Influence functions at work

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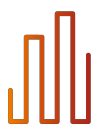
Introduction

- Illustration of practical uses of 'influence function' estimators with Stata
  1. Study structure of summary statistics (identification of 'influential observations')
  2. Variance estimation
  3. 'RIF regression'
- Application to income distribution analysis



Definition

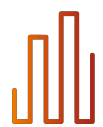
Let $v(F)$ be a statistic of interest (a functional) calculated in distribution F (the mean, a percentile, the Gini coefficient of inequality, etc.)

The *influence function* of v is a function of y and F and is defined as (Hampel, 1974)

$$\mathrm{IF}(y; v, F) = \lim_{\epsilon \downarrow 0} \frac{v((1-\epsilon)F + \epsilon \Delta_y) - v(F)}{\epsilon}$$

The IF captures the effect on $v(F)$ of an infinitesimal 'contamination' of F at point mass y.

(Note the expected value of the IF is zero: $\int IF(y, v(F))dF(y) = 0$)



Definition

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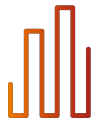


Definition (ctd.)

Expressions for $\mathrm{IF}(y; \upsilon, F)$ exist (or can be derived) for a wide range of statistics υ functionals:

... simple (linear) statistics, e.g., the mean

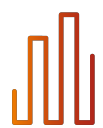
$$\mathrm{IF}(y; \mu, F) = y - \mu(F)$$

... and more complex (non linear) statistics, e.g., a quantile

$$\mathrm{IF}(y; Q_\theta, F) = \frac{-1}{f(Q_\theta(F))}(I(y \leq Q_\theta(F)) - \theta)$$

(Estimators of IF obtained by plugging estimates of the unknowns, e.g., $\mu(\hat{F})$, $Q_\theta(\hat{F})$, etc.

See e.g., Essama-Nssah and Lambert (2012) for a catalogue of IFs relevant to income distribution analysis.)



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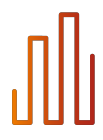

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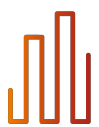
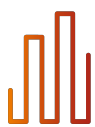


Practical use 1

Practical use 1:

- visualising the 'structure' of a (possibly complex) index
- comparison of indices (think of the many inequality measures!)
- identification of influential observations (and robustness of the index)



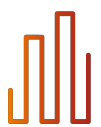
Income inequality indicators: the Atkinson index

The Atkinson inequality index (Atkinson, 1970):

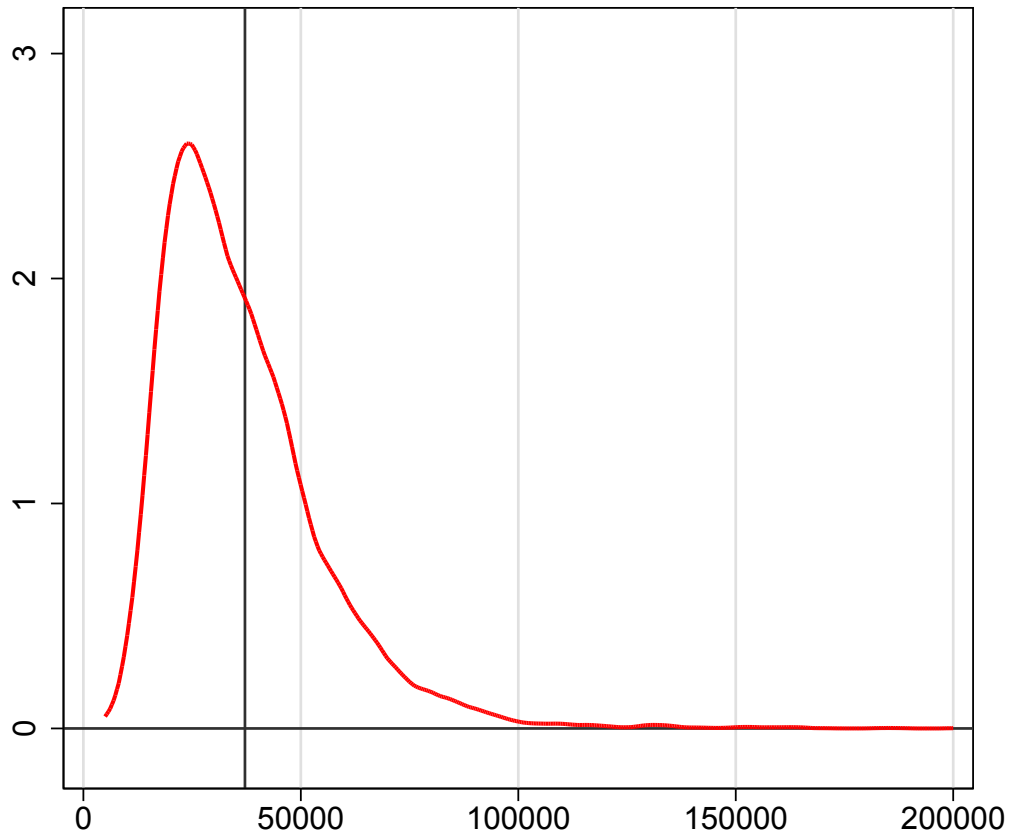
$$A(\epsilon) = 1 - \frac{1}{\mu}\left(\frac{1}{N}\sum_{i=1}^{N} y_i^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

