

Repeated half-sample bootstrap resampling

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Bootstrap variance estimation

- ▶ Bootstrap resampling provides convenient variance estimators (esp. for complicated statistics): resample, re-estimate, combine...
- ▶ Typical complex survey design features:
 - ▶ Stratification
 - ▶ Multi-stage sample selection
 - ▶ Unequal selection probabilities
 - ▶ Sampling without replacement
 - ▶ Imputation, non-response adjustments, post-stratification
- ⇒ More elaborate bootstrap procedure required (to preserve the dependence structure of the data as much as possible)

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Survey bootstrap

- ▶ Basic, naïve survey bootstrap: draw PSUs with replacement within each stratum
 - ▶ Variance estimates severely biased downwards if number of PSU/stratum is small
 - ▶ Specialized survey bootstrap resampling plans (or other resampling methods such as jackknife or BRR; see Kolenikov, *Stata Journal* 2010), such as
 - ▶ m out of n bootstrap resampling with rescaling (Rao and Wu, *JASA* 1988; see **bsweights**)
 - ▶ repeated half-sample bootstrap (Saigo, Shao and Sitter, *Survey Methodology* 2001)
- ⇒ simple, consistent for any PSU/stratum, applicable to wide array of estimators and allows re-imputation of bootstrap samples

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Repeated half-sample bootstrap mechanics

- ▶ When sample size N (PSUs per stratum) is even:
 - ▶ draw without replacement a sample of size $N/2$
 - ▶ duplicate each obs so the bootstrap sample has size N
- ▶ When N is odd, either (with probability $1/4$) ...
 - ▶ draw without replacement a sample of size $(N-1)/2$
 - ▶ duplicate each obs so the bootstrap sample has size $N-1$
 - ▶ triplicate one obs at random
- ▶ ... or (with probability $3/4$) ...
 - ▶ draw without replacement a sample of size $1 + (N-1)/2$
 - ▶ duplicate each obs so the bootstrap sample has size $N+1$
 - ▶ remove one obs at random

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The **rhsbSAMPLE** command: Syntax

The command **rhsbSAMPLE** (available on SSC shortly) clones official **bsample** but samples on the basis of the repeated half-sample algorithm

Syntax

```
rhsbSAMPLE [if] [in]  
[ , strata(varlist) cluster(varlist) idcluster() weight(varname) ]
```

The **rhsbsample** command: Usage example

Replace data in memory by a bootstrap sample:

Draw a (stratified clustered) bootstrap sample:

```
bsample      , strata(<strata>) cluster(<psu>)  
rhsbsample  , strata(<strata>) cluster(<psu>)
```

The **rhsbsample** command: Usage example

Generate 500 bootstrap replication weight variables:

Draw 500 (stratified clustered) bootstrap samples:

```
forvalues i=1/500 {  
    qui gen brw'i' = .  
    bsample      , strata(<strata>) cluster(<psu>) weight(nbrw'i')  
    qui gen rhsbrw'i' = .  
    rhsbsample  , strata(<strata>) cluster(<psu>) weight(rhsbrw'i')  
}
```

The **rhsbsample** command: Combination with **svy** prefix

Declare generated bootstrap replication weights in **svyset**:

Declare survey settings and use **svy** prefix:

```
svyset <psu> [pw=<wgt>], strata(<strata>) ///  
      vce(bootstrap) bsrweight(rhsbrw*)  
svy : logistic ...
```

Illustrative example on nhanes2 data

Load data

```
. use nhanes2.dta

. * collapsed strata -- Kolenikov 2010
. egen upsu = group( strata psu )

. gen cstrata = floor( sqrt( 2*strata-1) )

.
. svyset upsu [pw=finalwgt], strata(cstrata)

      pweight: finalwgt
           VCE: linearized
Single unit: missing
   Strata 1: cstrata
        SU 1: upsu
       FPC 1: <zero>
```

Illustrative example on nhanes2 data (ctd.)

Survey design

```
. svydes
```

Survey: Describing stage 1 sampling units

```
pweight: finalwgt
VCE: linearized
Single unit: missing
Strata 1: cstrata
SU 1: upsu
FPC 1: <zero>
```

Small stratum sizes



Stratum	#Units	#Obs	#Obs per Unit		
			min	mean	max
1	4	565	67	141.3	215
2	4	808	149	202.0	231
3	8	1364	105	170.5	270
4	8	1095	100	136.9	170
5	12	2215	144	184.6	215
6	10	1579	102	157.9	233
7	16	2725	116	170.3	288
7	62	10351	67	167.0	288

Illustrative example on nhanes2 data (ctd.)

