A Monte-Carlo analysis of multilevel binary logit model estimator performance

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Introduction: the context

- There is much regression-based analysis of harmonised individual-level data from multiple countries
  - Multilevel (a.k.a. hierarchical or mixed) models
  - Linear and non-linear (binary logit) models
  - Outcome modelled as function of individual-level and country-level variables (including unobserved country-level variables)
- Many social science researchers aim to quantify ‘country effects’ a.k.a. ‘contextual effects’:
  - *regression coefficients on level-2 (country-level) predictors*: extent to which differences in outcomes reflect differences in country-specific features of demographic structure, labour markets, tax-benefit systems etc, as distinct from the differences in outcomes associated with variations in characteristics of individuals
  - *level-2 variances, and ICC*: importance of ‘country effects’ also summarised in terms of variance of unobserved country-level factors (relative to the variance of unobserved individual-level factors)

Many multi-country datasets, much-used: small # countries, large # respondents/country

Data sources (in alphabetical order)	**Number of countries per wave (approx.)**
Eurobarometer	27
European Community Household Panel (ECHP)	15
European Quality of Life Survey (EQLS)	31
European Social Survey (ESS)	30
EU Statistics on Income and Living Conditions (EU-SILC)	27
European Values Study (EVS)	45
International Social Survey Program (ISSP)	36
Luxembourg Income Study (LIS)	32
Survey of Health, Ageing and Retirement in Europe (SHARE)	14

Notes: All datasets are based on cross-sectional surveys with the exception of ECHP and SHARE which are panel surveys.

Number of countries used in empirical studies is often smaller than the maximum possible

Many publications on wide range of topics using multi-country datasets

- Topics range from labour force participation and wages, to political and civic participation rates, and social and political attitudes:
- Many published papers:
  - Of 340 articles published in *European Sociological Review* between 2005 and 2012, 75 used regression-based analysis of multi-country data, of which 43 use multilevel modelling methods (13% of all published articles)
  - Significant number also in *Journal of European Social Policy* (14/111 between 2005 and 2009)
  - And, of course, publications elsewhere as well
- Project motivation: are the estimates of country effects likely to be reliable given the nature of the datasets?
  - Many applied social science researchers appear unaware of the issue …

Project Output #1: Bryan and Jenkins 'Regression analysis of country effects using multilevel data: a cautionary tale'

ISER Working Paper 2013-14

https://www.iser.essex.ac.uk/publications/working-papers/iser/2013-14

- Multilevel models (MLMs) are not the only way to analyse multi-country data
  - We review MLM and other approaches
- MLMs can yield unreliable estimates of country effects when there is only a small number of countries in the data set (as is typically the case: see Table above)
- Our conclusions draw on Monte-Carlo analysis of linear and binary logit mixed models with 2 specifications for each:
  - Basic: random country intercept and a country-level predictor;
  - Extended: as (i), plus 2 random slopes and cross-level interaction
    - **Extended model discussed in this talk**

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This talk: Issues arising in the Monte-Carlo analysis of a binary logit mixed model

- Computational issues: my experiences and tips from using simulate, with xtmelogit and runmlwin, and post-estimation processing using e.g. parmby, and eclplot
  - Few how-to-do-it guides for newbies; Cameron & Trivedi, Greene, ...
- Substantive issues: comparison of Stata's default adaptive quadrature estimator (7 quadrature points) with MLwiN's PQL2 estimator
  - Extremely long run times for Stata (version 11) compared to MLwiN (version 2.25)
    - E.g. for C = 20: Stata 19 days compared to MLwiN 1.5 hours!
    - Runtime problems with Stata jobs made worse: halted by Windows Update (office PC) and unknown gremlins (LSE's Windows server cluster used to run almost all jobs)
  - MLwiN very fast, but PQL2 estimators can perform poorly
    - 'Well-known'? … but only a few previous results (Rodriquez & Goldman 2001, Pinheiro & Chao 2006, Austin 2010), and not for the data structure of interest here

MC analysis design

- 2-level model with random intercept, 2 random slopes, country-level regressor, and cross-level interaction
  - More complex model than in the majority of applications, but interesting to explore (see WP for discussion of simpler model)
- Model specification and data generating process fixed over replications (as usual)
  - But uses a more realistic DGP than others – motivated by and derived from application that modelled women's labour force participation using EU-SILC data
- Number of replications, R = 1,000
- Fixed # persons/country: N_C =1000
- **Vary # countries: *C*: 5(5)50 100**

Monte-Carlo analysis: DGP reflects an EU-SILC application

- Data on women aged 18–64 years from EU-SILC cross-section for 2007 (26 countries)
- Logit model of probability of participation in labour market, as functions of
  - **individual-level**: age, age-squared, marital status (binary), number of children (integer), education level (4 categories derived from ISCED)
  - **country-level**: total childcare and pre-primary spending as a % of GDP (continuous)
- DGP: (a) *baseline parameters* derived from preliminary estimates of each of models (i) and (ii)
- DGP: (b) *joint distribution of the regressors* derived using a cell-based approach
  - Combinations of regressors define cells; Pr(individual in cell) derived from empirical frequency distribution in EU-SILC estimation samples
  - Age distribution fitted as Singh-Maddala for model (i), and uniform for model (ii) in EU-SILC data. Parameters used to generate age values that were then grouped into 5 classes in order to construct the cells
- DGP is same for each model examined; MC design varies *C*

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MC analysis: binary logit mixed model for ‘participation’ with random intercept, 2 random slopes, country-level regressor & interaction

Participation*_ic = b0		b0 = –9.1
+ b1 * age_ic		b1 = 0.5
+ b2 * age-squared_ic		b2 = – 0.006
+ b3 * cohab_ic + b3c * cohab_ic	← random slope	b3 = 0.02
+ b4 * nownch_ic + b4c * nownch_ic	← random slope	b4 = –0.27
+ b5 * isced3_ic		b5 = 0.7
+ b6 * isced4_ic		b6 = 0.9
+ b7 * isced56_ic		b7 = 1.4
+ c1 * chexp_c	← country-level	c1 = 0.7
+ c2 * (chexp_c X cohab_ic)	← country-individual interaction	c2 = 0.6
+ c3 * (chexp_c X nownch_ic)	← country-individual interaction	c3 = –0.1
+ u_c		
+ e_ic		

u_c ~ N(0, sig_u^2)		sig_u = 0.38
e_ic ~ cumlogit(0, sig_e^2)		sig_e = sqrt(_pi^2)/3
cov(u_c, e_ic) = 0		⇒ ICC ≈ 0.042
b3c ~ N(0, sig_b3c^2)	← variance of random slope	sig_b3c = 0.25
b4c ~ N(0, sig_b4c^2)	← variance of random slope	sig_b4c = 0.13

Stata 11 'driver' program (extract):

N_C and C are arguments

```
set seed 123456789
    <material omitted, including data generation>
program define mc_silc
    version 11
    args sig_e sig_u sig_b3c sig_b4c
    capture drop y u_c b3c b4c e_ic
    gen e_ic =  rnormal(0,`sig_e')
    gen u_c = rnormal(0, `sig_u') if tag
    bys country_id : replace u_c = u_c[1]
    gen b3c = rnormal(0, `sig_b3c') if tag
    bys country_id: replace b3c = b3c[1]
    gen b4c = rnormal(0, `sig_b4c') if tag
    bys country_id: replace b4c = b4c[1]
    ge y = cond(fixed + u_c + b3c*cohab + b4c*nownch + e_ic > 0, 1, 0 )
    // default estimation method is used (adaptive quadrature; 7 points), cov structure
    independent
    xtmelogit y age agesq cohab nownch isced3 isced4 isced56 ///
            chexp chexpXcohab chexpXnownch || country_id: cohab nownch , nolog  iter(250)
end

di "Time is: " c(current_time) " on " c(current_date)
simulate _b _se converged = e(converged) logRLL = e(ll) ///
    , reps(1000) saving(mc_hours_model3_v01_`Nc'_`C'_output.dta, replace double
    every(10)) ///
    : mc_silc (`sig_e') (`sig_u') (`sig_b3c') (`sig_b4c')
di "Time is: " c(current_time) " on " c(current_date)
```

