u/d\beta < 0$ if $\beta > \eta(\theta)$. To determine the effect of an increase in β on ε_e we use (26) which can be reexpressed as

$$\left(\frac{r+\delta}{\pi}\right)\varepsilon_e m = y - x + (\delta - s)\tilde{\gamma} + \lambda_e m \int_{\varepsilon_e}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon. \tag{41}$$

From (40) there is a negative relationship between ε_u and $\tilde{\gamma}$. Therefore, sign$(d\varepsilon_e/d\beta)$ =sign$[(s-\delta)d\varepsilon_u/d\beta]$. Following the proof of Proposition 7 we can show that κ_u increases with ε_u.

Proof of Proposition 11 Equation (25) is independent of y or s. Therefore, it is easy to show from (25) and (38) that both θ and ε_u increase following an increase in y or a decrease in s. From (28) one can show that

$$\frac{d\varepsilon_e}{dy} = \frac{d\varepsilon_u}{dy} + \frac{\pi\gamma}{m(1-\beta)}\left(\frac{-q'}{q^2}\right)\frac{d\theta}{dy} > 0.$$

Similarly, $\frac{d\varepsilon_e}{ds} < 0$. Following the proof in Proposition 7 one can establish that $\mathcal{V}_u$ and κ_u increase with y or $1/s$.

Proof of Proposition 9 The pair (ε_u, θ) is determined jointly by (25) and (31) where (31) is independent of δ and π. It is straightforward to show that $d\varepsilon_u/d\delta < 0$, $d\theta/d\delta < 0$ and $d\varepsilon_u/d\pi > 0$, $d\theta/d\pi > 0$. From (28), $d\varepsilon_e/d\delta < 0$ and $d\varepsilon_e/d\pi > 0$. From (32) if $\lambda_u = \lambda_o$ then $d\kappa_u/d\delta$ has the same sign as $d\theta/d\delta$, and $d\kappa_u/d\pi$ has the same sign as $d\theta/d\pi$.

Proof of Proposition 12 Differentiating (25) and (31) one can establish that $d\theta/d\lambda_e > 0$ and $d\varepsilon_u/d\lambda_e > 0$. From (28), $d\varepsilon_e/d\lambda_e > 0$. Following the proof in Proposition 7 we can show that $d\mathcal{V}_u/d\lambda_e > 0$ and $d\kappa_u/d\lambda_e > 0$.

Proof of Proposition 13 Differentiating (38) and (25), one can establish that $d\varepsilon_u/d\lambda_u > 0$ and $d\theta/d\lambda_u < 0$. According to (26), the sign of $d\varepsilon_e/d\lambda_u$ is the same as the sign of $s - \delta$. Following the proof in Proposition 7 we can show that $d\mathcal{V}_u/d\lambda_u > 0$ and $d\kappa_u/d\lambda_u > 0$.