Credit Market Frictions with Costly Capital Reallocation as a Propagation Mechanism*

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February 12, 2006

PRELIMINARY AND INCOMPLETE

Abstract

Empirical evidence suggests that capital separation is an important phenomenon over and beyond depreciation and that reallocation is a costly and time-consuming process. In addition, both separation and reallocation rates display substantial variation over the business cycle. We build a dynamic general equilibrium model where capital separation occurs endogenously because of credit constraints and capital (re)allocation is costly due to search frictions and capital specificity. Compared to the frictionless counterpart but also compared to models of financial frictions without costly capital reallocation, our model matches surprisingly well the persistence in U.S. output growth. Furthermore, our model implies that productive capital stocks vary more than reported in the data, which has the potential to substantially reduce the volatility of technology shocks inferred from the Solow residual.

*We thank Alain Delacroix, Etienne Wasmer and seminar participants of the Journées du CIRPÉE 2005, HEC Montréal and the Bank of Canada for helpful comments. Financial support from FQRSC, SSHRC (Kurmann) and PAFARC-UQAM (Petrosky-Nadeau) is gratefully acknowledged.

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1 Introduction

A large body of evidence suggests that credit market frictions play an important role for firm behavior. Empirically, panel data studies find that small firms with more difficult access to credit pay fewer dividends, take on more debt, and have investment rates that are more sensitive to cash flows even after controlling for future profitability.¹ Theoretically, numerous papers show how optimizing models of the firm with incomplete contract enforcement and asymmetric information in the lending process can rationalize the observed correlation of firm size and age with mean growth (negative) and survival rates (positive).²

While the relevance of credit market frictions is well established on the microeconomic level, their macroeconomic consequences for business cycle fluctuations are less obvious. Models of financial intermediation and agency costs by Bernanke and Gertler (1989) or Kiyotaki and Moore (1997) imply that the firm’s ability to finance investment varies inversely with the value of its collateral and thus with the business cycle. This financial accelerator mechanism has the potential to generate amplified and persistent output effects in response to small shocks. Yet, simulations in a dynamic stochastic general equilibrium (DSGE) context by Kocherlakota (2000), Chari, Kehoe and McGrattan (2002), or Petrosky-Nadeau (2005) suggest that for plausible calibrations, credit market frictions of this type alone fail to generate quantitatively important business cycle fluctuations.³

The lack of internal propagation can be traced back to the assumption in these models that capital from exiting firms is reallocated immediately and costlessly to new firms. In general equilibrium, this assumption implies that credit market frictions only affect the elasticity of aggregate investment with respect to average net worth of all firms. But quarterly investment as a share of fixed private non-residential capital stocks represents at most 3% in the national accounts, and the share of capital in production is about one third. It is therefore not surprising that more sensitive investment dynamics by themselves have only a very limited impact on the

¹See Hubbard (1998) and Stein (2000) for surveys.
³For example, Petrosky-Nadeau (2005) simulates the financial accelerator model of Bernanke, Gertler and Gilchrist (1998) in a New Keynesian context. He finds that the response of investment with the BGG friction is almost double than if no financial friction is present. However, the financial accelerator contributes only about 0.05% to the response of output and fails to generate persistence in output growth.
cyclical behavior of output.

In this paper, we investigate to what extent credit market frictions together with costly capital reallocation can generate more important business cycle fluctuations. Our investigation is motivated by firm-level observations in Ramey and Shapiro (1998), Eisfeldt and Rampini (2005) as well as Becker et al. (2005) who find that capital separation is an important phenomenon over and beyond depreciation, and that reallocation is a costly and time-consuming process. The national accounts miss these reallocation flows and thus, investment as a share of the capital stock is likely to be substantially larger than reported in the aggregate data. Furthermore, the same studies report important countercyclical and procyclical variations in capital separation and reallocation rates, respectively. This implies that capital stocks used for actual production may be much more volatile than previously assumed.

We formalize these observations with a DSGE model that extends the standard real business cycle (RBC) benchmark along three dimensions. First, firms must post projects at a cost and search for available capital to undertake investments. The probability of a match depends on how much capital is made available by households relative to the total number of projects posted. Second, matched capital remains with the same firm until separation occurs. Separated capital looses a fraction of its value due to specificity, and reallocation to another productive unit is time-consuming due to the aforementioned search friction. Third, separation occurs in part endogenously when the firm’s revenue falls short of covering factor payments, which are determined prior to the realization of an idiosyncratic productivity shock. We interpret the ex-post nature of this idiosyncratic shock as a simple form of a credit market constraint, in line with the large empirical literature on the importance of financial frictions for firm dynamics.

Allowing credit market frictions to impact capital reallocation implies that business cycle conditions and thus exogenous shocks directly affect the productive capital stock rather than just investment in new capital. For example, a positive permanent technology shock decreases the separation rate of capital from production. The consequent drop in the marginal value of capital leads to an important income effect that shifts up labor supply and thus reduces the response of hours worked. As a result, output reacts much more gradually to a permanent shock than the RBC benchmark, in line with empirical evidence from structural VARs.

Overall, this mechanism generates substantial internal propagation. In particular, our capital
matching model is capable of replicating the marked positive autocorrelation in U.S. output growth that, as Cogley and Nason (1995) emphasize, remains one of the great challenges for modern business cycle research. Indeed, neither the RBC benchmark nor the financial accelerator model of Bernanke, Gertler and Gilchrist (1998) manage to generate any of this persistence. And as we illustrate with our simulations, both the credit market friction and the costly reallocation of capital are necessary to generate this result in our model.