Create replication weights

```
. * STEP 1: generate sets of bootstrap weights:
. loc R 500

. * 01: Naive bootstrap
. forvalues i=1/'R' {
2.   qui gen nbrw`i' = .
3.   bsample , strata(cstrata) cluster(upsu) weight(nbrw`i')
4.   qui replace nbrw`i' = nbrw`i' * finalwgt
5. }

. * 02: RHS bootstrap
. forvalues i=1/'R' {
2.   qui gen rhsbrw`i' = .
3.   rhsbsample , strata(cstrata) cluster(upsu) weight(rhsbrw`i')
4.   qui replace rhsbrw`i' = rhsbrw`i' * finalwgt
5. }

. * 03: simple rescaled bootstrap (bsweights reads information from svyset)
. bsweights rsbrw , reps('R') n(-1) ← user-written command by Kolenikov
```


Illustrative example on nhanes2 data (ctd.)

Linearization-based variance

```
. svyset upsu [pw=finalwgt], clear strata(cstrata) vce(linearized)
```

```
    pweight: finalwgt
           VCE: linearized
Single unit: missing
  Strata 1: cstrata
    SU 1: upsu
    FPC 1: <zero>
```

```
. svy: mean highbp height weight
(running mean on estimation sample)
```

Survey: Mean estimation

Number of strata =	7	Number of obs =	10351
Number of PSUs =	62	Population size =	117157513
		Design df =	55

	Mean	Linearized Std. Err.	[95% Conf. Interval]	
highbp	.1058141	.0065151	.0927576	.1188706
height	168.4599	.136994	168.1853	168.7344
weight	71.90064	.1675056	71.56495	72.23632

Illustrative example on nhanes2 data (ctd.)

Declare replication weights

```
. svyset upsu [pw=finalwgt], clear strata(cstrata) vce(bootstrap) bsrweight(nbrw*) mse

    pweight: finalwgt
         VCE: bootstrap
         MSE: on
bsrweight: nbrw1 nbrw2 nbrw3 nbrw4 nbrw5 nbrw6 nbrw7 nbrw8 nbrw9 nbrw10 nbrw11 nbrw12
           nbrw13 nbrw14 nbrw15 nbrw16 nbrw17 nbrw18 nbrw19 nbrw20 nbrw21 nbrw22
           nbrw23 nbrw24 nbrw25 nbrw26 nbrw27 nbrw28 nbrw29 nbrw30 nbrw31 nbrw32
           nbrw33 nbrw34 nbrw35 nbrw36 nbrw37 nbrw38 nbrw39 nbrw40 nbrw41 nbrw42
           nbrw43 nbrw44 nbrw45 nbrw46 nbrw47 nbrw48 nbrw49 nbrw50 nbrw51 nbrw52
           nbrw53 nbrw54 nbrw55 nbrw56 nbrw57 nbrw58 nbrw59 nbrw60 nbrw61 nbrw62
           nbrw63 nbrw64 nbrw65 nbrw66 nbrw67 nbrw68 nbrw69 nbrw70 nbrw71 nbrw72
           nbrw73 nbrw74 nbrw75 nbrw76 nbrw77 nbrw78 nbrw79 nbrw80 nbrw81 nbrw82
           nbrw83 nbrw84 nbrw85 nbrw86 nbrw87 nbrw88 nbrw89 nbrw90 nbrw91 nbrw92
           nbrw93 nbrw94 nbrw95 nbrw96 nbrw97 nbrw98 nbrw99 nbrw100 nbrw101 nbrw102
           nbrw103 nbrw104 nbrw105 nbrw106 nbrw107 nbrw108 nbrw109 nbrw110 nbrw111
```

Illustrative example on nhanes2 data (ctd.)