Seed: I should've saved current value in c(seed) along with data

Convergence handling

Don't use Stata 's default for long-running jobs

Lessons regarding doing MC analysis

(the benefits of hindsight)

1. Save convergence status along with simulation output
2. Save simulation estimation frequently if runtimes are long
3. Save current value of seed along with data, in case wish to restart from where stopped
   - Bill Gould’s messages on Statalist
4. Think very seriously about how to split the MC analysis into smaller ‘packages’ (blocks of replications), and combining simulation output once all blocks have run
   - Stas Kolenikov’s messages on Statalist

MLwiN driver program (extract):

Code below replaces calls to xtmelogit on previous slide

- runmlwin is an excellent, highly recommended, wrapper program for calling MLwiN (almost all commands) and returning results in Stata format
  - by Charlton and Leckie, downloadable from SSC. (MLwiN is free to those with ac.uk email address.)
- Run first using **M**arginal **Q**uasi-**L**ikelihood, and then fit model using **P**artial **Q**uasi-**L**ikelihood using estimates as starting values (MLwiN manual)

```
// run twice as recommended in manual. NB don't display results in sd metric (-simulate-
    posts missing values if LB 95% CI missing)

runmlwin y age agesq cohab nownch isced3 isced4 isced56                    ///
    chexp chexpXcohab chexpXnownch cons                                    ///
    , level2(country_id: cohab nownch cons, diagonal ) level1(id) ///
      discrete( distribution(binomial) link(logit) denominator(cons) mql1 ) nopause   ///
    maxiterations(250)  tolerance(4) batch

runmlwin y age agesq cohab nownch isced3 isced4 isced56                    ///
    chexp chexpXcohab chexpXnownch cons                                    ///
    , level2(country_id: cohab nownch cons, diagonal ) level1(id) initsprevious ///
    discrete( distribution(binomial) link(logit) denominator(cons) pql2 ) nopause
    maxiterations(250)  tolerance(4) batch
```

Post-processing of simulation output

1. append simulation output produced for each value of *C*
2. Derive various summary statistics from the output, including relative bias, and coverage rates
   - mean …, over(C), followed by getmata, Mata calculation of summary statistics based on e(b) and e(V), putmata to return to Stata .dta files, listed, and also sent to rtf files for tabular summaries (using mkmat and Ben Jann's esttab on SSC)
3. Accompanying processing to produce summary graphs: the joys of parmby and eclplot (by Roger Newson, on SSC)

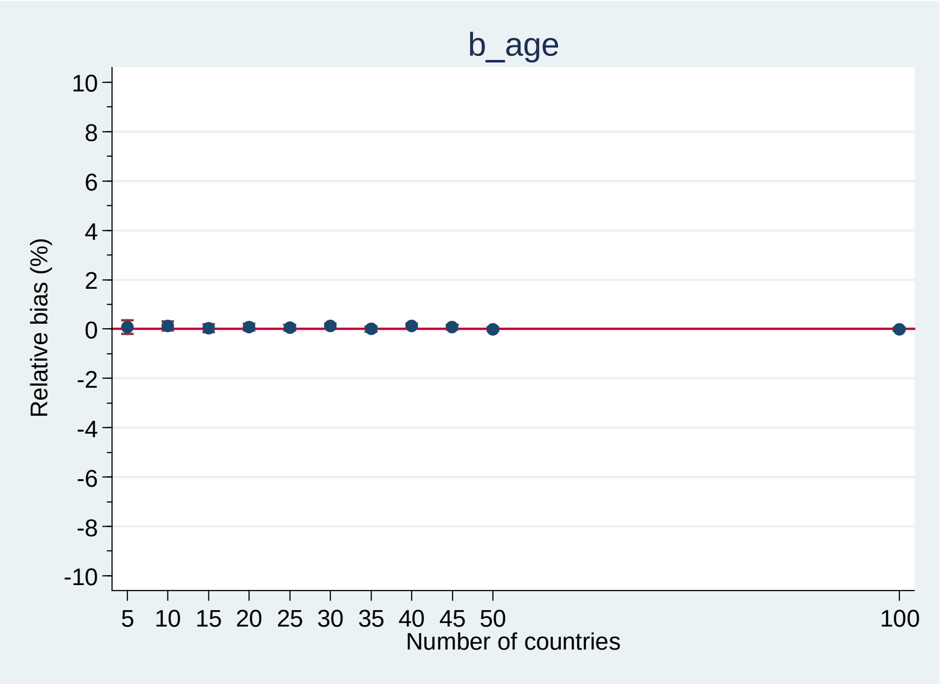
```
parmby "mean b_cons_noncover b_age_noncover b_agesq_noncover b_cohab_noncover
     b_nownch_noncover b_isced3_noncover b_isced4_noncover b_isced56_noncover
     c_chexp_noncover c_chexpXcohab_noncover c_chexpXnownch_noncover
     sig_u_noncover sig_b3c_noncover sig_b4c_noncover icchat_noncover "  ///
     , by(C) label saving(summary_partic_model3_s_ncover.dta, replace)
eclplot estimate min95 max95 C if parm == "sig_u_noncover" ///
 , xlab(5(5)50 100) ylab(0(.02).4, angle(0) format(%03.2f) ) ///
   yline(0.05) ymtick(0(.01).4) ytitle("Non-coverage rate")
```

Summarising MC analysis

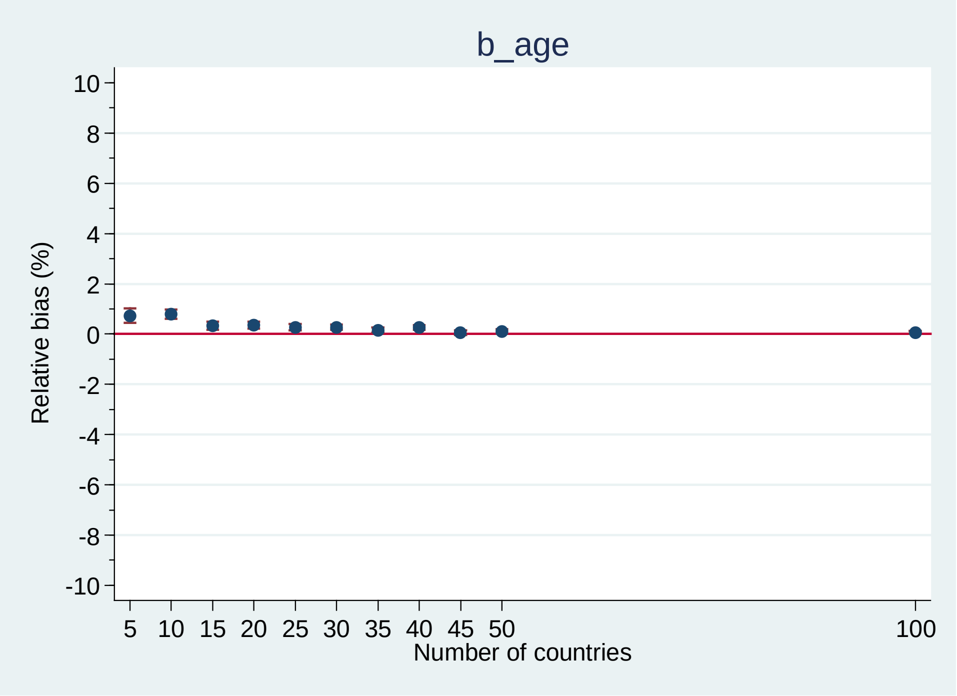
- *Relative parameter bias*: percentage difference between estimated parameter and true parameter, averaged over *R* replications
  - Ideal reference point: 0%
- *Non-coverage rate*: calculate 95% CI for each estimated parameter, assuming normality; calculate non-coverage indicator variable set equal to 0 if the CI included the true parameter, 1 if did not. Non-coverage rate is average over *R* replications
  - Ideal non-coverage rate for 95% CI is 0.05
  - Rates larger than 0.05 mean estimated CI is too narrow
- Charts to follow show estimates of above and 95% CI (summarising simulation variability)
- Look at 2 things: Stata versus MLwiN; performance relative to typical *C* (around 25 in multicountry datasets)
- For brevity, selected estimates only!

