Sequential Bargaining in the Screening Model

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Abstract

This research introduces the sequential bargaining to the standard screening model by allowing the agent to propose new contracts with strategic delay after the rejection of the principal's offer. We have found that if the difference between the types of agent are sufficiently large, the efficient outcome is achieved despite the incomplete information. Otherwise, we have characterized the unique sequential separating equilibrium and the unique simultaneous separating equilibrium with repect to the different parameter values. We've also defined more general "least-cost separating contracts" as well as "bargaining-proof contracts".

Keywords: Screening, Signalling, Bargaining.

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1 Introduction

The research introduces sequential bargaining into the standard screening or adverse selection model.

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The theory of screening or adverse selection is one of the major accomplishments of information economics. This theory is often cast in a framework with two parties, an uninformed principal and an informed agent, that is, the agent has private information about his "type" (some parameter of his utility function) which the principal doesn't know. The principal offers a menu of self-selection contracts, which the agent decides to accept or reject. This model has been applied to a variety interesting economic problems and generated a lot of classical literatures, e.g., optimal income taxation(Mirrlees (1971)), labor contracts (Hart (1983)), monopolistic nonlinear pricing (Mussa and Rosen (1978); Maskin and Riley (1984)), insurance contracts (Rothschild and Stiglitz (1976)), public goods provision (Clarke (1971); Groves (1973)), optimal regulation (Baron and Myerson (1982); Laffont and Tirole (1986)) and so on.

An important assumption of the standard screening model is that the principal has all of the bargaining power. She proposes take-it-or-leave-it offer and therefore requests a yes-or-no answer. The agent is not free to propose another one. Notice if the agent rejects the offer, the interaction stops between them in the standard model, whereas in the real world it should be expected to continue since many contracting involve bargaining. For example, bargaining is an important element in the labor contract negotiation, and in the bilateral trading and many other cases.

This assumption may be reasonable if there is competition between various agents. However, in this case, picking an agent (randomly) and making a take-it-or-leave-it offer is generally not the optimal mechanism. On the other hand, if there is indeed only one suitable agent, it may be reasonable to assume that he should also enjoy some bargaining power.

As Myerson (1983, pp.1767) said, "A good research strategy is to begin by just studying this case, where one individual has all of the bargaining ability." However, "since this is a bilateral monopoly situation, we cannot go very far unless we specify how the parties are going to bargain over the terms of exchange" (Salanie, 1997, P. 5).

This research attempts to introduce bargaining into the screening model. We deviate from convention by assuming that after rejecting the principal's offer, the agent can propose new contracts, which the principal can choose to accept or reject. By implementing alternating offer between the principal and the agent, this research gives them approximately equal bargaining powers. Now both the principal and the agent are contract "designers", but the principal moves first in the first period.

When the uninformed party proposes contracts, it is a screening or adverse selection problem, and when it is the informed party's turn to make offers, it is a signaling problem. So our model can be easily extend to signalling model bargaining, which we will also discuss later by allowing the informed party move first.

Besides the alternating offer assumption, we also allow the time between offers to be a strategic variable, following Admati and Perry (1987). In other words, each party, when it is his or her turn to move, can delay it as long as he or she wishes. We assume that until an offer has been made by the relevant player, the other player cannot revise previous offers.

It is well known that games with incomplete information in which the informed player makes a move tend to have multiple equilibria. We will implement "intuitive criterion" (Cho and Kreps, 1987) to constrain the out-of-equilibrium beliefs.

A summary of our main results follows. Suppose a seller (Principal)and a buyer (Agent) bargaining over the price and the quality of some goods. The seller's production function of quality is common knowledge, while the buyer has a high or a low marginal valuation for the quality, which is his private information. Suppose the seller makes the first offer, which could be a menu or a single contract. If the difference between the two types are sufficiently large, for all discount factors and all possible distributions of types, complete information Rubinstein outcome is achieved since the high type will have higher continuation payoff by telling the truth than mimic the low type. We can get this result without resort to any refinement. Then we discuss the case where the difference between those two types are not sufficiently large. That is, the high type has incentive to mimic the low type. If π^0,the initial probability that the buyer has low marginal valuation, is smaller and equal to the threshold $\widehat{\pi}$, then there exists a unique equilibrium path, the sequential separating equilibrium path (SEE). On this path, the high type buyer accepts an offer that is identical to the perfect equilibrium offer in the complete information game between him and the seller. The low valuation buyer does not accept the seller's first offer, and instead counteroffer his "least-cost-separating contract" (defined in section 3.3.1), which combines a downward quality distortion and a strategic delay. In other words, the low type combines both the quality distortion and the strategic delay to signal his type, which is optimal for him. Hence our newly defined "least-cost-separating contract" is more general than the usual static "Riley outcome", which only involves quality downward distortion in our case.

If π^0 is bigger than $\widehat{\pi}$, then there also exists a unique equilibrium path, the simultaneous separating equilibrium path (SIE), where the seller provides a menu of self-selection contracts in the first period, which satisfies the IRs and ICs constraints for the two type buyers. The buyer accepts the menu without delay and pick up his preferred contract according to his type. Contracts characterized in this case are what we call "bargaining-proof contracts" with incomplete information.