for $\epsilon \geq 0$

The higher ϵ, the higher 'inequality aversion'... Can we visualise that?

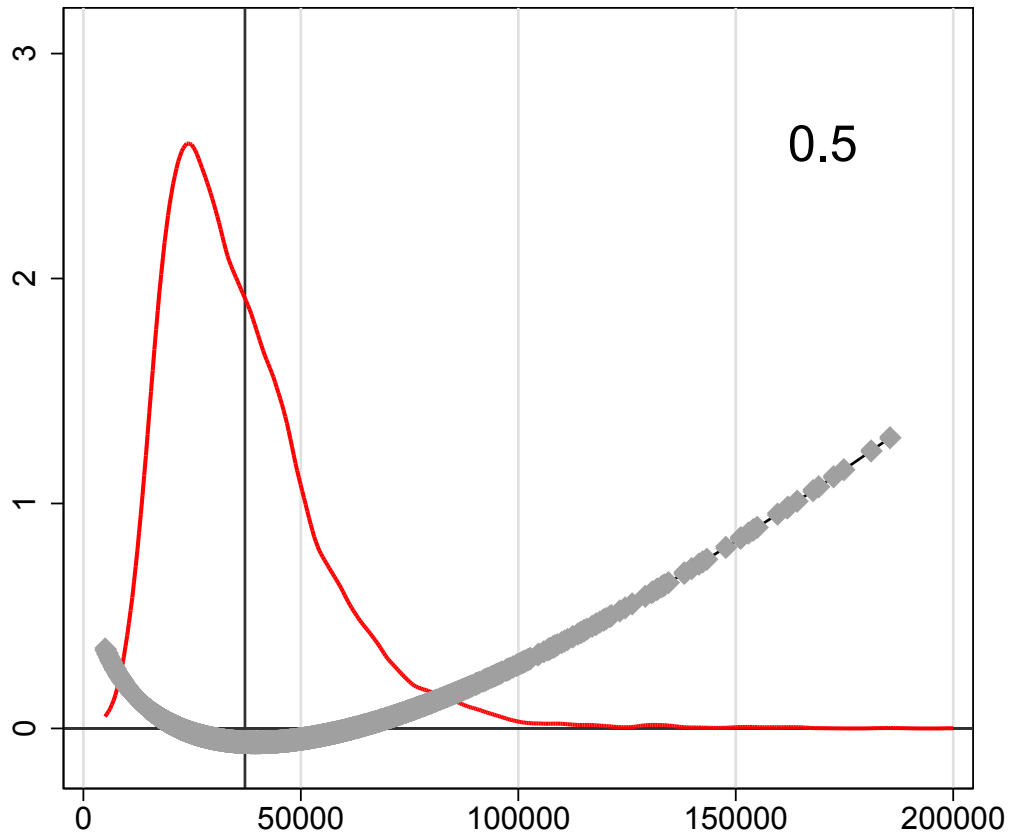


Income inequality indicators: the Atkinson index IF



(annual household income data for Luxembourg 2012)