Relative parameter bias, b_age

Stata AQ



MLwiN PQL2

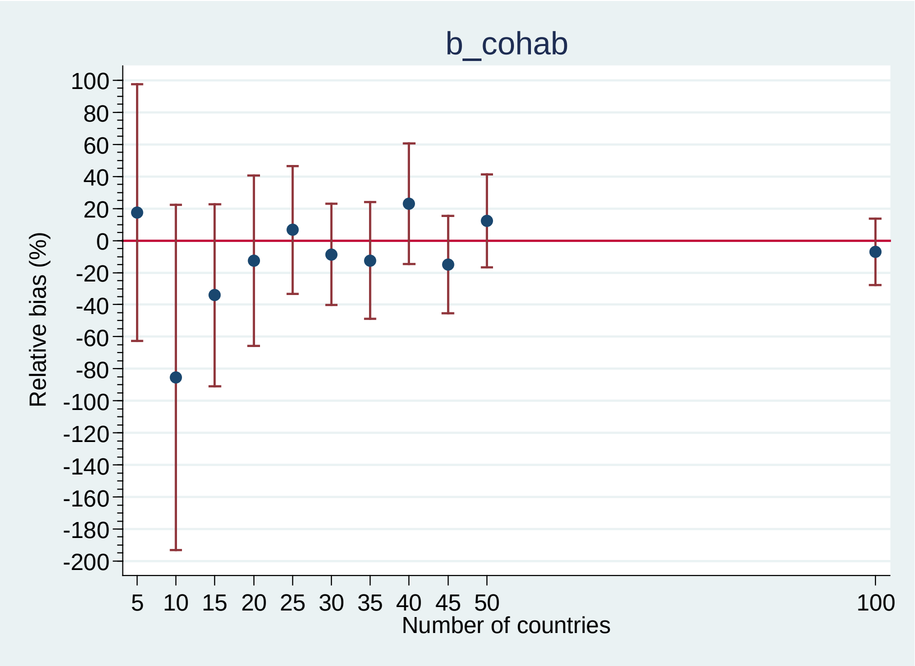


Individual fixed effect

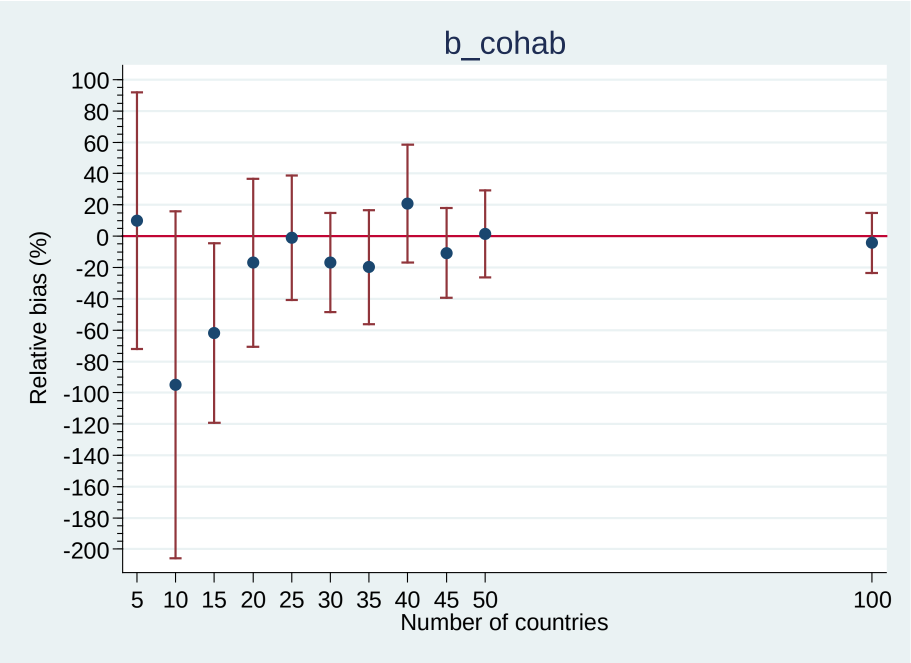
[Similar results for most other individual fixed effects and for individual-level variance]

Relative parameter bias, b_cohab

Stata AQ



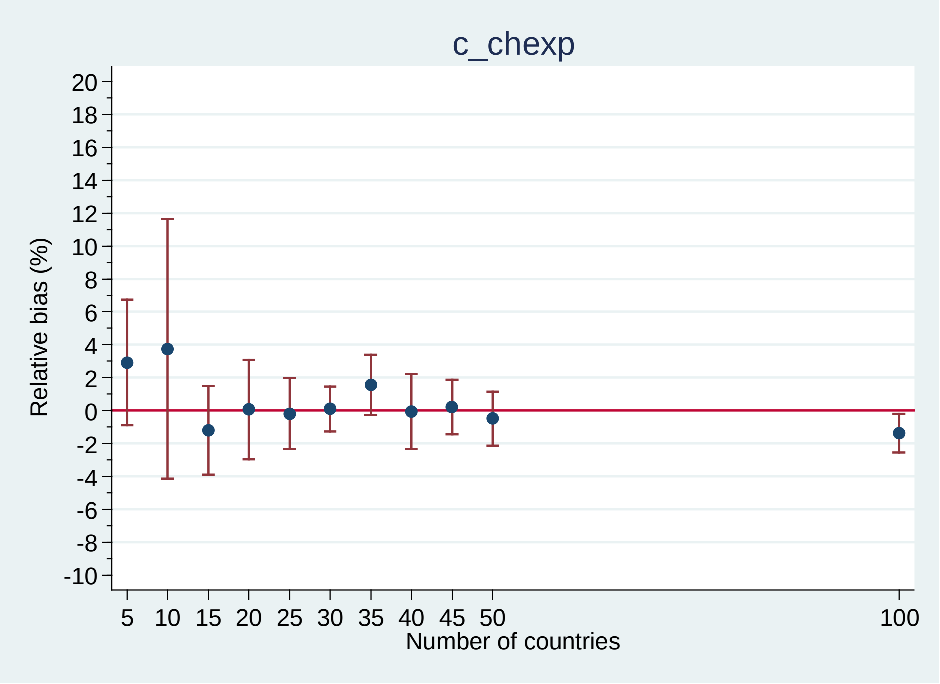
MLwiN PQL2



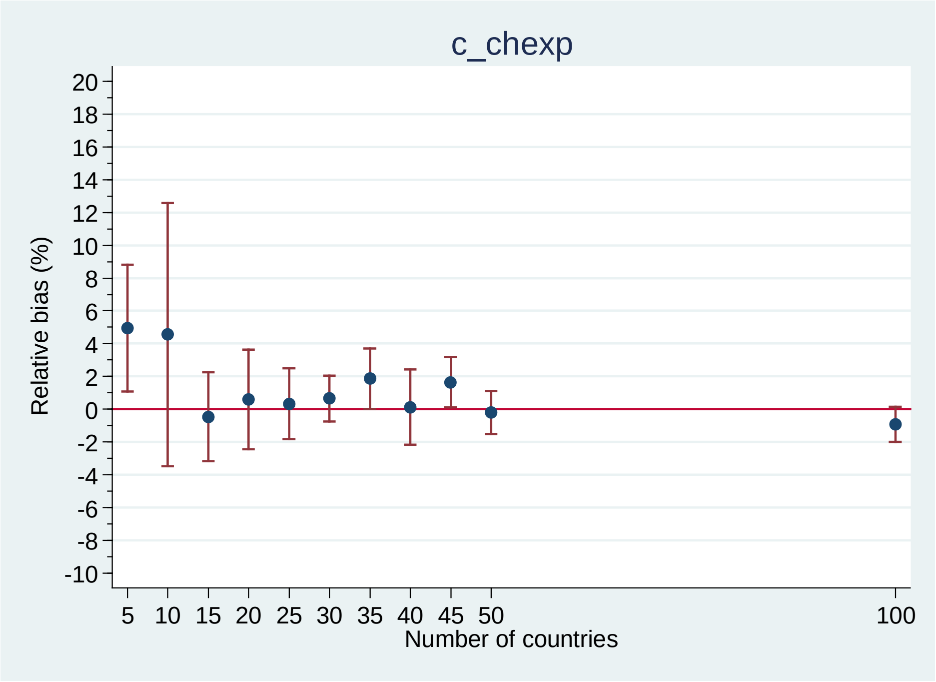
Individual fixed effect for which there's also cross-level interaction: note large degree of simulation variability