Our strategy to formalize capital allocation is inspired by the now widely employed search-and-matching approach to model labor market frictions, as pioneered by Diamond (1981) and Mortensen and Pissarides (1994). While this approach abstracts from the microfoundations for market incompleteness, it provides a dynamic mechanism that has proved tractable and encompasses different frictions encountered in the allocation of physical capital to productive units. Previously, Dell’ Aricia and Garibaldi (2000), den Haan, Ramey, and Watson (2003) and Wasmer and Weil (2004) have interpreted the same matching process as the result of firms soliciting financing for their capital expenditures. While such financing frictions may be highly relevant for new entrepreneurs and small firms, they seem less obvious for large firms that have ready access to liquid credit markets and account for the bulk of aggregate capital stocks. Aside from our different interpretation of the matching process, we also incorporate our model in a standard DSGE framework with endogenous labor supply and intertemporal consumption/savings decisions. The advantage of doing so is that the quantitative implications of our model can be readily compared to the RBC benchmark that our model nests as a special case, but also to the financial accelerator models mentioned above where credit market frictions only affect investment.4

To our knowledge, there are only very few papers that examine the business cycle implications of costly capital entry/exit together with credit market frictions. One of them is Cooley, Marimon and Quadrini (2003) who derive credit market frictions from limited contract enforceability and allow for heterogeneity in firm size. This heterogeneity makes aggregation and the computation of the equilibrium a non-trivial issue. By contrast, our modeling approach bypasses

4Moran (2005) and Pierrard (2005) also incorporate credit matching frictions into a business cycle context. However, they do not model endogenous capital separation and reallocation. In line with our results, their models fail to generate endogenous amplification and persistence.
the issue of firm size by assuming constant returns to scale production and the equilibrium is solved for a loglinear approximation around the balanced growth path. This greatly facilitates computation, allows for straightforward comparison with well-known business cycle models, and leaves us with plenty of flexibility to extend our analysis to more general descriptions of the rest of the economy.

The remainder of the paper is organized as follows. Section 2 reviews the empirical evidence on investment flows and capital stocks. Section 3 presents the model. Section 4 discusses functional specifications and calibration. Sections 5 and 6 report quantitative results and assess their robustness. Section 7 concludes.

2 Empirical evidence

To motivate our extension of the business cycle model, we first review the computation of investment flows and capital stocks in the U.S. national accounts. Second, we document firm-level evidence on separation and reallocation of existing capital.

2.1 NIPA investment flows and capital stocks

For the National Income and Production Accounts (NIPA), the Bureau of Economic Analysis (BEA) computes investment flows and aggregate capital stocks (called fixed reproducible tangible wealth) using a supply-side top-down approach. Investment flows by asset type are measured as the real value of shipments from capital goods producing industries after subtracting inventory changes, net exports abroad as well as private and government consumption of these assets. Capital stocks for each asset are then inferred from the respective investment flows using the perpetual inventory method

$$ K_{a,t} = \sum_{j=0}^{\infty} \omega_{ajt} I_{a,t-j}, $$

where $K_{a,t}$ is the capital stock of asset $a$ in period $t$; $I_{a,t-j}$ is the real investment flow into asset $a$ at $t-j$; and $\omega_{ajt}$ is the weight given to vintage $j$ of asset $a$. This weight embodies the

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Becker, Haltiwanger, Jarmin, Klimek and Wilson (2005) provide a more detailed description of these computations and discuss the associated problems.
depreciation (decline in value) of the asset due to wear and tear, obsolescence, accidental damage, and aging. For most asset types, depreciation schedules are assumed to decline geometrically over time.\(^6\)

For the sample 1950-1995, the annual investment flow for private non-residential assets averages 9.4% of its capital stock. The capital stock series is very smooth. Its Hodrick-Prescott filtered standard deviation for 1950-1995 is a mere 0.08. The low variability is one of the main reason why many business cycle researchers abstract from capital when computing Solow residuals (see for example King and Rebelo, 2000).

Aside from the many problems associated with measuring and appropriately deflating shipments of capital goods, there are three reasons why NIPA’s investment and capital stock series are problematic quantities for business cycle researchers. First, shipments of capital goods only provides information about investment flows of new assets and the BEA adjusts these series only for net transfers of used capital from consumers, government and foreign countries. Inter- and intra-industry transfers are completely missed. Second, the BEA’s depreciation schedules in \(\omega_{a jt}\) are supposed to reflect the service life of an asset, which implicitly assumes that capital from sales and exiting businesses is transferred costlessly to other productive units. To the extent that capital separation (i.e. exit of firms and sales) is an important phenomenon, \(\omega_{a jt}\) therefore underestimates the loss in capital value due to irreversibilities, specificity and reallocation frictions in the secondary market for capital. Both the first and second point imply that total annual investment in new and used capital goods may be substantially larger than 9.4%. Third and most importantly for our purpose, if capital separation and reallocation vary over the cycle, the NIPA capital stock measure may be too smooth.

### 2.2 Capital separation

To obtain a sense about gross investment flows, we need to adopt a bottom-up approach and look at actual firm-level data on investment expenditures and disinvestment. For example, Ramey and Shapiro (1998) use Compustat data to compute aggregate flows of gross capital additions and subtractions. For their full sample (1959-1995), on average 7% of the aggregate capital stock exits firms per year. This rate is trending upwards over time (from 4.2% for 1959-

\(^6\)See Katz and Herman (1997) for a description.
1969 to 9.5% in 1990-1995) and varies countercyclically around trend, resulting in a correlation coefficient with unemployment of 0.52. Ramey and Shapiro decompose this gross subtraction flow into three major components: retirement, sales and exits due to mergers and bankruptcies. Retirements – which can be interpreted as the physical result of depreciation – are the most important component (71%), followed by sales (21%). Pure exits, by contrast, account only for a small fraction (9%).

One potential problem with Ramey and Shapiro’s results is that their investment in new capital goods only averages an annual 6.9% of the capital stock, which is substantially less than reported in the NIPA tables. This suggests important measurement problems in Ramey and Shapiro’s calculations (we return to this issue below). Nevertheless, we retain that capital separation seems to be an important cyclical phenomenon over and beyond depreciation.

An additional problem with Compustat data is that it only covers corporations that file with the SEC. Other proprietorships and partnerships as well as establishments held by foreign firms not registered with the SEC are not part of their capital stock measure. Small and medium-size firms are thus underrepresented. Given that it is exactly these firms that are most likely to undergo major changes (merger/acquisition, bankruptcy, structural reorganisation), the share of separation due to sales and exits as well as the separation rate in general are likely to be larger for the economy as whole.