Income inequality indicators: the Atkinson index IF



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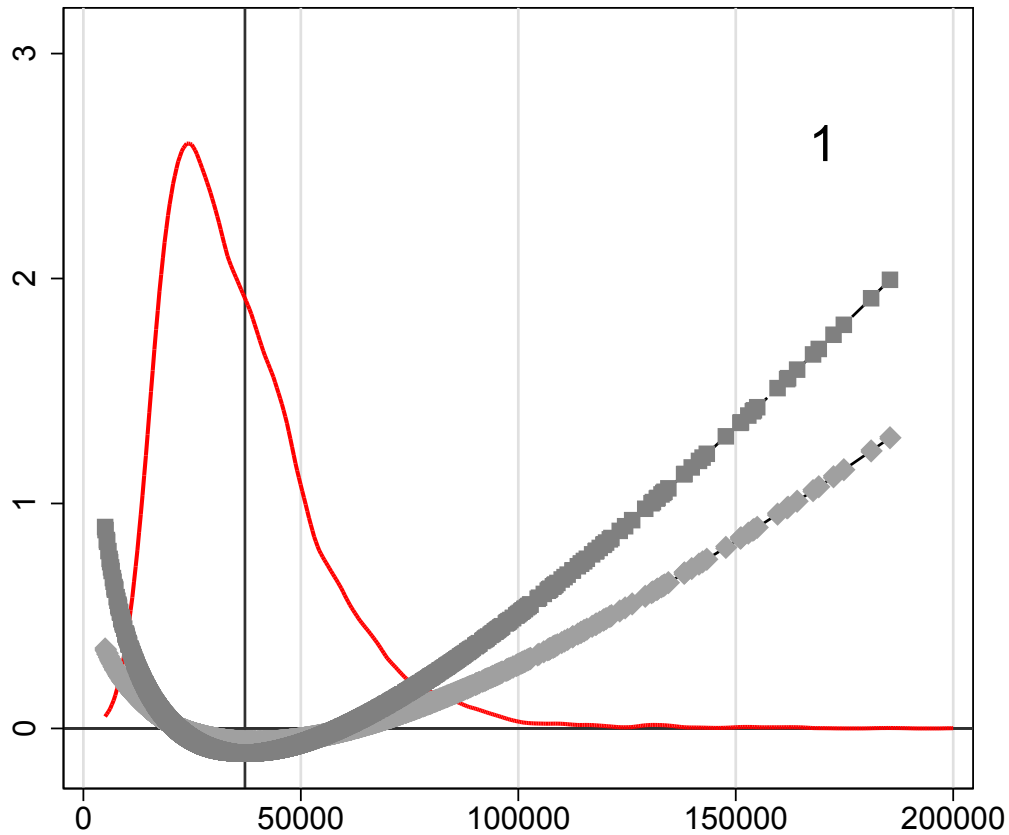

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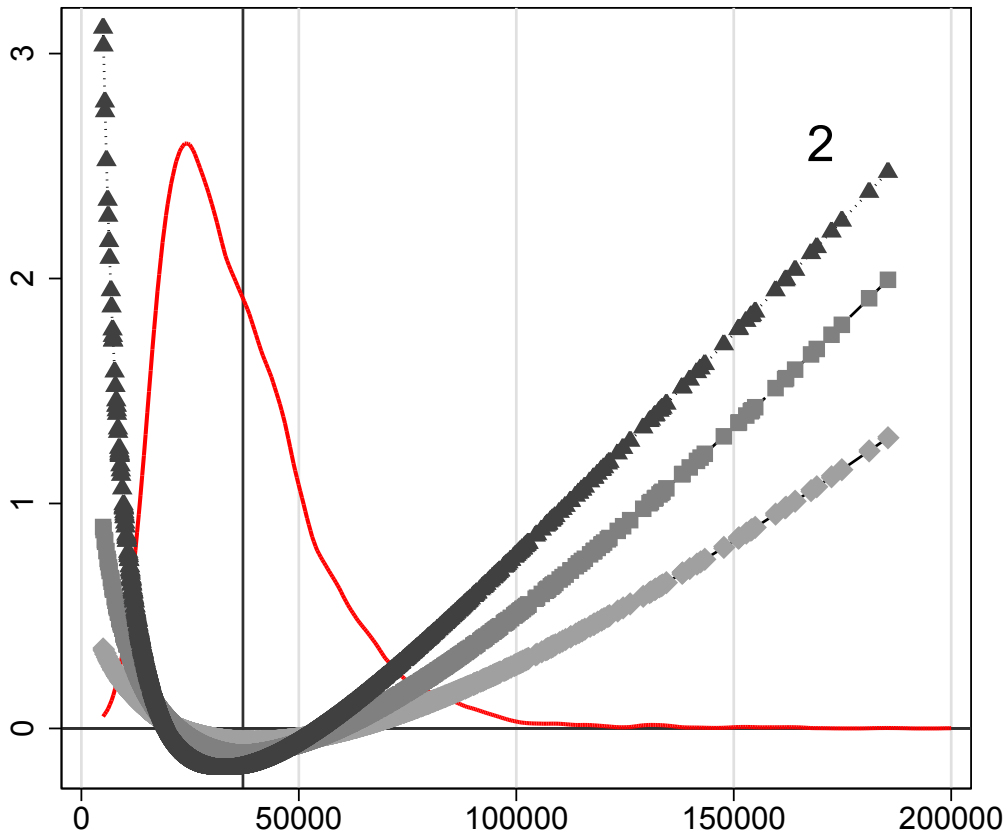
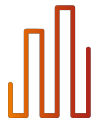
Income inequality indicators: the Atkinson index IF