Compared to the traditional bargaining literature with incomplete information (see Binmore, Osborne and Rubinstein (1992), Kenna and Wilson (1993), Ausubel, Cramton and Deneckere (2002) for surveys), there are several differences. First, the traditional bargaining involves only one dimension, usually a buyer and a seller bargaining over the price of a good, while multidimensional bargaining is the feature of this research. There are many multidimensional bargaining in real world, for example, a buyer and a seller bargaining over price and quantity or quality over some good, or a firm and a worker bargaining over wage and working load. Second, in the traditional bargaining, the "pie" or the total size of surplus is exogenously given, while here it is endogenously determined and itself is part of the equilibrium outcomes. Third, the major sorting variable in the traditional bargaining is time, here there are more sorting variables besides time. Especially, we assume the Spence-Mirrlees condition or single-crossing property holds here. For example, on one hand, by offering a menu of contracts, the uninformed party can induce the informed party to self-select. On the other hand, the informed party can use the contract to signal his own type. Specifically, in our model, the informed type can distort the quality traded downward as well as implement the strategic delay to signal his type. Fourth, in this research, it might be the interest of the high type to reveal his true information rather than mimic the low type. Because by revealing his true type, the total surplus could be much larger than mimic. In this case, the result could be the first-best outcomes, which significantly contrast the inefficient outcomes in either the standard screening model or the traditional bargaining literature.

There is a small literature on multidimensional bargaining. Wang (1998) has discussed one-sided offer. He shows that the uninformed party will propose the same take-it-or-leave-it menu every period and the game ends in the first period. Inderst (2003) and Sen (2000) have investigated exogenous alternating-offer cases. By a very strong assumption, Inderst (2003) shows that the game has a unique equilibrium where efficient contracts are implemented in the first period. Sen (2000) has got multiple equilibria and distinguished the "mimicking" and "no mimicking" regimes

The basic structure of our model is developed in section2. Section 3 is the characterization results. Section 4 shows the existence and uniqueness of the equilibrium path describe above. And section 5 proves the uniqueness of the equilibrium path. Section 6 is the extension and the conclusion.

2 The Model

We now describe the model. We assume as simple as possible environment, but the results can be carried on to more general setup as we will discuss the extension in section 5.

2.1 Objective functions and information

There are two parties, a seller (principal), denoted S, and a buyer (agent), denoted θ, bargaining over the quality q and the price p of some goods. A feasible trade is denoted by $(q, p) \in [\mathbb{R}_+ \times \mathbb{R}_+]$.$S$'s production cost of quality q is $C(q)$. And $C(.)$ is twice continuously differentiable with $C' > 0$ and $C'' > 0.C(0) = 0, C'(0) = 0$ and $\lim_{q\to\infty} C'(q) = \infty$.

S has no private information and her utility function is $V(q, p) = p - C(q)$. θ's utility function $U(q, p, \theta) = \theta q - p$, where the valuation for quality $\theta \in \{\underline{\theta}, \overline{\theta}\}$, is his private information. And we assume $0 < \underline{\theta} < \overline{\theta}$. Therefore, we have $U(q, p, \overline{\theta}) > U(q, p, \underline{\theta})$ for all (q, p), and $U(q, p, \overline{\theta}) - U(q', p, \overline{\theta}) > U(q, p, \underline{\theta}) - U(q', p, \underline{\theta})$ for any pair $q > q'$. This is the discrete form of Spence-Mirrlees condition: at any given quality level, the high type buyer is willing to pay more than the low type buyer for the same increase in quality. The proportion of $\underline{\theta}$ (the low type) is π^0, of $\overline{\theta}$ (the high type) is $1 - \pi^0$.

We define the first-best production level of q as that which maximizes the surplus under complete information about θ.To do so, consider the following maximization problem:

$$\max_q R(q, \theta) = \theta q - C(q)$$

By assumptions above, $q^*(\theta) = \arg\max R(q, \theta)$ exists and is uniquely determined by the first-order condition, $C'(q) = \theta$. $R^*(\theta) = R(q^*(\theta), \theta)$ is the maximum amount of surplus attainable when the seller bargains with a buyer of type θ. And also we have $\overline{R}^* > \underline{R}^* = R^*(\underline{\theta}) > 0$. We can see that even when the seller is dealing with the low valuation buyer $\underline{\theta}$, there are gains from trade.

2.2 The contract bargaining game

The contract bargaining game starts at time zero, when it is S's turn to make an offer. (The case in which θ makes the first offer is discussed in section 6). Subsequently, players make alternating offers until they reach an agreement. S can propose a menu of contracts $(\underline{m}, \overline{m})$ in the feasible finite set W,[1] where $\underline{m} = (\underline{q}, \underline{p})$, $\overline{m} = (\overline{q}, \overline{p})$,[2] from which, after acceptance, θ is free to choose one contract from the menu. After the rejection of the entire menu, θ can only offer a single contract. If S rejects the contract, she again proposes a menu, and so on so forth. A response to an offer involves either an acceptance or a rejection, and it can be made right away or at any time later. Acceptance ends the game. But if a rejection is made, the game moves on and the relevant player makes an counteroffer within no shorter than a given length of time, which is normalized to one. We assume until an counteroffer is made, the other party cannot change its previous offer. The first offer in the game can be made at any time $t \geq 0$.

[1] The technical reason for requiring the contracts in W to be finite is to ensure that a continuation equilibrium exists.

[2] Since we allow only two types of buyer, without loss of generality, we can assume the menu offered consisting of two contracts. The contracts in the menu could be identical, which means single contract.

We use $\tau \geq 0$ to denote delay, i.e. the length of the time at which the relevant player either accepts or rejects an existing offer and make an counteroffer. A relevant history for the game is a sequence of unacceptable offers and a sequence of times between offers. Formally, if $N \in \{1, 2, ...\infty\}$ is the number of rounds (offers), then a history of the N rounds is denoted by $H^N = \{m^n, \tau^n\}_{n=1}^N$. Note that τ^N denotes the time delay in round N since offer m^{N-1} was made; this history corresponds to the passage of $t = \sum_{n=1}^N \tau^n$ "real" time units.

A strategy for a player is a function that specifies for each history after which it is the player's turn to move, the length of time delay τ until the player respond, whether the latest offer is accepted and, if not accepted, a counteroffer. We denote the strategies of S, $\underline{\theta}$ and $\overline{\theta}$ by $\sigma_S, \sigma_{\underline{\theta}}$ and $\sigma_{\overline{\theta}}$, respectively. We consider only pure strategies.