A recent study by Becker, Haltiwanger, Jarmin, Klimek and Wilson (2005) provides additional information about the importance of capital separation due to exits. Among many other things, these authors use data from the Annual Capital Expenditure Survey (ACES) to quantify the importance of capital separation due to sales and exits. In existence since 1993, ACES is a nationally representative firm-level survey of capital investment in new and used structures and equipment. Every year, ACES selects a new probability sample that can be used to compute the capital stock of firms that disappear. These firms can be separated into two categories. Firms that cease to be active (called pure deaths) and firms that continue to operate under a different firm (called firmid deaths). Becker et al.’s data show that the amount of capital separated this way has risen from 2% of GDP in 1993 to 4% of GDP in 1999, with substantial variations over

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7Becker et al. also compute capital separation measures with data from the Annual Survey of Manufacturers (ASM)...to be added in a future draft.
the sample. Interestingly, roughly 90% of these separations come from firm deaths; i.e. continuing establishments.\footnote{The unimportance of pure firm deaths for capital separation suggests that using data on business failures alone for macroeconomic purposes is not very informative.} We consider this measure of capital separations from mergers/acquisition and bankruptcies as a lower bound. For one thing, it does not capture establishment deaths from multi-unit firms. In addition, the period under consideration is one with strong continuous growth in the U.S.. During slowdowns and recessions, the number of mergers/acquisitions and bankruptcies is expected to be substantially higher.

### 2.3 Capital reallocation

As mentioned above, Ramey and Shapiro (1998) also compute gross flows of capital additions. On average, these gross additions make up 9.7% of the total capital stock (without exhibiting an upward trend). Roughly 70% of the flows come from expenditures in new capital by existing firms (i.e. investment) while about 25% come from acquisitions of existing assets (i.e. capital reallocation). Similar to exit rates, entry of new firms contributes only a small fractions to capital additions. These addition rates are procyclical and exhibit large fluctuations.

As we noted previously, Ramey and Shapiro’s investment in new capital represents a substantially smaller share of capital stock than reported in the NIPA tables.\footnote{Ramey and Shapiro’s data also come with a number of other potential problems. First, their transformation from book to current-value data depends on the correct choice of price deflator and knowledge about the age of each capital acquisition. Second, capital addition and separation rates vary around roughly the same average starting in the mid 1980s. This would imply a constant capital stock, in contradiction with the data. Third, retirement rates are well below depreciation rates as assumed in the NIPA tables. We plan to investigate this issues about Compustat in the future.} One possibility for this discrepancy is that this investment share is computed without taking into account depreciation of capital in use. When doing so, the gross flow of additions jumps up to 17.3% of total capital stocks, with investment in new capital representing 12.3%. One has to keep in mind, that these depreciation rates represent accounting standards rather than actual decreases in the value-of-use. Nevertheless, it remains true that reallocation of used capital accounts for an important part of investment that is (almost) entirely missed in the NIPA tables.

This conclusion is confirmed by a more recent study by Eisfeldt and Rampini (2005) who also use Compustat data from 1971 to 2000 but base their calculations on book values rather
than current values as Ramey and Shapiro (1998) do. Eisfeldt and Rampini report that capital reallocated each year represents about 5% of total capital stocks and about 1/4 of total investment. Furthermore, this reallocation rate is positively correlated with output, with a correlation coefficient of 0.64.

The important share of reallocation in total investment provides indirect evidence for the importance of capital separation – even more so because capital reallocation seems to be associated, on average, with a substantial loss in value relative to its replacement cost at the original place of use. In particular, Ramey and Shapiro (2000) argue that reselling capital is a time-consuming and costly process because of thinness in used-capital markets and sectoral specificity of capital. Their argument is based on equipment level data about closures of aeronautical plants. They find that aerospace companies are overrepresented among buyers, and that even after taking into account age-related depreciation, the average resale value of equipment is only 28% relative to replacement cost.

11 Becker et al.’s ACES data corroborate this finding indirectly. They compare their series for capital separation due to firm death with the following year’s series of used capital expenditures and other additions and acquisitions. Over their 8-year sample, the thus defined absorption rate equals on average 64% of total separations. Since this measure also includes assets sold by continuing firms, the absorption of separated capital from firm death is likely to be lower.

3 The Model

As in the frictionless RBC benchmark, our model is populated by two agents: firms that produce using capital and labor; and households who decide on optimal consumption, leisure and investments in either riskless bonds or productive capital. There are two frictions in the model. First, the allocation of new and used capital to firms involves a costly and time-consuming search process. Second, firms face ex-post idiosyncratic productivity shocks that result in endogenous separation of loss-making capital units from production. For the sake of simplicity, we abstract

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10 The 5% reallocation rate is obtained when capital stocks are measure by the Compustat variable "property, plant and equipment", which is also what Ramey and Shapiro (1998) use.

11 Even for machine tools, which typically have a better resale value than specialized aerospace equipment, the resale value is only about 40% relative to the replacement cost.
from a distinct sector for capital allocation. Instead, households act directly as lenders.

3.1 Search and matching in the capital market

Capital is either in a productive state or in a liquid state. We define by $K_{it}$ the productive capital stock that enters the production function of firm $i$ in period $t$. Liquid capital $L_{it}$, in turn, is made up of two components: used capital that has been separated previously from other firms and new capital made available by households. As described below, we allow for the possibility that separation involves a loss of value of capital. But once this adjustment is made, our model does not distinguish between used and new capital. Hence, a negative flow of new capital simply implies that households reaffect used capital for consumption or investment in riskless bonds.

To undertake new investments, firms must post projects and search for liquid capital at cost $\kappa$ per project. We denote by $V_{it}$ the number of posted projects of firm $i$ in period $t$. The amount of liquid capital allocated to firms in a given period is subject to a technology that matches the total number of projects $V_t = \int V_{it} dt$ to available liquidity $L_t$. We describe this matching process with a function $m(L_t, V_t)$. A firm’s probability to find capital is therefore given by $p(\theta_t) = \frac{m(V_t, L_t)}{V_t}$ with $\partial p(\theta_t) / \partial \theta_t > 0$, where $\theta_t = \frac{L_t}{V_t}$ may be interpreted as a measure of relative capital market liquidity. Likewise, the probability of liquid capital being matched to a firm equals $q(\theta_t) = \frac{m(V_t, L_t)}{L_t}$ with $\partial q(\theta_t) / \partial \theta_t < 0$. We will assume that $m(L_t, V_t)$ exhibits constant returns to scale and thus $p(\theta_t) = \theta_t q(\theta_t)$.