(annual household income data for Luxembourg 2012)

Income inequality: the Quintile Share Ratio

How does it compare with the Quintile (Group) Share Ratio?



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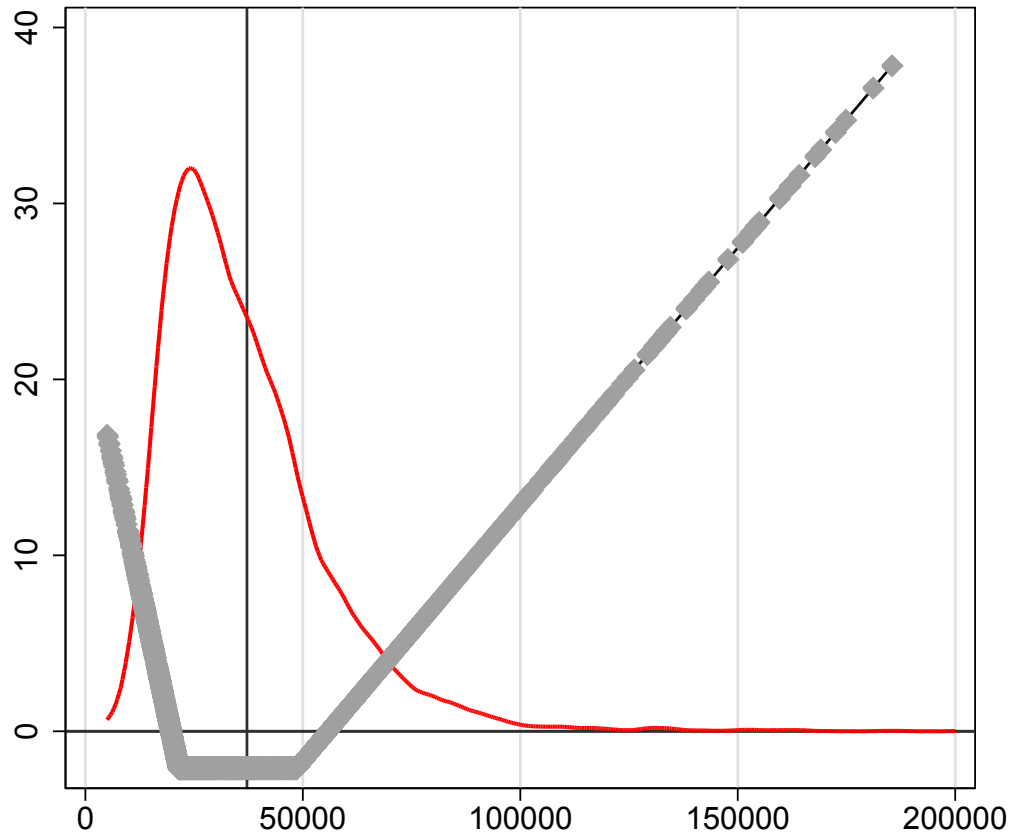
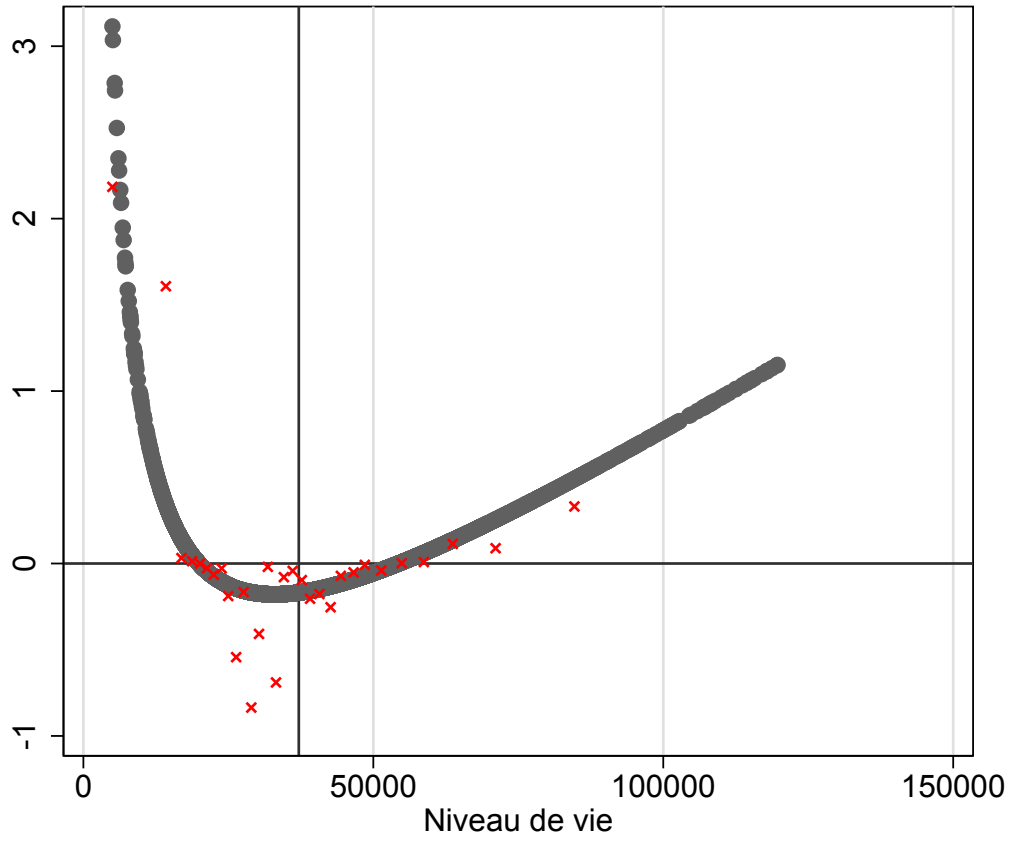