Relative parameter bias, c_chexp

Stata AQ



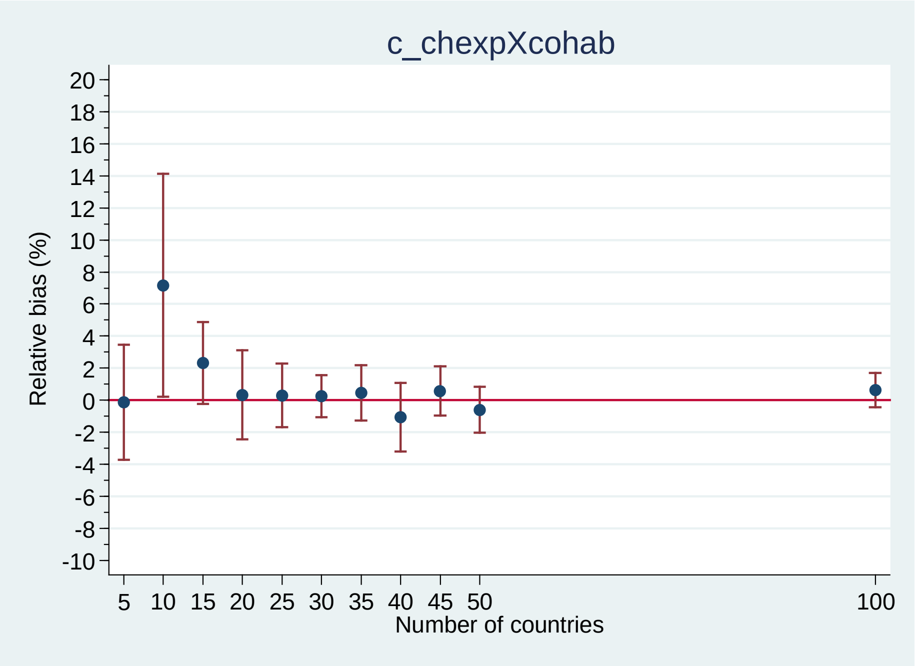
MLwiN PQL2



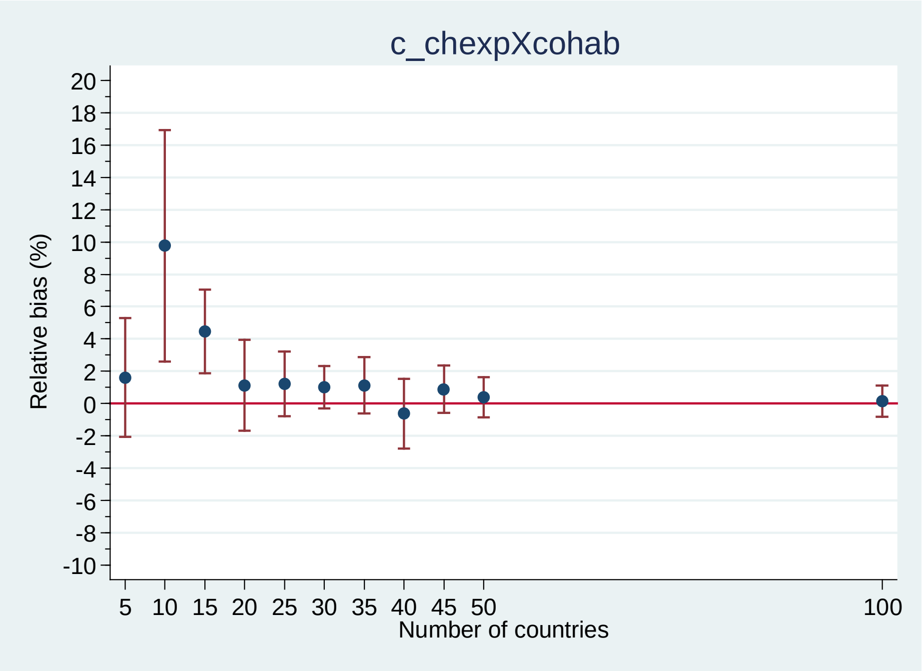
Country-level fixed effect

Relative parameter bias, c_chexpXcohab

Stata AQ



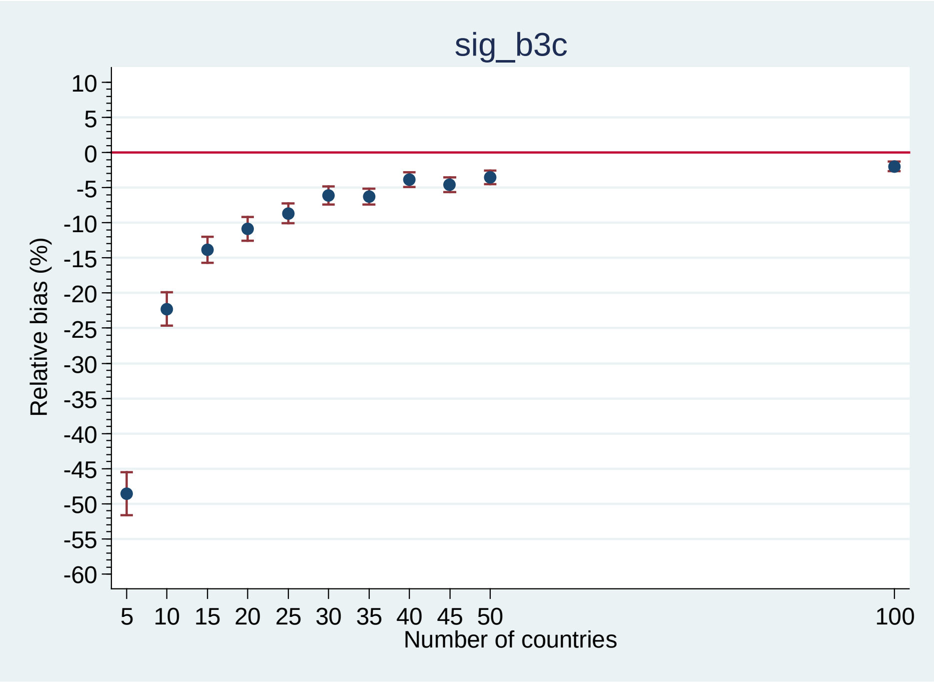
MLwiN PQL2



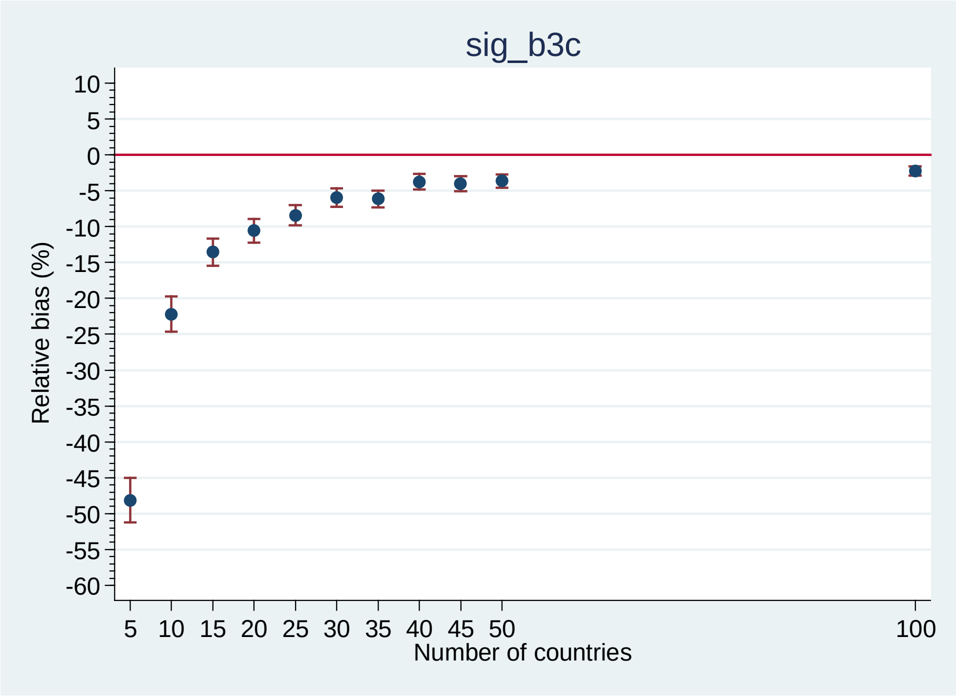
Cross-level interaction

Relative parameter bias, sig_b3c

Stata AQ



MLwiN PQL2



Random slope variance

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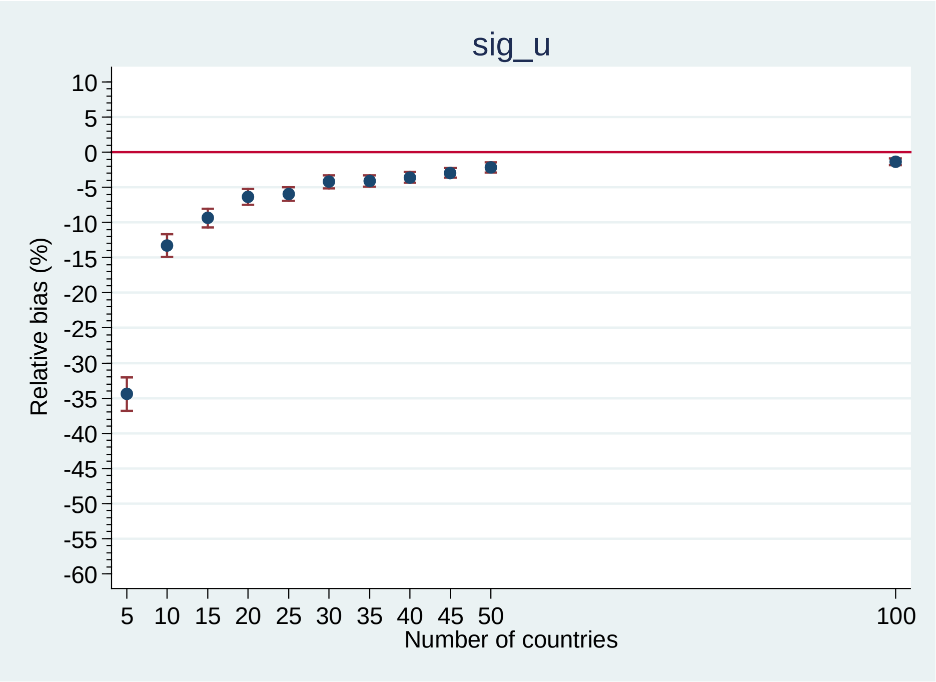
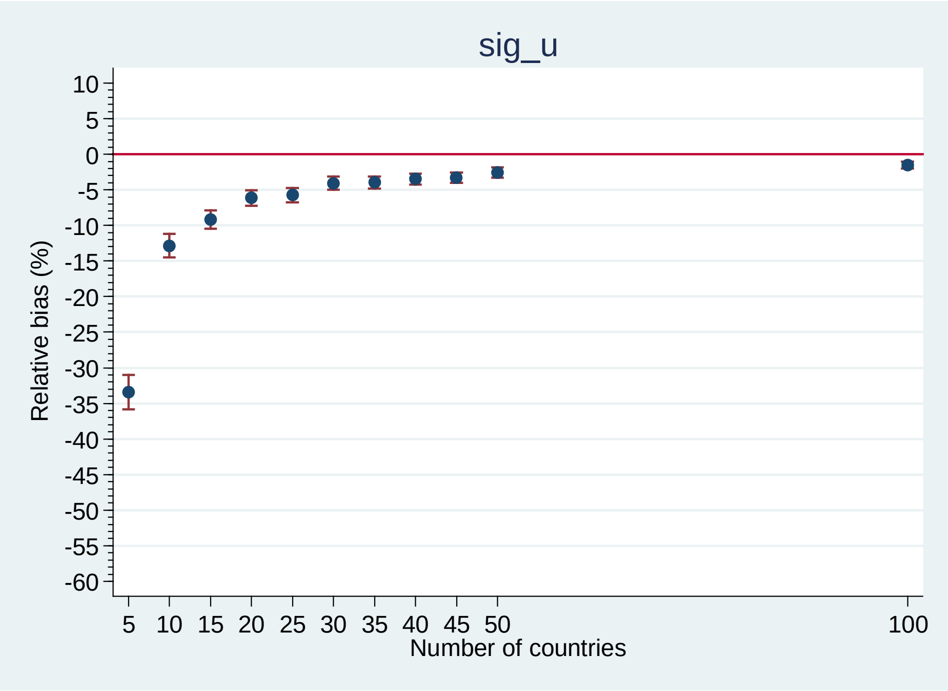
Relative parameter bias, sig_u