The identical treatment of used and new capital in liquidity implies that the search friction applies to all investment flows and not just to the reallocation of used capital. While the frictions involved in the allocation of new capital are certainly different than the frictions in the allocation of used capital, we believe that this simplifying assumption is as reasonable as imposing a convex investment adjustment cost function.$^{13}$

Capital matched to a firm in period $t-1$ enters production in period $t$. This relationship

\[ p(\theta) \text{ and } q(\theta) \text{ are between 0 and 1, we require that } m(l_t, v_t) \leq \min[l_t, v_t] \]

$^{12}$In addition, to ensure that $p(\theta)$ and $q(\theta)$ are between 0 and 1, we require that $m(l_t, v_t) \leq \min[l_t, v_t]$

$^{13}$As discussed in the introduction, the search friction for new capital could alternatively describe the process of firms soliciting lenders to finance their capital expenditures. This is the motivation pursued by Dell’ Aricia and Garibaldi (2000), Den Haan, Ramey, and Watson (2003) and Wasmer and Weil (2004). While such financing frictions make sense for small, bank-financed firms, it seems less plausible for larger firms (that account for the bulk of aggregate capital) with ready access to liquid capital markets.
between firm and capital continues to hold in \( t + 1 \) with probability \((1 - s_t)\) and so on for periods thereafter. If the relationship is terminated, which happens with probability \( s_t \), the capital is separated and returned to the lender net of depreciation \( \delta \). Both the matching probability and the separation rate are taken as exogenous by firms but depend on the state of the economy, as will be described below. Given these assumptions, firm \( i \)'s total capital stock used in existing projects evolves according to the following law of motion

\[
K_{it+1} = (1 - \delta)(1 - s_t)K_{it} + p(\theta_t)V_{it}.
\]

### 3.2 Households

The representative household chooses consumption \( C_t \), leisure \( 1 - N_t \), risk-free bond holdings \( B_{t+1} \), and the amount of liquidity \( L_t \) destined for capital investment in order to maximize the expected discounted flow of utility \( u(C_t, 1 - N_t) \). When liquidity gets matched with a project and is transformed into productive capital, it yields a net return of \( \rho_{t+1} \) in the following period. Any liquidity that remains unmatched yields zero return.

Given these assumptions, the optimization program of the household is described by the Bellman equation

\[
V(U_t, K_t, B_t) = \max_{C_t, N_t, L_t, B_{t+1}} \left[ u(C_t, 1 - N_t) + \beta E_t V(U_{t+1}, K_{t+1}, B_{t+1}) \right]
\]

\[
+ \Lambda_t \left[ W_t N_t + \rho_t K_t + \varphi(1 - \delta)s_t K_t + U_t + B_t + D_t - C_t - L_t - \frac{B_{t+1}}{(1 + r_t)} - G_t \right]
\]

s.t. \( K_{t+1} = (1 - \delta)(1 - s_t)K_t + q(\theta_t)L_t \)

where \( U_t = (1 - q(\theta_{t-1}))L_{t-1} \) is the quantity of unmatched liquidity in \( t - 1 \), \( D_t \) are firm profits transferred to households, \( \varphi(1 - \delta)s_t K_t \) is the value of capital separated from firms and returned into the budget constraint, \( r_t \) is the risk-free rate between \( t \) and \( t + 1 \), \( G_t \) is an exogenous government expenditure shock; and \( \Lambda_t \) is the shadow value of the budget constraint. The coefficient \( \varphi \) allows for the possibility that separated capital net of depreciation \((1 - \delta)s_t K_t\) suffers a loss in value due to specificity and/or costs related to separation. In particular, \( \varphi = 1 \) implies no loss while \( \varphi = 0 \) implies irreversibility. Also note that for now, both matching probability \( q(\theta_t) \) and separation rate \( s_t \) are taken as exogenous by households.
The first-order conditions of this optimization problem are

\[(C_t) : u_C = \Lambda_t \quad (1)\]
\[(N_t) : u_N = \Lambda_t W_t \quad (2)\]
\[(B_{t+1}) : \beta E_t[\Lambda_{t+1}(1 + r_t)] = \Lambda_t \quad (3)\]
\[(L_t) : \beta E_t[V_U(U_{t+1}, K_{t+1}, B_{t+1})(1 - q(\theta_t)) + V_K(U_{t+1}, K_{t+1}, B_{t+1})q(\theta_t)] = \Lambda_t \quad (4)\]

The first three conditions are standard. The fourth condition for the household’s choice of liquidity available for capital investment calls for some interpretation. It states that the discounted expected utility of the marginal unit of liquidity must equal the expected discounted return from investing in the riskless bond. With probability \((1 - q(\theta_t))\) a unit of liquidity remains unmatched and is worth \(V_U(U_{t+1}, K_{t+1}, B_{t+1})\) to the household, while with probability \(q(\theta_t)\) it is matched with a project and turned into productive capital with marginal value \(V_K(U_{t+1}, K_{t+1}, B_{t+1})\).

From the above Bellman equation, we can work out these marginal values as

\[V_U(U_t, K_t, B_t) = \Lambda_t \quad (5)\]
\[V_K(U_t, K_t, B_t) = \Lambda_t[\rho_t + \varphi(1 - \delta)s_t] + (1 - \delta)(1 - s_t)\beta E_t V_K(U_{t+1}, K_{t+1}, B_{t+1}) \quad (6)\]

Note that \(V_K\) is dynamic because with probability \(1 - s_t\) the investment relationship between household and firm continues into the next period.