An outcome of the game is $M = (m(q,p), t)$, with the interpretation that, contract m is agreed on at time t, the buyer pays p to the seller to obtain the goods with quality q. The seller's payoff is $\delta^t V(m)$, while the buyer $\delta^t U(m)$.

The equilibrium concept we use is sequential equilibrium. In a sequential equilibrium, one specifies the strategies σ and also the beliefs of S for every history H^N. We use $\pi(H^N)$ to denote S's belief at H^N that $\theta = \underline{\theta}$, and we use π to denote the entire system of beliefs. We require that the update belief be consistent and strategies be sequentially rational in the sense that, at every information set, a player's strategy maximizes this expected payoffs, given his or her beliefs and the opponent's strategy.

2.3 The standard screening model

For reference, we first examine the no bargaining case, that is, the standard model. We assume that outside options for both type buyers are zero. The principal solves the program:

$$V = \max_{\{\underline{p},\underline{q};\ \overline{p},\overline{q}\}} \{\pi^0[\underline{p} - C(\underline{q})] + (1 - \pi^0)[\overline{p} - C(\overline{q})]\}$$

$$s.t. \underline{\theta}\underline{q} - \underline{p} \geq \underline{\theta}\overline{q} - \overline{p} \tag{1}$$

$$\overline{\theta}\overline{q} - \overline{p} \geq \overline{\theta}\underline{q} - \underline{p} \tag{2}$$

$$\underline{\theta}\underline{q} - \underline{p} \geq 0 \tag{3}$$

$$\overline{\theta}\overline{q} - \overline{p} \geq 0 \tag{4}$$

We call the above program "no bargaining program (NBP)" in this research. We know (1) and (3) binding. And the results are standard:

$\underline{q}^N < \underline{q}^*$, $\overline{q}^N = \overline{q}^*$, where q^* is the first-best quality levels.

$\underline{p}^N = \underline{\theta}\underline{q}^N$, $\overline{p}^N = \overline{\theta}\overline{q}^* - (\overline{\theta} - \underline{\theta})\underline{q}^N$, where $(\overline{\theta} - \underline{\theta})\underline{q}^N$ is the information rent.

$V^N = \pi^0(\underline{\theta}\underline{q}^N - C(\underline{q}^N)) + (1 - \pi^0)\{(\overline{R}^* - (\overline{\theta} - \underline{\theta})\underline{q}^N\}$.

Note that first-order condition gives $C'(\underline{q}) = \underline{\theta} - \frac{1-\pi^0}{\pi^0}(\overline{\theta} - \underline{\theta})$

For $C'(\underline{q}) \geq 0$, we should have $\pi^0 \geq 1 - \frac{\underline{\theta}}{\overline{\theta}}$. This condition ensures that $\underline{\theta}$ can be included in the menu. Furthermore, we should have $\pi^0 \geq \frac{(\overline{\theta}-\underline{\theta})\underline{q}^N}{\overline{\theta}\underline{q}^N - C(\underline{q}^N)}$, this condition guarantees that while including $\underline{\theta}$ in the menu, the surplus S gets from $\underline{\theta}$ can at least compensate the information rent he gives to $\overline{\theta}$. Otherwise, it is optimal for S to propose a single contract designed for $\overline{\theta}$. In other words, to sustain the above standard results, π^0 should be sufficiently large.

3 Characterization Results

3.1 Rubinstein Outcome

As a benchmark, first consider complete information Rubinstein (1982) outcomes. We can obtain Rubinstein contracts, that is, the complete information first-best contracts with alternating offer offered by the seller and the buyer respectively: $\overline{m}_S^R = (\overline{q}^*, C(\overline{q}^*) + \frac{1}{1+\delta}\overline{R}^*)$, $\underline{m}_S^R = (\underline{q}^*, C(\underline{q}^*) + \frac{1}{1+\delta}\underline{R}^*)$,

$\overline{m}_B^R = (\overline{q}^*, C(\overline{q}^*) + \frac{\delta}{1+\delta}\overline{R}^*)$, $\underline{m}_B^R = (\underline{q}^*, C(\underline{q}^*) + \frac{\delta}{1+\delta}\underline{R}^*)$.

Proposition 1 *When the seller has complete information about the buyer, there is a unique subgame perfect equilibrium. In this equilibrium, the seller offers $\overline{m}_S^R$ ($\underline{m}_S^R$) to $\overline{\theta}$ ($\underline{\theta}$) whenever it is her turn to make an offer, and accept an offer $\overline{m}_B$ ($\underline{m}_B$) of buyer $\overline{\theta}$ ($\underline{\theta}$)if and only if*

$V(\overline{m}_B) \geq \frac{\delta}{1+\delta}\overline{R}^*$ $(V(\underline{m}_B) \geq \frac{\delta}{1+\delta}\underline{R}^*)$*; buyer* $\overline{\theta}$ $(\underline{\theta})$ *always proposes* $\overline{m}_B^R$ $(\underline{m}_B^R)$*, and accepts an offer* $\overline{m}_S$ $(\underline{m}_S)$ *if and only if* $\overline{U}(\overline{m}_S) \geq \frac{\delta}{1+\delta}\overline{R}^*$ $(\underline{U}(\underline{m}_S) \geq \frac{\delta}{1+\delta}\underline{R}^*)$*. The outcome is that the seller proposes* $\overline{m}_S^R$ $(\underline{m}_S^R)$ *at time zero, and the buyer* $\overline{\theta}$ $(\underline{\theta})$*immediately accepts this offer.*

In the equilibrium of the complete information contracts bargaining game, the seller gets $\frac{1}{1+\delta}$share of total surplus $\overline{R}^*$ or $\underline{R}^*$, and the buyer $\overline{\theta}$ $(\underline{\theta})$ will get $\frac{\delta}{1+\delta}$share of total surplus $\overline{R}^*(\underline{R}^*)$. And the contract specifies the first-best quality $\overline{q}^*(\underline{q}^*)$.