Stata AQ



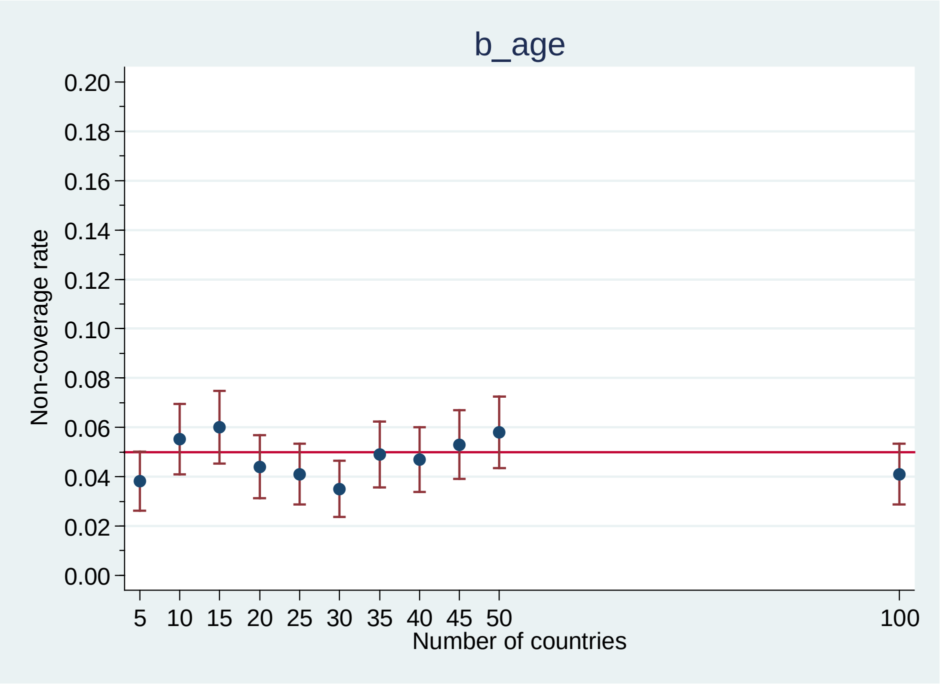
MLwiN PQL2



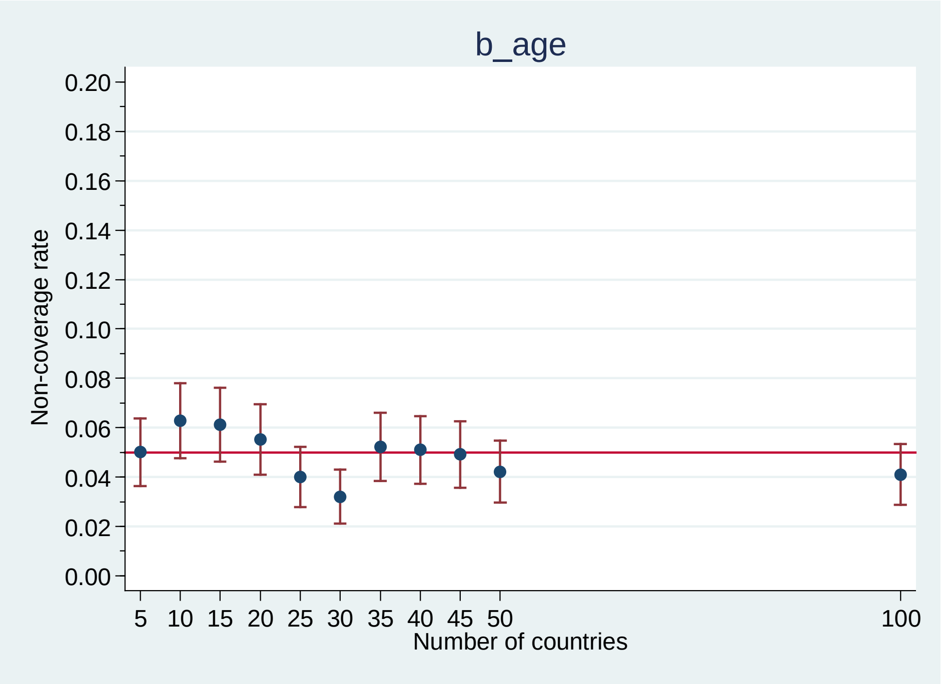
Country-level variance
[And hence also estimates of ICC]

Non-coverage rate, b_age

Stata AQ



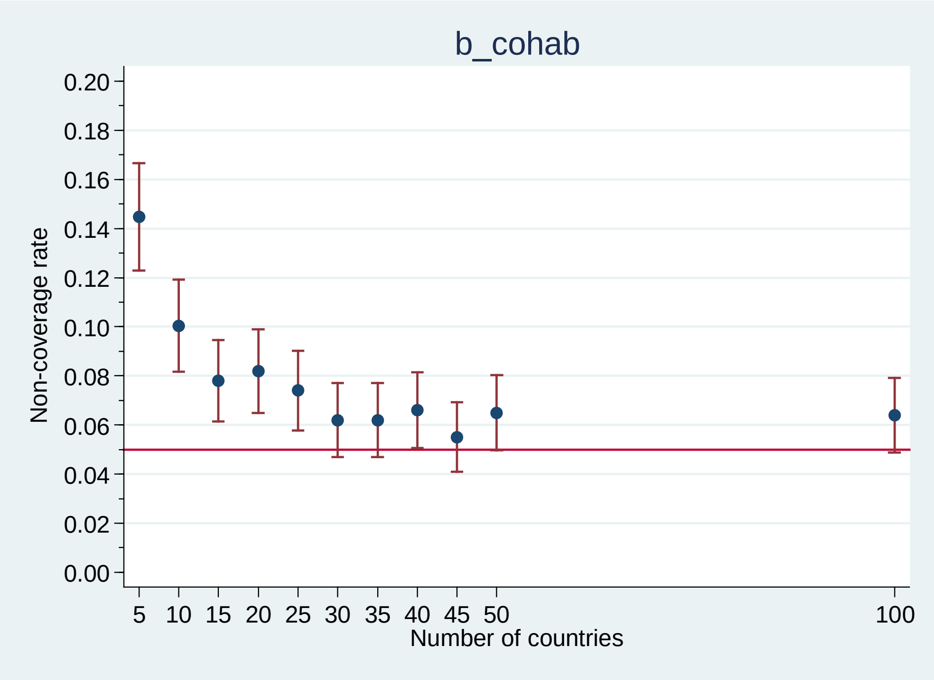
MLwiN PQL2



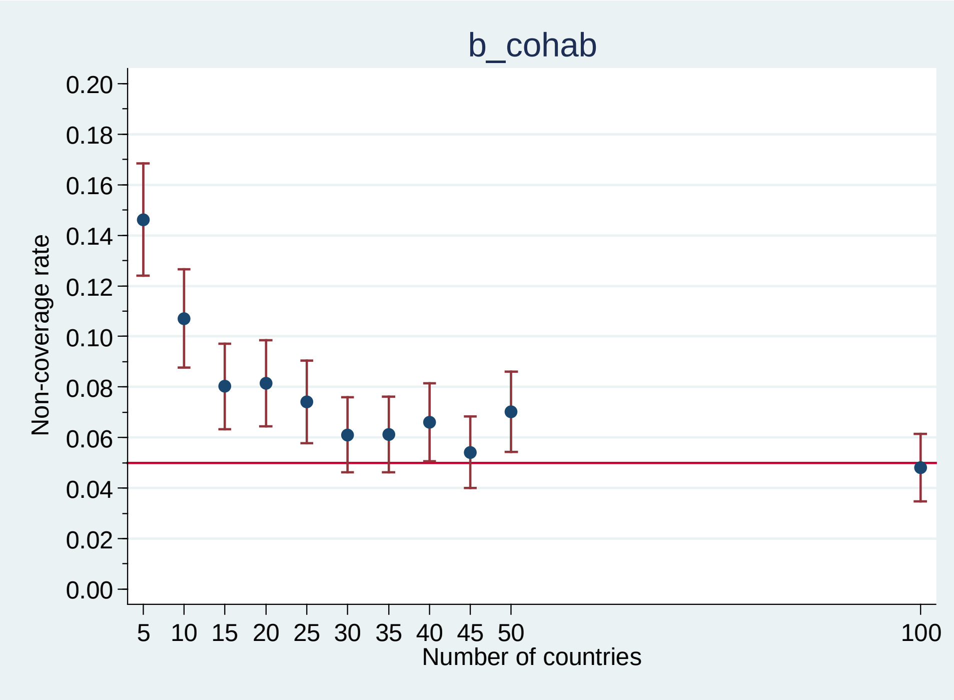
Individual fixed effect
[Similar results for most other individual fixed effects and individual-level variance]