### 3.3 Firms

At the beginning of each period, firm \(i\) observes exogenous aggregate technology \(X_t\) and hires labor \(N_{it}\) given the capital stock of its existing projects \(K_{it}\) to produce with technology

\[a_{it}f(X_t N_{it}, K_{it}), \quad (7)\]

with \(f_N, f_K > 0\) and \(f_{NN}, f_{KK} < 0\). The variable \(a_{it} > 0\) denotes an idiosyncratic productivity shock to firm \(i\) that is independently distributed over time with cumulative density \(F(a_{it})\) and mean \(E(a_{it}) = 1\). The realization of \(a_{it}\) takes place after all input decisions and factor price equilibria are established. The ex-post nature of this shock represents the credit market friction of our model and will give rise to endogenous separation. By making the shock known to both
the firm and households, we bypass, however, any agency problems that are usually emphasized in the literature on financial frictions.

Aside from the optimal amount of labor to hire, the firm needs to decide on new project postings \( V_{it} \), which come at unit cost \( \kappa \). The profit maximization problem of the firm is thus described by the following Bellman equation

\[
J(K_{it}) = \max_{N_{it}, V_{it}} \left[ f(X_{it}N_{it}, K_{it}) - W_{it}N_{it} - \rho_{it}K_{it} - \kappa V_{it} + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J(K_{it+1}) \right]
\]

s.t. \( K_{it+1} = (1 - \delta)(1 - s_{it})K_{it} + p(\theta_t)V_{it}, \)

where \( W_t \) and \( \rho_t \) are the wage rate and the rental rate of capital, respectively; and \( \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \) is the discount factor of future cash flows. This discount factor is a function of \( \Lambda \) because the firm transfers all profits to the households. Note that we dropped the idiosyncratic productivity shock \( a_{it} \) from the production function because the firm’s optimal decision occurs before the realization of the shock, which is expected to equal \( E(a_{it}) = 1 \). Furthermore, both \( W_t \) and \( \rho_t \) are taken to be exogenous by the firm. The exogeneity of \( W_t \) is a direct consequence of our assumption of competitive labor markets. The exogeneity of \( \rho_t \), in turn, will be further discussed below.

The resulting first-order conditions of the optimization problem are

\[
(N_{it}) : f_N(X_{it}N_{it}, K_{it}) = w_t
\]

\[
(V_{it}) : \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{it+1}) = \kappa \frac{p(\theta_t)}{p(\theta_t)}
\]

where \( J_K(K_{it}) \) is the marginal value to the firm of an additional matched project that has been transformed into capital. In addition, differentiating the firm’s value function with respect to productive capital yields

\[
J_K(K_{it}) = f(X_{it}N_{it}, K_{it}) - \rho_{it} + (1 - \delta)(1 - s_{it})\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{it+1}).
\]

This equation simply states that the value to the firm of an additional unit of capital is worth today’s marginal product of capital net of the rental rate plus its expected future value net of depreciation in case the project is continued.
3.4 Rental rate of capital

To determine the rental rate of capital, we assume that once matched, households and firms split the surplus of their relationship according to a Nash bargaining process. As discussed above, this bargaining process takes place before the idiosyncratic shock \( a_{it} \) is realized. The surplus is the sum of marginal benefits to each party, \( S_{it} = J_K(K_{it}) + \frac{V_k(U_{it}, K_{it})}{\lambda_{it}} - V_U(U_{it}, K_{it}) \). Define \( \eta \) as the household’s relative bargaining power. It then receives \( V_k(U_{it}, K_{it}) - V_U(U_{it}, K_{it}) = \eta S_{it} \), while the firm’s share is \( J_K(K_{it}) = (1 - \eta)S_{it} \). After some algebraic manipulations (see the appendix) we obtain the following expression for the rental rate

\[
\rho_{it} = \eta \left[ f_K(X_t N_{it}, K_{it}) + (1 - \delta)(1 - st) \frac{K}{\theta_t} \right] + (1 - \eta)[\delta + (1 - \varphi)(1 - \delta)s_t].
\]

The first term in brackets on the right hand side is the maximum amount the firm is willing to pay per unit of capital. It equals the marginal product of capital plus the average search cost for capital expenditures that is saved by continuing the relationship into next period. The second term in brackets is the opportunity cost of the lender, which equals the fraction not lost to depreciation when capital remains liquid \( \delta \) plus the value not lost to specificity when capital is separated \( (1 - \varphi)(1 - \delta)s_t \).

From the optimality conditions on liquidity and bond holdings, results from the firm’s problem and Nash bargaining, a relationship between the economy’s risk free rate and the tension on credit markets can be borne out (see again the appendix for details on this derivation)

\[
\beta E_t[\Lambda_{t+1}r_t] = \frac{\eta}{1 - \eta} \frac{\kappa}{\theta_t} \Lambda_t.
\]

All else being equal, an increase in the economy’s risk free rate \( r_t \) implies a decrease in capital market liquidity rate \( \theta_t \) because households find it less profitable to set aside funds for capital investments.

3.5 Separations

Capital separation can occur for a variety of reasons and we do not want to impose in our model that all separation is due to our credit market friction. We therefore model the separation rate \( s_t \) as

\[ s_t = s^x + s^e_t, \]
where $s^x$ denotes (constant) exogenous separation in the sense of being related to reasons other than the credit market; and $s^e_t$ denotes endogenous separation due to our credit market friction. To model this latter part, we assume for now that any firm with negative profits after the realization of the idiosyncratic shock $a_{it}$ is terminated. Given that the firm profits after the realization of $a_{it}$ are $D_{it} = a_{it} f(XN_{it}, K_{it}) - W_i N_{it} - \rho_t K_{it} - \kappa V_{it}$, the threshold value $\tilde{a}_{it}$ up to which separation occurs equals $\tilde{a}_{it} = (W_i N_{it} - \rho_t K_{it} - \kappa V_{it})/f(XN_{it}, K_{it})$ and the endogenous part of separation equals $s^e_t = F(\tilde{a}_{it})$. From this formula, it is clear that separation depends on the state of the economy.