Now we go back to the incomplete information case. We will show that our results depend on $\frac{\overline{\theta}}{\underline{\theta}}$ and π^0.

3.2 Non-costly Separating

Proposition 2 *For all* $(\pi^0, \delta) \in (0,1) \times (0,1)$*, there exists* $c^0 \in (1, \infty)$ *such that if* $\frac{\overline{\theta}}{\underline{\theta}} \geq c^0$*, the contracts bargaining game has a unique equilibrium outcome where the seller proposes* $(\overline{m}_S^R, \underline{m}_S^R)$ *at time zero, and the buyer* $\overline{\theta}$ $(\underline{\theta})$*immediately accepts this offer and picks up* $\overline{m}_S^R$ $(\underline{m}_S^R)$*.*

That is to say, if the difference between the two type is sufficiently large, the high type has no incentive to mimic the low type, and the separating is non-costly, and the outcome is identical to the complete information Rubinstein outcome and the first-best quality level is achieved. The reason is like the following: the alternating offers create type-dependent continuation payoff for the buyer. If the difference is sufficient large, the high type buyer essentially has a much higher continuation payoff or reservation value. Thus, he won't mimic the low type.

Proof. There exists some c to let the following inequalities hold: $\overline{U}(\overline{m}_S^R) \geq \overline{U}(\underline{m}_S^R) \Longrightarrow \overline{U}(\overline{m}_B^R) \geq \overline{U}(\underline{m}_B^R)$.And let $\overline{U}(\overline{m}_S^R) = \overline{U}(\underline{m}_S^R)$, we can find c^0. ■

From now on, we let $\frac{\overline{\theta}}{\underline{\theta}} < c^0$, or $\overline{U}(\overline{m}_S^R) < \overline{U}(\underline{m}_S^R)$, that is to say, the high type has the incentive to mimic the low type. So far we haven't implement any refinement. From now on, we make Assumption 1 in the spirit of intuitive criterion, and for the rest of the paper, we only discuss the sequential equilibrium that survives Assumption 1.

Assumption 1 (A1). Suppose that, according to the equilibrium strategies, either (i) at time t, S accepts an offer m made by θ, or (ii) at time t, $\overline{\theta}$ accepts an offer m made by S. Suppose (m', τ) satisfies $\underline{U}(m', t+\tau) > \underline{U}(m, t)$, $\overline{U}(m', t+\tau) < \overline{U}(m, t)$ and $V(m', t+\tau) \geq V(\underline{m}_B^R, t+\tau)$. If θ offers m' at time $t+\tau$, then the belief of S upon getting this offer must be $\pi = 1$.

And we also assume $\delta \to 1$ from now on.

3.3 Costly Separating

3.3.1 Least-cost-separating contracts

If it is $\underline{\theta}$'s interest to separate himself, he has many ways to signal his type: distortion q downward or the strategic delay or combining both. We first define the "least-cost-separating contracts".

The usual "least-cost-separating contracts"(Riley outcome) are obtained by successively solving the following programs:

Program for $\overline{\theta}$:

$$\max_{\{\overline{q},\, \overline{p}\}} \overline{U} = \overline{\theta}\overline{q} - \overline{p}$$

$$s.t. \overline{p} - C(\overline{q}) \geq \frac{\delta}{1+\delta}\overline{R}^*$$

Program for $\underline{\theta}$:

$$\max_{\{\underline{q},\, \underline{p}\}} \underline{U} = \underline{\theta}\underline{q} - \underline{p}$$

$$s.t.\ \underline{p} - C(\underline{q}) \geq \frac{\delta}{1+\delta}\underline{R}^*$$

$$\overline{\theta}\overline{q} - \overline{p} \geq \overline{\theta}\underline{q} - \underline{p}$$

The solution to $\overline{\theta}$'s program gives $\overline{m}_B^R$. Plug it into $\underline{\theta}$'s program, we can get $\underline{m}_B^S = (\underline{q}^S, C(\underline{q}^S) + \frac{\delta}{1+\delta}\underline{R}^*)$ with $\underline{q}^S < \underline{q}^*$.

Since $\underline{\theta}$ can also separate himself by strategic delay, so his problem now becomes:

$$\max_{\{\underline{q},\, \underline{p},\, \tau\}} \underline{U} = \delta^{\tau}\{\underline{\theta}\underline{q} - \underline{p}\}$$

$$s.t.\ \underline{p} - C(\underline{q}) \geq \frac{\delta}{1+\delta}\underline{R}^{*}$$

$$\overline{\theta}\overline{q} - \overline{p} \geq \delta^{\tau}(\overline{\theta}\underline{q} - \underline{p})$$

Solving the problem, we can get $\underline{M}_B^{S^*} = \{\underline{m}_B^{S^*}(\underline{q}^{S^*}, C(\underline{q}^{S^*}) + \frac{\delta}{1+\delta}\underline{R}^{*}), \tau^{*}\}$. And we can show that $\underline{q}^S \leq \underline{q}^{S^*} \leq \underline{q}^{*}$ and $\tau^{*} \geq 0$. We have $\underline{U}(\underline{m}_B^{S}) \leq \underline{U}(\underline{M}_B^{S^*})$.

3.3.2 Sequential vs. Simultaneous separating

There are two possible separating equilibrium outcomes: the sequential separating equilibrium outcome (SEE) and the simultaneous separating equilibrium outcome (SIE).

Definition 1 *Under the SEE, S first provide $\overline{m}_S^R$, and $\overline{\theta}$ accept it without delay, but $\underline{\theta}$ doesn't accept it, instead counteroffer $\underline{m}_B^{S^*}$ with strategic delay τ^{*}.*

In this case, the seller's payoff is $V^{SEE} = \pi^0\delta^{\tau^*}\frac{\delta}{1+\delta}\underline{R}^{*} + (1-\pi^0)\frac{1}{1+\delta}\overline{R}^{*}$.