An aside: comparison with delete-one jackknife influence measure

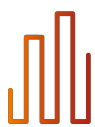


Not the same when observations are weighted!

Practical use 2

Practical use 2:

- estimation of the sampling variance of the index
- asymptotic approximation that works with complex non-linear statistics
- valid with complex survey design!
- (... it is all in the Stata manuals already)



Variance estimation

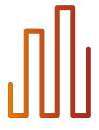
An asymptotic approximation of the variance of υ is given by (Hampel, 1974)

$$V(\upsilon, F) \approx \int \mathrm{IF}(y; \upsilon, F)^2 dF(y)$$

Practically boils down to estimation of a total (Deville, 2000):

$$V(\hat{\upsilon}, F) \approx V\left(\sum_{i=1}^{N} w_i \mathrm{IF}(y_i; \upsilon, \hat{F})\right)$$

... and formula well-known for the variance of a total even with complex survey design: implemented in Stata!



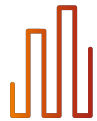
Variance estimation

Code template

```
svyset ...
generate rif= ...  // note:  point estimate is added to
IF
svy:  mean rif
```

Silly example with the mean:

```
svyset [pw=W] , ...
su y [aw=W]
gen rifmean = r(mean) + (y - r(mean))
svy:  mean rifmean
svy:  mean y // yes, it works!
```



Variance estimation

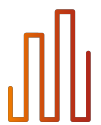
Built in some user-written commands

```
.         svy : inequaly nivie  , atkinson(0.5 1 2)  s80s20
(running inequaly on estimation sample)

Survey data analysis

Number of strata   =         4                  Number of obs      =       5,988
Number of PSUs     =     5,988                  Population size    = 515,407.14
                                                 Design df          =       5,984
                                                 F(   0,   5984)    =           .
                                                 Prob > F           =           .

------------------------------------------------------------------------------
             |             Linearized
       nivie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
atkp5        |
       _cons |   .0621767   .0039672    15.67   0.000     .0543995     .069954
-------------+----------------------------------------------------------------
atk1         |
       _cons |   .1174055   .0061389    19.12   0.000     .1053711      .12944
-------------+----------------------------------------------------------------
atk2         |
       _cons |   .2146223   .0083138    25.82   0.000     .1983242    .2309204
-------------+----------------------------------------------------------------
s80s20       |
       _cons |   3.953448   .1226658    32.23   0.000     3.712978    4.193917
------------------------------------------------------------------------------
```

Practical use 3

Practical use 3:

- 'Recentered IF regression' (Firpo et al. 2009)
- evaluate impact of covariates on distribution statistics



RIF regression

The effect of interest

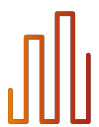
For example, how do foreign households affect $v(F)$?

$$F(y) = \sum_{x \in \Omega_X} s_x F_x(y)$$

Consider an infinitesimal variation: swap native for foreign workers

$$G_t^{F,t,k}(y) = (s_k + t) F_k(y) + (s_r - t) F_r(y) + \sum_{x \in \Omega_X \setminus \{k,r\}} s_x F_x(y).$$

What is the impact of this swap on the statistic of interest?



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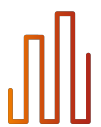

RIF regression

The effect of interest

For example, how do foreign households affect $\upsilon(F)$?

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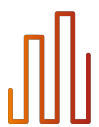
Consider an infinitesimal variation: swap native for foreign workers

$$G_r^{F,t,k}(y) = (s_k + t) \, F_k(y) + (s_r - t) \, F_r(y) + \sum_{x \in \Omega_X \setminus \{k,r\}} s_x \, F_x(y).$$

What is the impact of this swap on the statistic of interest?



Methods (ctd.)

Recentered influence function estimator

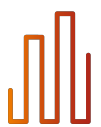
Firpo et al. (2009) show that effect of interest is given by:

$$\mathrm{E}[\mathrm{RIF}(y;v,F)|X=k]-\mathrm{E}[\mathrm{RIF}(y;v,F)|X=r]$$

where $\mathrm{RIF}(y;v,F)=v(F)+\mathrm{IF}(y;v,F)$

Regression-based estimator, β in :

$$\mathrm{E}[\mathrm{RIF}(y;v,F)|X=x]=\alpha+x\beta$$



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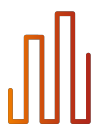
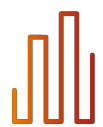


Interpretation of RIF regression coefficients

- The RIF at y gives the influence on $v(F)$ of an infinitesimal increase in the density of the data at y
- Regression coefficients reveal how much the average influence of observations vary with X (holding other covariates constant)
- It also reveals how much $v(F)$ would respond to a change in the distribution of X in the population holding distribution of other covariates constant
  - linear approximation valid only for *marginal* changes in X!

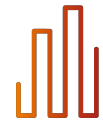


Illustrative example 1

Data and problem

- Panel Study Liewen zu Letzebuerg 2011 (official source for poverty and inequality statistics in Luxembourg)
- Permanent panel (started in 2003) gradually converted into rotating panel from 2010:
  - new samples ('rotation groups') now added for four years (so dataset for 2010, 2011, 2012 is mix of old and new samples)
  - Concern about how much 'new samples' differ from 'old sample' (due, e.g., to attrition (and slightly different sampling frame))
  - Impact on trends in inequality and poverty indicators? Break in series when 'old sample' abandoned?