Non-coverage rate, b_cohab

Stata AQ



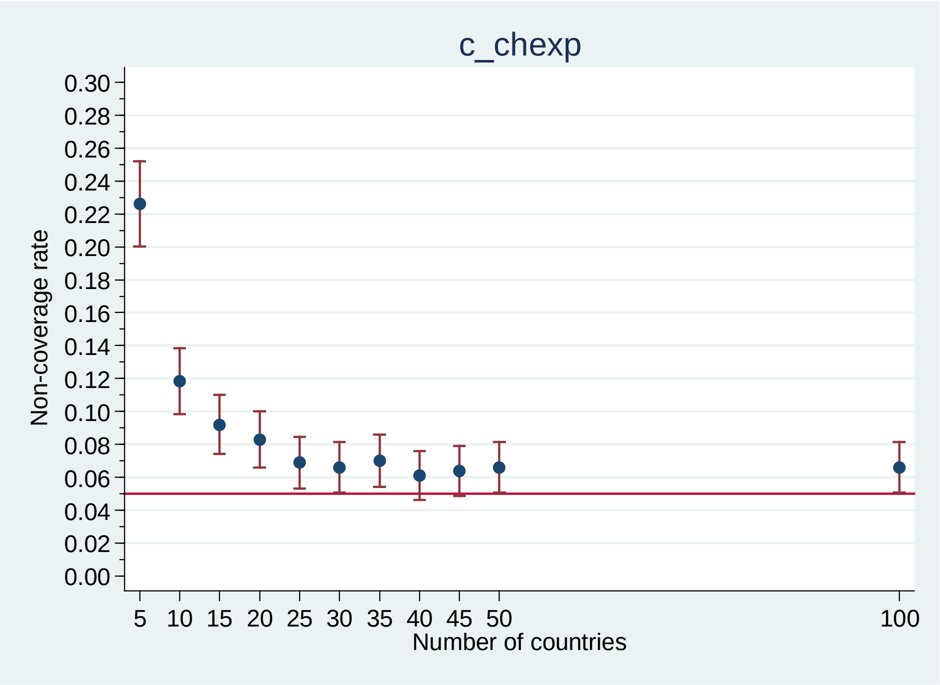
MLwiN PQL2



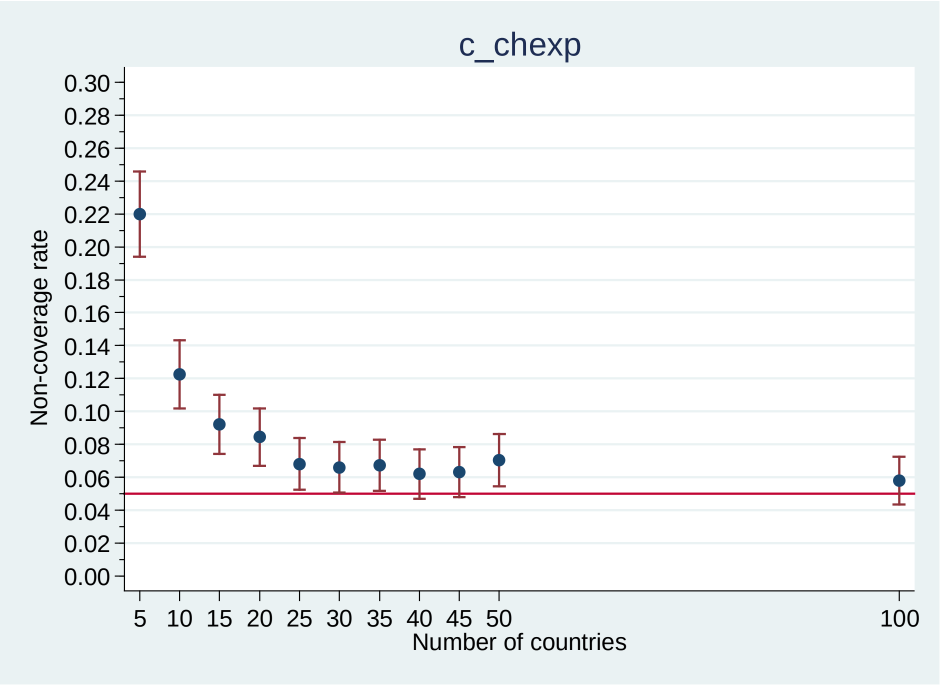
Individual fixed effect for which there’s also cross-level interaction