Definition 2 *Under the SIE, S first provide a menu $(\overline{m}_S^{S^*}, \underline{m}_S^{S^*})$, and each type buyer accepts the menu without delay, and pick up his own contract.*

Thus, the seller's problem is:

$$V^{SIE} = \max_{\{(\underline{q},\underline{p});(\overline{q},\overline{p})\}}\{\pi^0[\underline{p} - C(\underline{q})] + (1-\pi^0)[\overline{p} - C(\overline{q})]\}$$

$$s.t. \underline{\theta}\underline{q} - \underline{p} \geq \underline{\theta}\overline{q} - \overline{p}$$

$$\overline{\theta}\overline{q} - \overline{p} \geq \overline{\theta}\underline{q} - \underline{p}$$

$$\underline{\theta}\underline{q} - \underline{p} \geq \delta \underline{U}(\underline{M}_B^{S^*})$$

$$\overline{\theta}\overline{q} - \overline{p} \geq \frac{\delta}{1+\delta}\overline{R}^*$$

Solving the above problem, we can get contracts $\overline{m}_S^{S^*}(\underline{q}^N, \underline{\theta}\underline{q}^N - \delta\underline{U}(\underline{M}_B^{S^*}))$, $\underline{m}_S^{S^*}(\overline{q}^*, \overline{\theta}\overline{q}^* - (\overline{\theta} - \underline{\theta})\underline{q}^N - \delta\underline{U}(\underline{M}_B^{S^*}))$, and the seller's payoff $V^{SIE} = \pi^0(\underline{\theta}\underline{q}^N - C(\underline{q}^N)) + (1-\pi^0)\{(\overline{R}^* - (\overline{\theta} - \underline{\theta})\underline{q}^N\} - \delta\underline{U}(\underline{M}_B^{S^*})$.

Proposition 3 *There exists a $\widehat{\pi}$ such that if $0 < \pi^0 \leq \widehat{\pi}$, S is better off in SEE than in the SIE; on the other hand, if $\pi^0 > \widehat{\pi}$, S is better off in SIE than SEE.*

Proof. Let $V^{SEE} \geq V^{SIE}$, we can find $\pi^0 \leq \widehat{\pi} = \frac{(\overline{\theta}-\underline{\theta})\underline{q}^N + \delta\underline{U}(\underline{M}_B^{S^*}) - \frac{\delta}{1+\delta}\overline{R}^*}{\underline{\theta}\underline{q}^N - C(\underline{q}^N) - \frac{\delta^{\tau^*+1}}{1+\delta}\underline{R}^* - \frac{\delta}{1+\delta}\overline{R}^*}$. ■

To guarantee that $\widehat{\pi}$ is a reasonable number, we assume $(\overline{\theta} - \underline{\theta})\underline{q}^N + \delta\underline{U}(\underline{M}_B^{S^*}) - \frac{\delta}{1+\delta}\overline{R}^* > 0$, and $\underline{\theta}\underline{q}^N - C(\underline{q}^N) - \frac{\delta^{\tau^*+1}}{1+\delta}\underline{R}^* \geq \delta\underline{U}(\underline{M}_B^{S^*})$.

The idea is quite simple, if the probability of low type is sufficiently low, it is better for the seller just exclude the low type in the first offer rather than including it. Because by including it, the seller has to pay the high type buyer information rent.

4 Existence of the equilibrium

Proposition 4 *If $0 < \pi^0 \leq \widehat{\pi}$, then there exists the SEE.*

Proof. Suppose that $0 < \pi^0 \leq \widehat{\pi}$, the following is an equilibrium.

- S's beliefs: suppose in response to the last offer of m by S, θ offer m' after an additional delay of τ. (1) Suppose $\overline{U}(m) \geq \overline{U}(\overline{m}_S^R)$. If $\overline{U}(m) \geq \overline{U}(m', \tau)$, $\underline{U}(m', \tau) > \underline{U}(m)$, and $V(m', \tau) \geq V(\underline{m}_B^R, \tau)$, then $\pi = 1$. Otherwise $\pi = 0$. (2) Suppose $\overline{U}(m) < \overline{U}(\overline{m}_S^R)$. If $\overline{U}(\overline{m}_B^R, \tau) \geq \overline{U}(m', \tau)$, $\underline{U}(m', \tau) > \underline{U}(m)$, and $V(m', \tau) \geq V(\underline{m}_B^R, \tau)$, then $\pi = 1$. Otherwise $\pi = 0$.

- S's strategy: Offer $\overline{m}_S^R$ at time zero. If in response to an offer m by S, θ makes a counteroffer m' after additional delay τ, and S's belief at this node is that $\theta = \underline{\theta}$ with probability π, then:

  (1) If $\pi = 0$, then accepts m' if $V(m', \tau) \geq V(\overline{m}_B^R, \tau)$, otherwise rejects it without delay and counteroffers $\overline{m}_S^R$ right after.

  (2) If $\pi = 1$, then accepts m' if $V(m', \tau) \geq V(\underline{m}_B^R, \tau)$, otherwise rejects it without delay and counteroffers $\underline{m}_S^R$ right after.

- $\overline{\theta}'$s strategy: at any point, if S makes an offer m, accept without delay if $\overline{U}(m) \geq \overline{U}(\overline{m}_S^R)$; otherwise rejects it without delay and counteroffers $\overline{m}_B^R$ right after.

- $\underline{\theta}'$s strategy: at any point, if S offers m, accepts it without delay if $\underline{U}(m) \geq \delta \underline{U}(\underline{M}_B^{S^*})$; otherwise, rejects it and counteroffers $\underline{m}_B^{S^*}$ with delay τ^*.

