Instrumental variables estimation using heteroskedasticity-based instruments

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This presentation is based on the work of Arthur Lewbel, “Using Heteroskedasticity to Identify and Estimate Mismeasured and Endogenous Regressor Models,” *Journal of Business & Economic Statistics*, 2012. The contributions of Baum, Schaffer and Talavera are the development of Stata software to implement Lewbel’s methodology.

Motivation

Instrumental variables (IV) methods are employed in linear regression models, e.g., $y = \mathbf{X}\beta + u$, where violations of the zero conditional mean assumption $\mathrm{E}[u|\mathbf{X}] = 0$ are encountered.

Reliance on IV methods usually requires that appropriate instruments are available to identify the model: often via exclusion restrictions.

Those instruments, $\mathbf{Z}$, must satisfy three conditions: (i) they must themselves satisfy orthogonality conditions ($\mathrm{E}[u\mathbf{Z}] = 0$); (ii) they must exhibit meaningful correlations with $\mathbf{X}$; and (iii) they must be properly excluded from the model, so that their effect on the response variable is only indirect.

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Finding appropriate instruments which simultaneously satisfy all three of these conditions is often problematic, and the major obstacle to the use of IV techniques in many applied research projects.

Although textbook treatments of IV methods stress their usefulness in dealing with endogenous regressors, they are also employed to deal with omitted variables, or with measurement error of the regressors ('errors in variables') which if ignored will cause bias and inconsistency in OLS estimates.

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Lewbel's approach

The method proposed in Lewbel (*JBES*, 2012) serves to identify structural parameters in regression models with endogenous or mismeasured regressors in the absence of traditional identifying information, such as external instruments or repeated measurements.

Identification is achieved in this context by having regressors that are uncorrelated with the product of heteroskedastic errors, which is a feature of many models where error correlations are due to an unobserved common factor.

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In this presentation, we describe a method for constructing instruments as simple functions of the model's data. This approach may be applied when no external instruments are available, or, alternatively, used to supplement external instruments to improve the efficiency of the IV estimator.

Supplementing external instruments can also allow 'Sargan–Hansen' tests of the orthogonality conditions to be performed which would not be available in the case of exact identification by external instruments.

In that context, the approach is similar to the dynamic panel data estimators of Arellano and Bond (*Review of Economic Studies*, 1991) et al., as those estimators customarily make use of appropriate lagged values of endogenous regressors to identify the model. In contrast, the approach we describe here may be applied in a purely cross-sectional context, as well as that of time series or panel data.

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The basic framework

Consider Y_1, Y_2 as observed endogenous variables, X a vector of observed exogenous regressors, and $\varepsilon = (\varepsilon_1, \varepsilon_2)$ as unobserved error processes. Consider a structural model of the form:

$$
\begin{aligned}
Y_1 &= X'\beta_1 + Y_2\gamma_1 + \varepsilon_1 && (1) \\
Y_2 &= X'\beta_2 + Y_1\gamma_2 + \varepsilon_2 && (2)
\end{aligned}
$$

This system is triangular when $\gamma_2 = 0$ (or, with renumbering, when $\gamma_1 = 0$). Otherwise, it is fully simultaneous. The errors $\varepsilon_1, \varepsilon_2$ may be correlated with each other.

If the exogeneity assumption, $E(\varepsilon X) = 0$ holds, the reduced form is identified, but in the absence of identifying restrictions, the structural parameters are not identified. These restrictions often involve setting certain elements of β_1 or β_2 to zero, which makes instruments available.

In many applied contexts, the third assumption made for the validity of an instrument—that it only indirectly affects the response variable—is difficult to establish. The zero restriction on its coefficient may not be plausible. The assumption is readily testable, but if it does not hold, IV estimates will be inconsistent.

Identification in Lewbel's approach is achieved by restricting correlations of $\varepsilon\varepsilon'$ with X. This relies upon higher moments, and is likely to be less reliable than identification based on coefficient zero restrictions. However, in the absence of plausible identifying restrictions, this approach may be the only reasonable strategy.

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The parameters of the structural model will remain unidentified under the standard homoskedasticity assumption: that $\mathrm{E}(\varepsilon\varepsilon'|X)$ is a matrix of constants. However, in the presence of heteroskedasticity related to at least some elements of X, identification can be achieved.

In a fully simultaneous system, assuming that $\mathrm{cov}(X, \varepsilon_j^2) \neq 0, j = 1, 2$ and $\mathrm{cov}(Z, \varepsilon_1\varepsilon_2) = 0$ for observed Z will identify the structural parameters. Note that Z may be a subset of X, so no information outside the model specified above is required.

The key assumption that $\mathrm{cov}(Z, \varepsilon_1\varepsilon_2) = 0$ will automatically be satisfied if the mean zero error processes are conditionally independent: $\varepsilon_1 \perp \varepsilon_2 | Z = 0$. However, this independence is not strictly necessary.