It is important to realize that this separation rule is not optimal from the point of view of the household. In fact, the i.i.d. nature of idiosyncratic shock implies that the household would like to continue the relationship with firms just below the zero profit threshold, because separation entails loss of value $(1 - \varphi)$ and because matching capital with a new firm is costly (there is a probability of no match at which the liquid capital unit yields zero return). Only for idiosyncratic productivity shock so low that the household needs to inject money to cover for wage payments is there a point at which separating becomes more profitable than injecting money and continuing the relationship. As preliminary calculations reveal (not reported here), this would result in an additional time-varying risk premium in the formula for the rental rate that takes into account the fact that households bear an asymmetric risk of non-repayment. We will investigate the quantitative effects of optimal separation and this risk premium in a future version of the paper.

3.6 Aggregation and equilibrium

The micro literature on firm dynamics usually assumes decreasing returns to scale production (see for example Cooley and Quadrini, 2001 or Esteban-Rossi and Wright, 2005). Here, for reasons of tractability, we follow the traditional macro literature and assume that the production function $f(\cdot)$ exhibits constant returns to scale. Under this assumption, it is straightforward to show that the capital labor ratio of all firms is the same and thus, all optimality conditions are independent of firm size and the rental rate is identical for all firms; i.e. $\rho_{it} = \rho_t$.

With the constant returns assumption, we bypass any issues that arise from firm size heterogeneity. These issues are admittedly important but taking them into account would greatly
complicate aggregation and quantitative analysis of the model. In particular, it allows us to draw direct comparisons with other representative agents models such as the frictionless RBC benchmark or the financial accelerator model of Bernanke, Gertler and Gilchrist (1998).

To compute the equilibrium, we aggregate over all capital units. The dynamics for the aggregate stock of productive capital become

\[ K_{t+1} = (1 - \delta)(1 - s_t)K_t + m(L_t, V_t). \]  
(13)

The aggregate equilibrium dynamics of our model are defined by the system of equations (7), (1)-(3), (8)-(13) plus aggregate profits \( D_t = A_t f(n_t, k_t) - W_t N_t - \rho_t K_t - \kappa V_t \). This last equation assumes that there exists a complete insurance market for shortfalls in wage and rental payments, which are assumed to be covered by the higher than average profits of surviving firms.

### 3.7 Comparison with the baseline RBC model

Before continuing to the quantitative evaluation of our model, it is useful to compare our model with the baseline RBC model (as described in King and Rebelo, 2000) in which both credit market frictions and costly capital allocation are absent. In particular, the RBC model describes a world in which the cost of project postings \( \kappa \) is zero and thus, firms post an infinity of projects. Moreover, all capital is returned to the household (net of depreciation) at the end of each period and is reallocated at no cost at the beginning of following period.

In terms of our model, these assumptions translate into \( s_t = 1 \), \( q(\theta_t) = 1 \) and \( U_t = 0 \). Furthermore, it can easily be shown that \( \rho_t = Af_k(n_t, k_t) \): they repayment on liquidity is equal to the marginal product of capital.\(^{14}\) Finally, from the law for productive capital one sees that to choose liquidity then amounts to choosing capital in the following period; i.e. \( L_t = K_{t+1} - (1 - \delta)K_t \). This implies a value of matched liquidity \( V_K(U_t, K_t, B_t) = \Lambda_t[\rho_t + (1 - \delta)] \), and the optimality condition for the choice of liquidity becomes a standard Euler equation:

\[ \beta E_t \Lambda_{t+1}[\rho_{t+1} + (1 - \delta)] = \Lambda_t. \]

\(^{14}\)The value of bargaining power \( \eta \) is irrelevant in the RBC setting as the competitive nature of the capital market rules out any positive surplus between matched firms and lenders.
4 Shocks, functional forms and calibration

4.1 Shocks

Following much of the RBC literature we assume that our model economy is perturbed by two exogenous processes: an aggregate technology shock $X_t$ and a government expenditure shock $G_t$. The technology shock follows a random walk with drift

$$\log X_t = \mu + \log X_{t-1} + \varepsilon^X_t,$$

with $\varepsilon^X_t \sim (0, \sigma^2_X)$. The technology shock thus has a permanent effect on the different real aggregates with the exception of labor.

The government expenditure shock $G_t$ is also persistent but stationary process

$$\log G_t = (1 - \rho_G) \log G + \rho_G \log G_{t-1} + \varepsilon^G_t$$

with $\varepsilon^G_t \sim (0, \sigma^2_G)$ and $0 < \rho_G < 1$.

The two shocks are assumed to be uncorrelated. We can therefore compare the impulse response functions of the two shocks to impulse response functions obtained from structural VARs that identify a permanent and a transitory shock (e.g. Blanchard and Quah, 1989).

4.2 Functional forms

For household preferences, we assume indivisible labor as in Hansen (1985) and Rogerson (1988) and specify the family’s expected utility as

$$u(C, N) = \frac{1}{1-\gamma} \left[ C^{1-\gamma} v^*(N)^{1-\gamma} - 1 \right],$$

where $v^*(1-N) = \left[ N v(1-H) \frac{1}{1-\gamma} + (1 - \frac{N}{H}) v(1) \frac{1}{1-\gamma} \right]$ and $H$ is the number of hours worked by the employed (see King and Rebelo, 2000 for a detailed discussion). $N$ therefore represents the average fraction of hours worked in the economy. The linearity of hours worked implies an infinite Frisch elasticity of labor supply. This allows us to sidestep the issue of insufficient labor supply elasticity in standard Walrasian models of the labor market, thus enabling the RBC benchmark to better match the relatively volatilities of employment and real wages in the data.

For production, we assume a Cobb-Douglas function with constant returns to scale of the form

$$af(XN, K) = a(XN)^{1-\alpha}K^\alpha$$

with $0 < \alpha < 1$. The idiosyncratic shock is assumed to follow a log-normal distribution (which guarantees $a > 0$) with variance $\sigma^2_a$ and mean equal to $-\frac{\sigma^2_a}{2}$. 

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(so as to satisfy $E(a) = 1$). Finally, the matching technology takes the form similar to the one used in the labor literature, $m(V, L) = \chi V^\epsilon L^{1-\epsilon}$ with $0 < \epsilon < 1$.