  To verify this equilibrium note first that S would not offer m such that $\overline{U}(m) < \overline{U}(\overline{m}_S^R)$, since given θ's strategy this leads to a lower payoff than offering $\overline{m}_S^R$. Since $\pi^0 \leq \widehat{\pi}$, by proposition 3, offering $\overline{m}_S^R$ is optimal for S. The rest of the verification is straightforward. Note that the belief above satisfy (A1).

■

Proposition 5 *If $\widehat{\pi} < \pi^0$, the SIE exists.*

Proof. Suppose $\widehat{\pi} < \pi^0$. Then the following is an equilibrium.

- S's beliefs: suppose in response to the last offer of m by S, θ offer m' after an additional delay of τ. (1) Suppose $\overline{U}(m) \geq \overline{U}(\overline{m}_S^R)$. If $\overline{U}(m) \geq \overline{U}(m', \tau)$, $\underline{U}(m', \tau) > \underline{U}(m)$, and $V(m', \tau) \geq V(\underline{m}_B^R, \tau)$, then $\pi = 1$. Otherwise $\pi = 0$. (2) Suppose $\overline{U}(m) < \overline{U}(\overline{m}_S^R)$. If $\overline{U}(\overline{m}_B^R, \tau) \geq \overline{U}(m', \tau)$, $\underline{U}(m', \tau) > \underline{U}(m)$, and $V(m', \tau) \geq V(\underline{m}_B^R, \tau)$, then $\pi = 1$. Otherwise $\pi = 0$..

- S's strategy: Offers $(\overline{m}_S^{S^*}, \underline{m}_S^{S^*})$ at time zero. If in response to the offer by S, θ makes a counteroffer m' after additional delay τ, and S's belief at this node is that $\theta = \underline{\theta}$ with probability π, then:

(1) If $\pi = 0$, then accepts m' if $V(m', \tau) \geq V(\overline{m}_B^R, \tau)$, otherwise rejects it without delay and counteroffers $\overline{m}_S^R$ right after.

(2) If $\pi = 1$, then accepts m' if $V(m', \tau) \geq V(\underline{m}_B^R, \tau)$, otherwise rejects it without delay and counteroffers $\underline{m}_S^R$ right after.

- $\overline{\theta}'$s strategy: at any point, if S makes a menu offer including a contract m, accepts it and chooses m without delay if $\overline{U}(m) \geq \overline{U}(\overline{m}_S^R)$; otherwise, rejects it without delay and counteroffers $\overline{m}_B^R$ right after.

- $\underline{\theta}'$s strategy: at any point, if S offers a menu including a contract m, accepts it and chooses m without delay if $\underline{U}(m) \geq \underline{U}(\underline{m}_S^{S^*})$; otherwise, rejects it and counteroffers $\underline{m}_B^{S^*}$ with delay τ^*.

  To verify this equilibrium note first that S would not offer $(\overline{m}, \underline{m})$ such that $\overline{U}(\overline{m}) < \overline{U}(\overline{m}_S^{S^*})$ and $\underline{U}(\underline{m}) \geq \underline{U}(\underline{m}_S^{S^*})$, since given θ's strategy this leads to a lower payoff than offering $(\overline{m}_S^{S^*}, \underline{m}_S^{S^*})$. Since $\widehat{\pi} < \pi^0$, by proposition 3, offering $(\overline{m}_S^{S^*}, \underline{m}_S^{S^*})$ is optimal for S. The rest of the verification is straightforward. Note that the belief above satisfy (A1).

■

5 Uniqueness of the equilibrium

Before proving the uniqueness, we first derive a series of lemmas with respect to the boundaries of the players.

Lemma 1 *In the bargaining game,* $\sup(V) \leq \frac{1}{1+\delta}\overline{R}^*$ *and* $\inf(\overline{U}) \geq \frac{\delta}{1+\delta}\overline{R}^*$.

Proof. We prove this lemma by contradiction. Suppose $V^H = \sup(V) > \frac{1}{1+\delta}\overline{R}^*$. We show $\overline{\theta}$ can profitably deviate by offering $\overline{m}'(\overline{q}, \overline{p})$ to let $V(\overline{m}') = \delta(V^H) + \zeta$ for some $\zeta > 0$. S will accept this offer. And $\overline{\theta}'$s payoff will be $\delta[\overline{R}^* - \delta(V^H) - \zeta]$. Observe since $V^H > \frac{1}{1+\delta}\overline{R}^*$, we can get $\delta[\overline{R}^* - \delta(V^H)] > [\overline{R}^* - V^H]$. Then there exist an $\zeta > 0$ such that the above deviation is profitable for $\overline{\theta}$. This proves $\sup(V) \le \frac{1}{1+\delta}\overline{R}^*$. Given this, it is easy for us to get $\inf(\overline{U}) \ge \frac{\delta}{1+\delta}\overline{R}^*$. Simply by telling the truth, $\overline{\theta}$ can guarantee himself at least $\frac{\delta}{1+\delta}\overline{R}^*$. ■

Lemma 2 *$\overline{\theta}$ can get no more than $\frac{1}{1+\delta}\overline{R}^*$ when he makes an offer.*

Proof. We show $\sup(\overline{U}) \le \frac{1}{1+\delta}\overline{R}^*$ when $\overline{\theta}$ makes an offer. Suppose $\sup(\overline{U}) = \frac{1}{1+\delta}\overline{R}^* + \alpha$, for some $\alpha > 0$.This implies that S can realize in the respective continuation game a payoff not bigger than $\frac{\delta}{1+\delta}\overline{R}^* - \alpha$. We show that S can profitably deviate by rejecting and offering a contract $\overline{m}(\overline{q}^*, \overline{p})$ such that

$$\overline{\theta}\overline{q}^* - \overline{p} = \delta(\frac{1}{1+\delta}\overline{R}^* + \alpha) + \alpha\frac{1-\delta^2}{2\delta}$$