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Unobserved single-factor models

A class of models satisfying the assumptions underlying Lewbel's method are those in which cross-equation error correlations are due to the presence of an unobserved common factor:

$$\begin{aligned} Y_1 &= X'\beta_1 + Y_2\gamma_1 + \varepsilon_1, \quad \varepsilon_1 = \alpha_1 U + V_1 && (3) \\ Y_2 &= X'\beta_2 + Y_1\gamma_2 + \varepsilon_2, \quad \varepsilon_2 = \alpha_2 U + V_2 && (4) \end{aligned}$$

where U, V_1, V_2 are unobserved, uncorrelated with X and conditionally uncorrelated with each other when conditioned on X. V_1, V_2 are idiosyncratic errors, while U is an omitted variable that may directly influence both Y_1, Y_2.

This general framework subsumes the case of classical measurement error, where $\gamma_2 = 0$ and $\alpha_2 = 1$. In this context, the unobserved common factor U is the measurement error in Y_2.

These models also include constructs where an omitted variable causes bias and inconsistency. For instance, in wage and schooling equations, the unobserved factor may represent an individual's ability and initiative, which influences both her schooling and labor productivity.

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To deal with measurement error or omitted variables, we would usually impose identification restrictions which provide instruments. Imagine there are no available instruments, and let Z be a vector of observed exogenous variables: a subvector of X, or X itself. Assume X is uncorrelated with (U, V_1, V_2); that Z is uncorrelated with (U^2, UV_j, V_1V_2); and that Z is correlated with V_2^2 (or, in a simultaneous system, with V_1^2 as well).

Given these assumptions, it can be shown that

$$\begin{aligned} \mathrm{cov}(Z, \varepsilon_1\varepsilon_2) &= 0 \quad (5) \\ \mathrm{cov}(Z, V_2^2) &\neq 0 \quad (6) \end{aligned}$$

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However, the errors need not actually arise from a factor model of this form; it is sufficient that the conditions

$$\begin{aligned} \mathrm{E}(X\varepsilon_j) &= 0,\ j = 1, 2 \qquad (7) \\ \mathrm{cov}(Z, \varepsilon_1\varepsilon_2) &= 0 \qquad (8) \end{aligned}$$

hold, along with some heteroskedasticity of ε_j. Identification is achieved whether or not Z is a subvector of X.

Single-equation estimation

In the most straightforward context, we want to apply the instrumental variables approach to a single equation, but lack appropriate instruments or identifying restrictions. The auxiliary equation or 'first-stage regression' may be used to provide the necessary components for Lewbel's method.

In the simplest version of this approach, generated instruments can be constructed from the auxiliary equations' residuals, multiplied by each of the included exogenous variables in mean-centered form:

$$Z_j = (X_j - \overline{X}) \cdot \epsilon \tag{9}$$

where ϵ is the vector of residuals from the 'first-stage regression' of each endogenous regressor on all exogenous regressors, including a constant vector.

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These auxiliary regression residuals have zero covariance with each of the regressors used to construct them, implying that the means of the generated instruments will be zero by construction. However, their element-wise products with the centered regressors will not be zero, and will contain sizable elements if there is clear evidence of 'scale heteroskedasticity' with respect to the regressors. Scale-related heteroskedasticity may be analyzed with a Breusch–Pagan type test: `estat hettest` in an OLS context, or `ivhettest` (Schaffer, SSC; Baum et al., *Stata Journal*, 2007) in an IV context.

The greater the degree of scale heteroskedasticity in the error process, the higher will be the correlation of the generated instruments with the included endogenous variables which are the regressands in the auxiliary regressions.

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Stata implementation

An implementation of this simplest version of Lewbel's method, `ivreg2h`, has been constructed from Baum, Schaffer, Stillman's `ivreg2` and Schaffer's `xtivreg2`, both available from the SSC Archive. The panel-data features of `xtivreg2` are not used in this implementation: only the nature of `xtivreg2` as a 'wrapper' for `ivreg2`.

In its current version, `ivreg2h` can be invoked to estimate

- a traditionally identified single equation, or
- a single equation that fails the order condition for identification: either (i) by having no excluded instruments, or (ii) by having fewer excluded instruments than needed for traditional identification.

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In the former case, of external instruments augmented by generated instruments, the program provides three sets of estimates: the traditional IV estimates, estimates using only generated instruments, and estimates using both generated and excluded instruments.

In the latter case, of an underidentified equation, the only the estimates using generated instruments are displayed. Unlike `ivreg2` or `ivregress`, `ivreg2h` allows the syntax

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An empirical example

In Lewbel's 2012 *JBES* paper, he illustrates the use of his method with an Engel curve for food expenditures. An Engel curve describes how household expenditure on a particular good or service varies with household income (Ernst Engel, 1857, 1895).[1] Engel's research gave rise to **Engel's Law**: while food expenditures are an increasing function of income and family size, food budget shares decrease with income (Lewbel, *New Palgrave Dictionary of Economics*, 2d ed. 2007).

In this application, we are considering a key explanatory variable, total expenditures, to be subject to potentially large measurement errors, as is often found in applied research: due in part to infrequently purchased items (Meghir and Robin, *Journal of Econometrics*, 1992).