4.3 Calibration

We calibrate our model to quarterly data. For the parameters that are common with the RBC benchmark, we use calibrations that are standard in the literature. The annual trend growth rate is set to 1.6%, which implies $\mu = 0.004$. The household’s discount factor is set to $\beta = 0.992$ in order to match an average annual real yield on a riskless 3-month treasury bill of 4.95%. We set $\gamma = 2.5$, in line with the survey of asset pricing studies by Kocherlakota (1996). This has the realistic implication that more consumption in the family is allocated to working individuals than to non-working individuals (see King and Rebelo, 2000 for details). Furthermore, $H$ on the utility part for leisure is set such that the average fraction of total hours worked equals $n = 0.214$. The rate of depreciation of capital is set to $\delta = 0.025$, which corresponds to an annual decline of productive use of capital of 10%. The value of $\alpha = 1/3$ implies an average labor share in production of two thirds. Finally, we use estimates from Christiano and Eichenbaum (1992) to set the average share of government consumption in output to $G/Y = 0.177$.

For the remaining parameters that are proper to our capital matching model here, the calibration strategy consists of matching a number of salient long-run averages from the firm-level data discussed in Section 2 and other sources. First, we set the steady state credit spread to 1.87% on an annual basis, which corresponds to the average spread of the Aaa corporate bond yield over the 3-month Treasury bill. This implies a steady state rate of return on capital of 6.82% in annualized terms.

Second, based on corporate profits after taxes obtained from the NIPA tables, we set the ratio of profits to output to an annual 5%. This lets us pin down the steady state capital-output ratio, output, consumption and real wages.

Third, we choose a quarterly steady state separation rate of $s = 0.02$. Together with $\delta = 0.025$, this rate implies the following steady state gross investment rate (using the capital accumulation equation (13))

$$\frac{m(V, L)}{K} = [\exp(\mu) - (1 - \delta)(1 - s)] = 0.0485,$$
which translates into a yearly investment rate of 19.4% — a value that is in between of what Ramey and Shapiro (1998) and Eisfeldt and Rampini (2005) report from their Compustat data.

To calibrate endogenous separation, we assume that the exogenous part of separation is constant and accounts for half of total separations; i.e. $s^x = 0.01$. Together with the steady state zero profit threshold $\tilde{a}$ (computed from the above calibrations), the variance of the lognormal distribution $\sigma_a^2$ is then such that $F(\tilde{a}) = s - s^x = 0.01$. Furthermore, we leave $\varphi = 1$ for now (no loss in value-of-use for separated capital). Later on, we will set $\varphi$ such that roughly 25% of steady state investment comes from used capital.

For the matching probabilities $p(\theta)$, $q(\theta)$ and the matching function elasticity $\epsilon$, we have admittedly the least information because for capital goods, there is no equivalent to the unemployment rate and the help wanted index that are used to calibrate search models of the labor market. However, it turns out that the firm’s probability of finding capital for a project $p(\theta)$ hardly affects the dynamics of the model and only influences the cost of project postings $\kappa$, which is endogenously pinned down by the system of steady state equation (the same is true for the bargaining weight $\eta$). For now, we thus choose $p(\theta) = 0.5$. The probability for a unit of capital to be allocated to a project $q(\theta)$, in turn, can be linked to the average duration of used capital goods before reallocation: $q(\theta) = 1/d$ where $d$ equals to duration in quarters. From various industry surveys of used capital markets, it appears that the average duration for property and equipment is somewhere in between one and two quarters. We therefore set $q(\theta) = 0.75$. Finally, for lack of evidence, we set $\epsilon = 0.5$ and assess the robustness of our results when changing this parameter.

Finally, we need to calibrate the parameters of the exogenous driving processes. For the permanent technology shock, we extract a Solow residual from the data and demean its growth rate. The resulting volatility implies $\sigma_x = 0.72$. For the transitory government spending shock, we take the estimates from Christiano and Eichenbaum (1992) and set $\rho_G = 0.96$ and $\sigma_G = 0.089$.

## 5 Simulation results

We analyze the empirical performance of our model in two stages. First, we consider impulse response functions (IRFs) of different aggregates with respect to a permanent technology shock
and with respect to a temporary government spending shock. The goal of this exercise is to graphically highlight the effects of our credit market friction with costly reallocation. Second, we report a variety of unconditional second moments. To put the different results in perspective, we compare them to the benchmark RBC model, which is a special case of our model, and the non-monetary version of Bernanke, Gertler and Gilchrist’s (1998) financial accelerator model.

The permanent nature of the technology shock implies that we need to normalize all aggregates by $X_t$ to obtain a stationary system that we can simulate using log-linear solution techniques (except for the predetermined variables, which we normalize by $X_{t-1}$). Once normalized, we compute the rational expectations solution of the log-linear system of equations with the numerical mechanism described in King and Watson (1998).\(^{15}\)

### 5.1 Impulse response functions

#### 5.1.1 Permanent aggregate technology shock

Figure 1 plots the IRFs of prominent macro aggregates to a permanent technology shock.

![Figure 1](image)

As is immediately apparent from the top-left panel, our capital matching model (solid lines) generates a more gradual response of output to the shock than the RBC benchmark (dotted lines).

\(^{15}\)We thank Bob King for providing us with the relevant Matlab code.
lines). In fact, this response is remarkably close to the empirical IRF from the 2-variable structural VAR reported in Cogley and Nason (1995).

The main reason of this gradual response in output is the decrease on impact in aggregate hours worked. This decrease occurs because the positive income effect that shifts up the inelastic labor supply curve is stronger than the substitution effect coming from the technology induced outward shift of labor demand. By contrast, in the RBC benchmark, hours worked increase upon impact.

This result is interesting in its own right since Gali (1999) famously argued that the negative response of hours after a permanent shock is evidence of nominal rigidities. Our model shows that this decrease in hours can be obtained without nominal rigidities as long as the income effect in the labor market outweighs the substitution effect. But why is this income effect so large in our model compared to the RBC benchmark? The main reason is the different effect of the shock in the capital market. In the RBC model, the marginal value of investing in capital decreases relatively little. By contrast, the marginal value of investing in our model decreases much more because the rate of separation drops precipitously (see bottom-left panel of Figure 2 below). This leads to a larger response in the capital stock (see top-right panel of Figure 1), which is one of the main results of our model: endogenous separation can lead to substantially more volatile capital stocks.