That is, $\overline{U}(\overline{m}) \ge \delta(\frac{1}{1+\delta}\overline{R}^* + \alpha)$, $\overline{\theta}$ won't reject. It remains to show that this strategy is strictly profitable to S. To see this, note that the difference

$$\begin{aligned} & \delta[\overline{p} - C(\overline{q}^*)] - [\frac{\delta}{1+\delta}\overline{R}^* - \alpha] \\ = & \frac{\alpha(1-\delta^2)}{2} > 0 \end{aligned}$$

■

Lemma 3 *In the bargaining game,* $\sup(\underline{U}) \le \frac{1}{1+\delta}\underline{R}^*$ *and* $\inf(V) \ge \frac{\delta}{1+\delta}\underline{R}^*$

Proof. Suppose $\underline{U}^H = \sup(\underline{U}) > \frac{1}{1+\delta}\underline{R}^*$. We show S can profitably deviate by offering $\underline{m}'(\underline{q}, \underline{p})$ to let $\underline{U}(\underline{m}') = \delta(\underline{U}^H) + \xi$ for some $\xi > 0$. $\underline{\theta}$ will accept this offer. And S's payoff will be $\delta[\underline{R}^* - \delta(\underline{U}^H) - \xi]$. Observe since $\underline{U}^H > \frac{1}{1+\delta}\underline{R}^*$, we can get $\delta[\underline{R}^* - \delta(\underline{U}^H)] > [\underline{R}^* - \underline{U}^H]$. Then there exist an $\xi > 0$ such that the above deviation is profitable for S. This proves $\sup(\underline{U}) \le \frac{1}{1+\delta}\underline{R}^*$. Given this, it is easy to get $\inf(V) \ge \frac{\delta}{1+\delta}\underline{R}^*$. Note instead of accepting an offer m' giving him $V(m') < \frac{\delta}{1+\delta}\underline{R}^*$, S can always reject it right away and propose $\underline{m}_S^R$. Both buyers will accept it with probability 1. S can make at least $\frac{\delta}{1+\delta}\underline{R}^*$ in this worst case. ■

Lemma 4 *If S's belief specifying $\pi = 1$, S accepts any offer no less than $\frac{\delta}{1+\delta}\underline{R}^*$ for her .*

Proof. We want to show $\sup(V) = \frac{\delta}{1+\delta}\underline{R}^*$ when $\pi = 1$ and when it is S's turn to reject or accept $\underline{\theta}$'s offer. The proof is similar as lemma 3. ■

Combining previous two lemmas, when $\pi = 1$ and when it is S's turn to reject or accept $\underline{\theta}$'s offer, $\inf(V) = \sup(V) = \frac{\delta}{1+\delta}\underline{R}^*$.

Lemma 5 *If $\underline{\theta}$ signals his type, the maximum payoff for him is $\underline{U}(\underline{M}_B^{S^*})$.*

Proof. Since $\underline{U}(\underline{M}_B^{S^*})$ is the least-cost separating contract for $\underline{\theta}$ and $\inf(V) = \frac{\delta}{1+\delta}\underline{R}^*$ if S's belief specifying $\pi = 1$, thus, $\underline{U}(\underline{M}_B^{S^*})$ is the maximum payoff for $\underline{\theta}$ when he signals his type. ■

Lemma 6 *If $\underline{\theta}$ signals his type, the minimum payoff for him is $\underline{U}(\underline{M}_B^{S^*})$.*

Proof. Because of lemma 4. and because $\underline{U}(\underline{M}_B^{S^*})$ is the only contract selected by the intuitive criterion. ■

Lemma 7 *There exists a $\widetilde{\pi} > 0$ such that if $0 < \pi^0 \leq \widetilde{\pi}$, S will offer a single contract to $\overline{\theta}$ rather than a self-selection menu at time zero. If $\pi^0 > \widetilde{\pi}$, S will offer a self-selection menu at time zero.*

Proof. We show the existence of such a $\widetilde{\pi}$. First, to include $\underline{\theta}$ in the menu, the surplus S gets from $\underline{\theta}$ should at least compensate the information rent he gives to $\overline{\theta}$. Otherwise, it is optimal for S to propose a single contract designed for $\overline{\theta}$. This condition will give us a threshold π_1^0. For example, in the SIE program, this condition gives $\pi_1^0 = \frac{(\overline{\theta}-\underline{\theta})\underline{q}^N+\delta\underline{U}(\underline{M}_B^{S^*})}{\overline{\theta}\underline{q}^N-C(\underline{q}^N)}$. Second, in the sequential separating case, S can also get some surplus from $\underline{\theta}$. These two conditions will define a $\widetilde{\pi} > \pi_1^0$. ■

Lemma 8 *When $0 < \pi^0 \leq \widetilde{\pi}$, S's payoff is bounded from below by V^{SEE}.*

Proof. For any $\varepsilon > 0$, at time zero S can offer a single contract $\overline{m}_0(\overline{q}^*, C(\overline{q}^*) + \frac{1}{1+\delta}\overline{R}^* - \varepsilon)$. By lemma 1, $\overline{\theta}$ accepts the contract. Combining with lemma 5 and ε is arbitrarily small, V^{SEE} is a lower boundary for S's payoff. ■

Lemma 9 *When $0 < \pi^0 \leq \widetilde{\pi}$, S's payoff is bounded from above by V^{SEE}.*

Proof. When $0 < \pi^0 \leq \widetilde{\pi}$, S can only provide a single contract. Under SEE, she proposes $\overline{m}_S^R$ at time zero, and $\overline{\theta}$ accepts because of lemma 2. And this contract gives the seller $\frac{1}{1+\delta}\overline{R}^*$. When $\underline{\theta}$ counteroffer $\underline{U}(\underline{M}_B^{S^*})$, this offer gives the seller $\frac{\delta}{1+\delta}\underline{R}^*$. From Lemma 1, 4 and 5, we know V^{SEE} indeed is the highest payoff for S. ■