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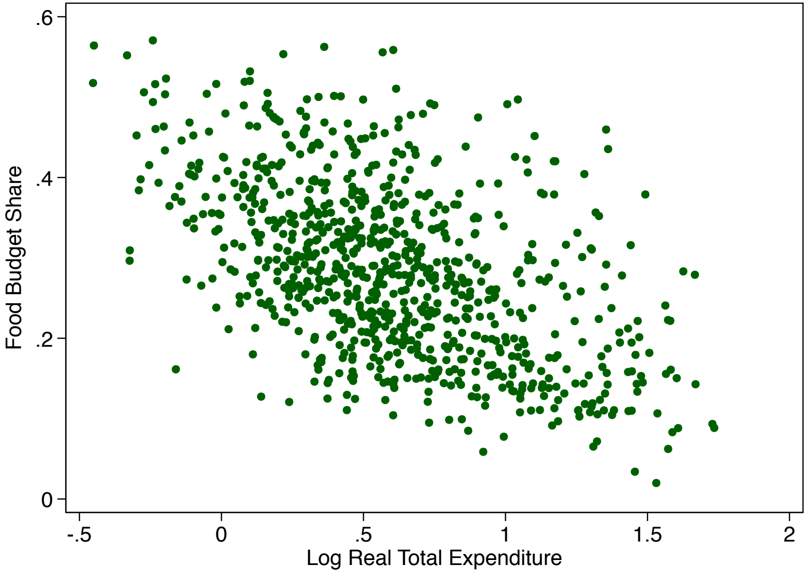
The data are 854 households, all married couples without children, from the UK Family Expenditure Survey, 1980–1982, as studied by Banks, Blundell and Lewbel (*Review of Economics and Statistics*, 1997). The dependent variable is the food budget share, with a sample mean of 0.285. The key explanatory variable is log real total expenditures, with a sample mean of 0.599. A number of additional regressors (age, spouse's age, ages^2, and a number of indicators) are available as controls. The coefficients of interest in this model are those of log real total expenditures and the constant term.


Food Budget Share
.6
.4
.2
0
-.5
0
.5
1
1.5
2
Log Real Total Expenditure

We first estimate the model with OLS regression, ignoring any issue of mismeasurement. We then reestimate the model with log total income as an instrument using two-stage least squares: an exactly identified model. As such, this is also the IV-GMM estimate of the model.

In the following table, these estimates are labeled as OLS and TSLS1. A Durbin–Wu–Hausman test for the endogeneity of log real total expenditures in the TSLS1 model rejects with p-value=0.0203, indicating that application of OLS is inappropriate.

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Table: OLS and conventional TSLS

	(1) OLS	(2) TSLS,ExactID
lrtotexp	-0.127	-0.0859
	(0.00838)	(0.0198)
Constant	0.361	0.336
	(0.00564)	(0.0122)

Standard errors in parentheses

These OLS and TSLS results can be estimated with standard `regress` and `ivregress 2sls` commands. We now turn to estimates produced from generated instruments via Lewbel's method.

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We produce generated instruments from each of the exogenous regressors in this equation. The equation may be estimated by TSLS or by IV-GMM, in each case producing robust standard errors. For IV-GMM, we report Hansen's J.

Table: Generated instruments only

	(1) TSLS,GenInst	(2) GMM,GenInst
lrtotexp	-0.0554	-0.0521
	(0.0589)	(0.0546)
Constant	0.318	0.317
	(0.0352)	(0.0328)
Jval		12.91
Jdf		11
Jpval		0.299

Standard errors in parentheses

The greater efficiency available with IV-GMM is evident in the precision of these estimates. However, reliance on generated instruments yields much larger standard errors than identified TSLS.[2]

As an alternative, we augment the available instrument, log total income, with the generated instruments, which overidentifies the equation, estimated with both TSLS and IV-GMM methods.

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Table: Augmented by generated instruments

	(1) TSLS,AugInst	(2) GMM,AugInst
lrtotexp	-0.0862	-0.0867
	(0.0186)	(0.0182)
Constant	0.336	0.337
	(0.0114)	(0.0112)
Jval		16.44
Jdf		12
Jpval		0.172

Standard errors in parentheses

Relative to the original, exactly-identified TSLS/IV-GMM specification, the use of generated instruments to augment the model has provided an increase in efficiency, and allowed overidentifying restrictions to be tested. As a comparison:

Table: With and without generated instruments

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Further developments

We have illustrated this method with one endogenous regressor, but it generalizes to multiple endogenous (or mismeasured) regressors. It may be employed as long as there is at least one included exogenous regressor. If there is only one, the resulting equation will be exactly identified.

As this estimator has been implemented within the `ivreg2` framework, all of the diagnostics and options available in that program (Baum, Schaffer, Stillman, *Stata Journal*, 2003, 2007) are available in this context.

The extension of this method to the panel fixed-effects context is relatively straightforward, and we are finalizing a version of Schaffer's `xtivreg2` which implements Lewbel's method in this context. These routines will be made available via the SSC Archive and announced on Statalist.

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