Naive bootstrap variance

```
. svy , nodots : mean highbp height weight
```

```
Survey: Mean estimation      Number of obs   =      10351
                             Population size  =  117157513
                             Replications     =         500
```

	Observed Mean	Bstrap * Std. Err.	[95% Conf. Interval]	
highbp	.1058141	.0061895	.0936828	.1179453
height	168.4599	.1348131	168.1957	168.7241
weight	71.90064	.1567976	71.59332	72.20795

```
. estimates store mn_naive
```

works with non-svy-aware, user-written commands

```
. svy bootstrap (r(coeff)) , nodots : sgini height
```

```
Bootstrap results
```

```
Number of obs   =      1035
Population size  =  11715751
Replications     =         50
```

```
command:  sgin height
         _bs_1:  r(coeff)
```

	Observed	Bstrap *
--	----------	----------

Illustrative example on nhanes2 data (ctd.)

RHS replication weights

```

. svyset upsu [pw=finalwgt], clear strata(cstrata) vce(bootstrap) bsrweight(rhsbrw*) mse
    pweight: finalwgt
      VCE: bootstrap
      MSE: on
bsrweight: rhsbrw1 rhsbrw2 rhsbrw3 rhsbrw4 rhsbrw5 rhsbrw6 rhsbrw7 rhsbrw8 rhsbrw9
            rhsbrw10 rhsbrw11 rhsbrw12 rhsbrw13 rhsbrw14 rhsbrw15 rhsbrw16 rhsbrw17
            rhsbrw18 rhsbrw19 rhsbrw20 rhsbrw21 rhsbrw22 rhsbrw23 rhsbrw24 rhsbrw25
            rhsbrw26 rhsbrw27 rhsbrw28 rhsbrw29 rhsbrw30 rhsbrw31 rhsbrw32 rhsbrw33
            rhsbrw34 rhsbrw35 rhsbrw36 rhsbrw37 rhsbrw38 rhsbrw39 rhsbrw40 rhsbrw41
            rhsbrw42 rhsbrw43 rhsbrw44 rhsbrw45 rhsbrw46 rhsbrw47 rhsbrw48 rhsbrw49
            rhsbrw50 rhsbrw51 rhsbrw52 rhsbrw53 rhsbrw54 rhsbrw55 rhsbrw56 rhsbrw57

```

Illustrative example on nhanes2 data (ctd.)

RHS bootstrap variance

```
. svy , nodots : mean highbp height weight
```

```
Survey: Mean estimation      Number of obs   =      10351
                             Population size  =  117157513
                             Replications    =         500
```

	Observed Mean	Bstrap * Std. Err.	[95% Conf. Interval]	
highbp	.1058141	.0067235	.0926363	.1189918
height	168.4599	.1400124	168.1855	168.7343
weight	71.90064	.1677603	71.57183	72.22944

```
. estimates store mn_rhs
```

```
. svy bootstrap (r(coeff)) , nodots : sgini height
```

```
Bootstrap results      Number of obs   =      10351
                       Population size  =  117157513
                       Replications    =         500
```

```
command:  sgini height
         _bs_1:  r(coeff)
```

Illustrative example on nhanes2 data (ctd.)

SE estimates compared

```
. estimates tab mn_lin mn_naive mn_rescaled mn_rhs , se
```

Variable	mn_lin	mn_naive	mn_rescaled	mn_rhs
highbp	.10581408 .00651508	.10581408 .00618952	.10581408 .00628382	.10581408 .00672347
height	168.45989 .13699399	168.45989 .13481314	168.45989 .1400032	168.45989 .14001242
weight	71.900636 .16750564	71.900636 .15679756	71.900636 .17280145	71.900636 .16776031

legend: b/se

```
. estimates tab gini_naive gini_rescaled gini_rhs , se
```

Variable	gini_naive	gini_rescaled	gini_rhs
_bs_1	.03282964 .00023417	.03282964 .00026128	.03282964 .00024105

legend: b/se

References

- ▶ Kolenikov S. (2010). Resampling variance estimation for complex survey data. *Stata Journal*, 10(2): 165–199.
- ▶ Rao, J.N.K. and Wu, C.F.J. (1988). Resampling inference with complex survey data. *Journal of the American Statistical Association*, 83: 231–241.
- ▶ Saigo, H., Shao, J. and Sitter, R.R. (2001). A Repeated Half-Sample Bootstrap and Balanced Repeated Replications for Randomly Imputed Data. *Survey Methodology*, 27(2): 189–196.

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