Lemma 10 *When $\pi^0 > \widetilde{\pi}$, S's payoff is bounded from below by V^{SIE}.*

Proof. For any $\varepsilon > 0$, S can end the game in time zero by offering a menu $\{\underline{m}_1(\underline{q}^N, \underline{\theta}\underline{q}^N - \delta\underline{U}(\underline{M}_B^{S*}) - \varepsilon), \overline{m}_1(\overline{q}^*, \overline{\theta}\overline{q}^* - (\overline{\theta} - \underline{\theta})\underline{q}^N - \delta\underline{U}(\underline{M}_B^{S*}) - \varepsilon)\}$. To see this, note that $\underline{\theta}$ will accept it by lemma 4, and $\overline{\theta}$ will accept by lemma 2. As $\varepsilon > 0$ can be chosen arbitrarily small, V^{SIE} indeed constitutes a lower boundary for S's payoff. ■

Lemma 11 *When $\pi^0 > \widetilde{\pi}$, S's payoff is bounded from above by V^{SIE}.*

Proof. When $\pi^0 > \widetilde{\pi}$, S will propose a menu. The SIE program uses the lower boundaries for the buyer's payoff and makes a take-it-or-leave-it offer, thus V^{SIE} is the highest payoff S can get. ■

Lemma 12 $\widetilde{\pi} = \widehat{\pi}$.

Proof. By lemma 8, 9 and 10, 11 with proposition 3, we can quickly get this result. ■

Proposition 6 *If $0 < \pi^0 \leq \widehat{\pi}$, then the only equilibrium path is the SEE.*

Proof. Combining lemma 8, 9 and 12, V^{SEE} is the unique equilibrium payoff when $0 < \pi^0 \leq \widehat{\pi}$. Also under the equilibrium path, both type buyers get their unique equilibrium payoff. Thus, the SEE is the only candidate for an equilibrium path. The existence of an equilibrium that supports this path was established in proposition 4. ■

Proposition 7 *If $\widehat{\pi} < \pi^0$, the only equilibrium path is the SIE.*

Proof. Combining lemma 10, 11 and 12, V^{SIE} is the unique equilibrium payoff when $\widehat{\pi} < \pi^0$. Also under the equilibrium path, $\overline{\theta}$ gets more than what he can when he proposes and $\underline{\theta}$ gets his unique equilibrium payoff. Thus, the SIE is the only candidate for an equilibrium path. The existence of an equilibrium that supports this path was established in proposition 5. ■

5.1 Bargaining-proof contracts

We define the menu of the SIE the "bargaining-proof contracts" in the sense that the dynamic game ends up at the first period. We can compare the SIE program defined in section 3.3.2 to the NBP defined in section 2.3. These two programs look alike accept that the right hand sides of the IRs of SIE is the continuation payoffs of the buyers while the outside options of buyers of NBP are normalized to zero. The qualities specified in both menus are exactly the same for the high type and the low type, respectively, while the prices as well as the income distribution are different. This is quite intuitive because in SIE the agent has bargaining power, he should expect to get more, that is, satisfy his IRs, while the ICs still need to be satisfied, thus the quality for the low type is distorted downward. In other words, given the right parameters, in our case, $\widehat{\pi} < \pi^0$, the contracts bargaining game can be treated as the endogenous type-dependent IRs problem.

6 Extension and Conclusion

There are many extensions on the model. Let's briefly discuss several of them.

(1) The informed buyer moves first.

Consider the game in which the buyer makes an offer at time zero. It can be shown, using the similar arguments to those used in the previous sections, given $\frac{\overline{\theta}}{\underline{\theta}} < c^0$, there are one unique equilibrium path, the sequential separating equilibrium path where $\overline{\theta}$ offers $\overline{m}_S^R$ at time zero while $\underline{\theta}$ offers $\underline{m}_B^{S*}$ with strategic delay t^*, and S accepts either of it without delay.

(2) Two-sided private information.

Another important hypothesis of the standard screening model is that the principal is "uninformed," i.e., does not possess private information when contracting. Following Myerson (1983), Maskin and Tirole (1990, 1992), however, this research can also consider "informed principal" case.

Besides the buyer's private information, we can assume that the production cost could high $C_H(q)$ or low $C_L(q)$, that is, for the same q, $C_H(q) > C_L(q)$, and $C'_H(q) > C'_L(q)$, which is the seller's private information. And we assume that the probability of low cost is v. We also assume that even the low valuation buyer deals with the high cost seller, there are still gains from trade. Many interesting cases will arise in this two-sided private information situation.

(3) More than two types.

The model can also be extend to more than two-type cases. For finite many type case, as long as the two adjacent types satisfy $\frac{\bar{\theta}}{\underline{\theta}} > c^0$, the non-costly separating still holds, but it is not true for the continuous types case. For finite many type case and continuous type case, the buyer can still combine both the time and the quality distortion to signal his type if necessary, that is, by the newly defined "least-costly separating equilibrium".

Besides the above extensions, many other bargaining rules as well as various bargaining power distributions can be implemented rather than the one we discuss above. Also, the extension to without commitment can be also very interesting.

Despite the fact that many contract negotiation involves bargaining, there is no bargaining in the standard contract theory, no matter in the screening, signalling or moral hazard models. The standard theory abstracts away bargaining by assuming the principal has all of the bargaining power. This research introduces the sequential bargaining into the standard screening model by allowing alternating offer and strategic delay and establishes some clear-cut results, for example, the non-costly separating equilibrium outcome, the sequential separating equilibrium and the simultaneous separating equilibrium according to the different parameter values. We have proved the existence and the uniqueness of the SEE and SIE. We've also defined the more general "least-cost separating contracts" as well as "bargaining-proof contracts".

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