16To be fair, there has been a substantial amount of controversy about the empirical robustness of Gali's findings. See for example the debate between Chari, Kehoe and McGrattan (2005) and Christiano, Eichenbaum and Vigfusson (2005). What is not contested in the literature, however, is that hours worked respond relatively little on impact in response to a permanent shock.

17Note the 25% deviation from steady state means that the separation rate drops from its steady value of 2% per quarter to 1.5%.
Figure 2

Figure 2 also displays the IRFs of the other variables related to separation and reallocation of capital. An interesting feature of these plots is that both project postings $V_t$ and liquidity put up for investment in capital $L_t$ drop. On the liquidity side, the reason for this drop is that households see less of a need to set aside funds for new capital investments given that less capital is being returned through separation. On the firm side, firms find it optimal to wait with new project postings (for which they incur cost $\kappa V_t$) until more funds are available. The outcome of these reactions is that the capital market tension $\theta_t = L_t/V_t$ drops, which means that the probability of locating funds for a project $p(\theta_t)$ drop while the probability of locating a project $q(\theta_t)$ increases.

5.1.2 Transitory government expenditure shock

Figure 3 displays the IRFs to a expansionary government expenditure shock.
As before, we observe that the capital stock reacts much stronger in our model (solid lines) than in the RBC benchmark (dotted lines). This is again due to endogenous separation, which increases greatly on impact of the shock (see Figure 4 below). Parallel to the above explanation, however, this drop in capital implies that the marginal value of capital jumps up. The drop in labor supply from the resulting income effect outweighs the leftward shift of labor demand from the drop in capital stocks. Hence, hours jump up strongly upon impact and do not display the humpshaped response pattern commonly found in structural VARs in response to non-permanent shocks. Likewise, output does not display a humpshaped response pattern. We will investigate in a future version of the paper how this result is affected by optimal separation and time-varying risk-premium (as discussed in Section 3).

For further reference, Figure 4 displays the IRFs of the variables related to separation and reallocation in response to the government expenditure shock.
5.2 Unconditional second moments

5.2.1 Autocorrelation of output growth

One of the great challenges in business cycle macroeconomics is the positive autocorrelation of output growth over several quarters in the data. As Cogley and Nason (1995) document, the RBC model completely misses to generate such positive autocorrelation and researchers have proposed different theories that could potentially explain this pattern. However, the results so far have been mixed at best.\(^{18}\)

Figure 5 displays the autocorrelation function for output growth for the data (green line), our model (blue line), the RBC model (dotted line) and a non-monetary version of Bernanke, Gertler and Gilchrist’s (1998) financial accelerator model (solid lines with stars).\(^{19}\)

\(^{18}\)See Gilchrist and Williams (1999) or Chiang, Gomes and Schorfheide (2003) for two of the more promising attempts.

\(^{19}\)See Petrosky-Nadeau (2005) for a description of the BGG model. The calibration of this model is similar to the one reported in BGG.
As is immediately apparent, our capital matching model tracks the empirical autocorrelation of output in the data surprisingly well. What is especially remarkable is the high value for the correlation at the first lag. To our knowledge, very few parsimonious models manage to come as close to the empirical counterpart.

By contrast, neither the RBC model nor BGG’s financial accelerator model generate any autocorrelation at all. This goes to show that credit market frictions together with costly capital allocation generates substantial internal propagation. What is also interesting in our model is that neither the credit market friction nor costly capital reallocation would have lead to this result.

5.2.2 Volatilities and cross-correlations

Table 1 presents unconditional second moments for the growth rates of different prominent macro aggregates for quarterly U.S. data, our capital matching model, the RBC benchmark and BGG’s financial accelerator model.

There are several striking features. First, our model creates more volatile capital growth rates relative to the growth rate of output (columns a). By contrast, in the RBC and BGG’s financial accelerator model, capital stocks can only be affected through variations in investment.
Changes in capital stocks due to endogenous separation thus represents a second channel through which capital stocks are affected. As discussed in Section 2, this channel seems to be very much present in the data. It could have important consequences for the measurement of the Solow residual and the thus resulting technology shocks. In particular, technology shocks as computed in Section 4 have been criticized for their large volatility that imply a substantial probability of technological regress (see for example the discussion in King and Rebelo, 2000). A more volatile capital stock as generated in our model has the potential to reduce the size of the technology shock, thus addressing one of the main criticism of the RBC paradigm.

A second interesting result is that our model succeeds in generating markedly more volatile movements in labor. The model also generates real wage growth that is less highly correlated than in the RBC model or BGG’s financial accelerator model (columns b). At the same time, our model implies that real wage growth becomes even more volatile, which is a step away from what we observe in the data.

Finally, our model generates fluctuations in normalized output $Y_t/X_t$ that are substantially larger than in the RBC benchmark. However, this internal amplification mechanism does not translate into larger fluctuations in output growth. In fact, the standard deviation of output growth becomes smaller relative to the RBC benchmark. We plan to investigate this issue further in a future version of the paper.

<table>
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<tr>
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<th>U.S. Data a</th>
<th>RBC a</th>
<th>Matching Model a</th>
<th>Bernanke et al. a</th>
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<td>b</td>
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<td>-0.015</td>
</tr>
</tbody>
</table>

Notes: (a) Std. Dev. relative to output. (b) Contemporaneous correlation with output.

Tab. 1 - Unconditional second moments

Figure 1:
6 Robustness of results

The purpose of this section is twofold. First, we quantify the robustness of our model with respect to alternative combinations of parameter calibrations. Second, we turn off endogenous separation and then substantially reduce the costly capital reallocation so as to illustrate that both credit market frictions and costly capital reallocation are needed for the internal propagation in our model.

Details and discussion of these results to be added.

7 Conclusion

To be added.

References


[22] Hall (1987),