Non-coverage rate, c_chexp

Stata AQ



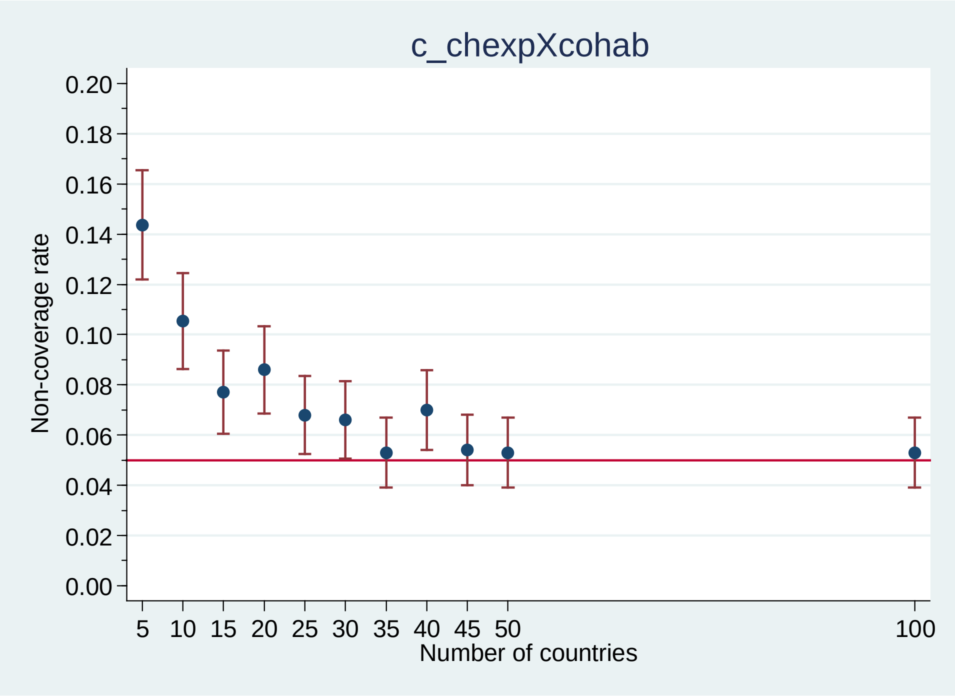
MLwiN PQL2



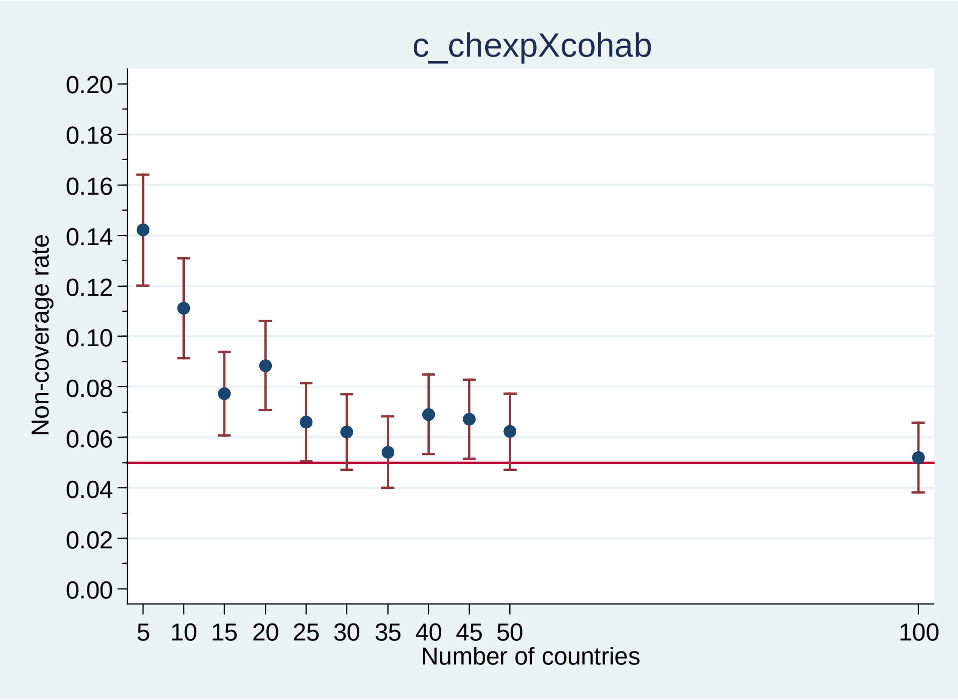
Country-level fixed effect

Non-coverage rate, c_chexpXcohab

Stata AQ



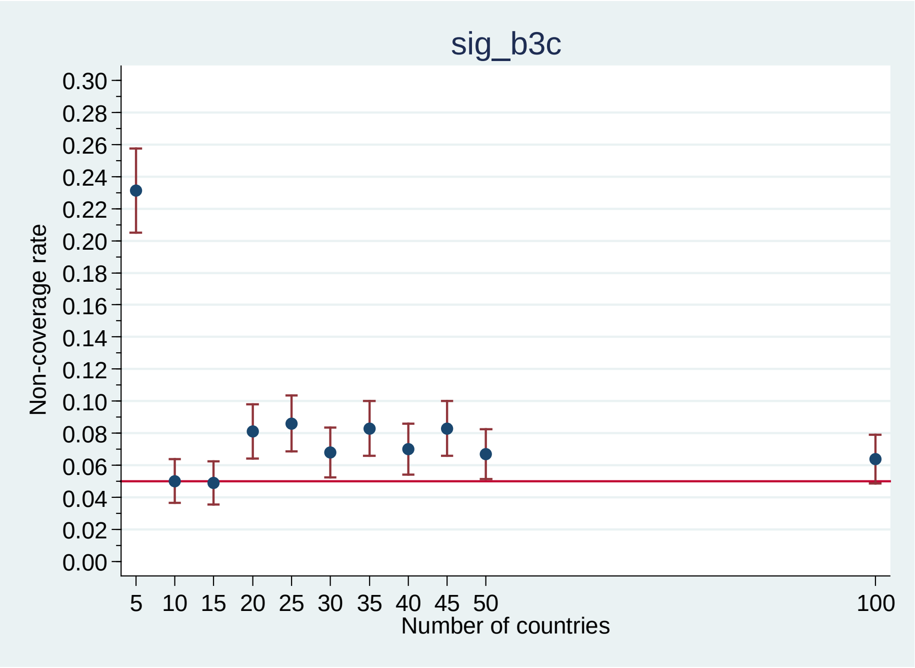
MLwiN PQL2



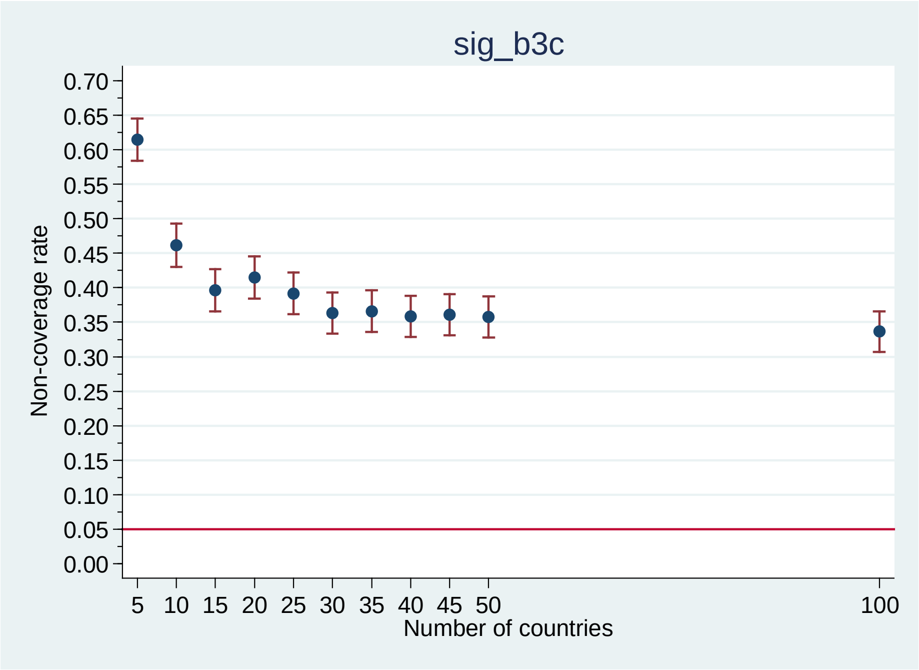
Cross-level interaction

Non-coverage rate, sig_b3c

Stata AQ



MLwiN PQL2

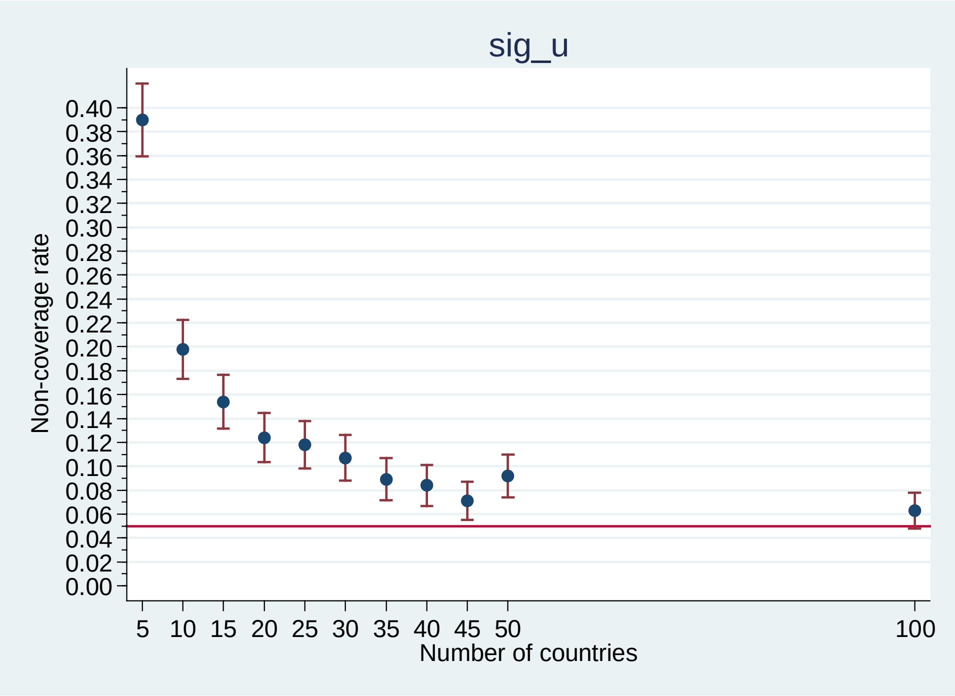


Random slope variance

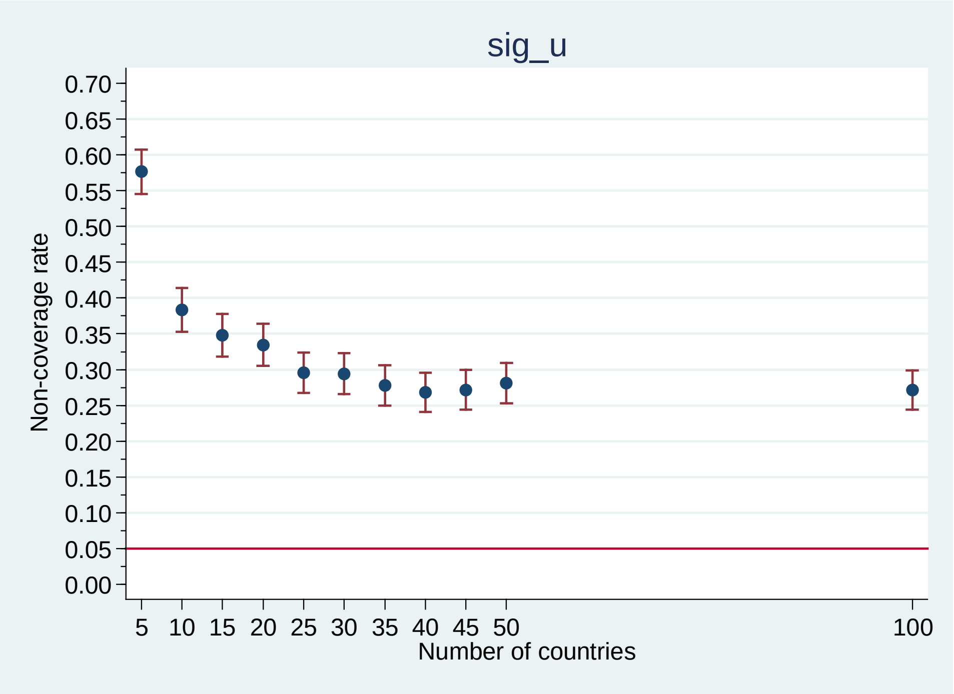
Note different scales, LHS and RHS

Non-coverage rate, sig_u

Stata AQ



MLwiN PQL2



Country-level variance

[And hence also estimates of ICC]

Note different scales, LHS and RHS

Conclusions

- Computational (1): Lessons about how to implement Monte-Carlo analyses using Stata
- Computational (2): Stata 13's speedier mixed logit model estimators will help!
- Substantive (1): general problem of reliability of estimates when one has multi-country data with small number of countries
  - Apparently not realised by many applied social scientists
- Substantive (2): Adaptive quadrature performs better than PQL for mixed binary logit models, notably for random effect variances
- Substantive (3): Would be useful to explore other (less familiar) approaches to estimation and inference, e.g. …
  - Bayesian approach (e.g. MCMC in MLwiN, BUGS)
    - Does relatively well in the small-*C* case, suggests research by Browne & Draper (2006), Moineddin et al. (2007), Stegmuller (2013)