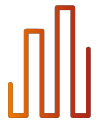
$\Longrightarrow$ Use RIF regression to check if rotation group impacts on inequality



Illustrative example 1

Code

```
svy:  inequaly nivie , atkinson(0.5 1 2)
predict rif* , rif // predict after -inequaly- gives (R)IF
svy:  regress rif1 ib9.(rot)
svy:  regress rif2 ib9.(rot)
svy:  regress rif3 ib9.(rot)
svy:  inequaly nivie , s80s20
predict rifs80s20 , rif
svy:  regress rifs80s20 ib9.(rot)
svy:  newpoverty nivie , fracmedian(.6)
predict rifh , rif
svy:  regress rifh ib9.(rot)
```



Results: Atkinson(0.5)

```
.         svy: regress rif1 ib9.(rot)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs     =       5,988
Number of PSUs     =     5,988                  Population size   = 515,407.14
                                                Design df         =       5,984
                                                F(   3,   5982)   =        2.96
                                                Prob > F          =      0.0312
                                                R-squared         =      0.0023

------------------------------------------------------------------------------
             |             Linearized
        rif1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .0157444   .0186107     0.85   0.398    -.0207392     .052228
           2 |  -.0048659   .0043721    -1.11   0.266    -.0134369     .003705
           3 |   .0111881   .0059868     1.87   0.062    -.0005481    .0229243
             |
       _cons |   .0575857   .0033763    17.06   0.000     .0509669    .0642044
------------------------------------------------------------------------------
```

Results: Atkinson(2)

```
.         svy: regress rif3 ib9.(rot)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs      =       5,988
Number of PSUs     =     5,988                  Population size    = 515,407.14
                                                Design df          =       5,984
                                                F(   3,   5982)    =        2.04
                                                Prob > F           =      0.1056
                                                R-squared          =      0.0023

------------------------------------------------------------------------------
             |             Linearized
        rif3 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .0246292   .0347917     0.71   0.479    -.0435751    .0928336
           2 |   -.006993   .0135387    -0.52   0.606    -.0335338    .0195478
           3 |   .0339101   .0171911     1.97   0.049     .0002094    .0676108
             |
       _cons |   .2036115    .009367    21.74   0.000     .1852488    .2219741
------------------------------------------------------------------------------
```

Results: S80/S20

```
.         svy: regress rifs80s20 ib9.(rot)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs     =       5,988
Number of PSUs     =     5,988                  Population size   = 515,407.14
                                                Design df         =       5,984
                                                F(   3,   5982)   =        2.44
                                                Prob > F          =      0.0622
                                                R-squared         =      0.0027

------------------------------------------------------------------------------
             |             Linearized
   rifs80s20 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .4599947   .5221966     0.88   0.378    -.5636989    1.483688
           2 |  -.0630366   .2020467    -0.31   0.755    -.4591209    .3330477
           3 |   .5306184   .2375919     2.23   0.026     .0648526    .9963842
             |
       _cons |   3.753404   .1405622    26.70   0.000     3.477851    4.028956
------------------------------------------------------------------------------
```

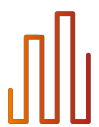
Results: Poverty rate

```
.         svy: regress rifh ib9.(rot)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs     =       5,988
Number of PSUs     =     5,988                  Population size   = 515,407.14
                                                Design df         =       5,984
                                                F(   3,   5982)   =        1.32
                                                Prob > F          =      0.2659
                                                R-squared         =      0.0020

------------------------------------------------------------------------------
             |             Linearized
        rifh |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .0146267   .0233548     0.63   0.531    -.0311571    .0604105
           2 |   .0069663   .0211085     0.33   0.741     -.034414    .0483466
           3 |    .043081   .0221201     1.95   0.052    -.0002823    .0864443
             |
       _cons |   .1342025   .0129755    10.34   0.000     .1087658    .1596393
------------------------------------------------------------------------------
```

Illustrative example 2

Effect of foreign households on inequality and poverty?

- Effect of a marginal increase in share of foreign-headed households on indicators
  - assuming no change in income structure otherwise
- Condition on age of foreign households



Results: Atkinson(0.5)

```
.         svy: regress rif1 ib9.(rot) i.(chme11)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs     =       5,987
Number of PSUs     =     5,987                  Population size   = 515,119.97
                                                Design df         =       5,983
                                                F(   6,   5978)   =        7.42
                                                Prob > F          =      0.0000
                                                R-squared         =      0.0099

------------------------------------------------------------------------------
             |             Linearized
        rif1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .0142397   .0183501     0.78   0.438    -.0217331     .0502126
           2 |  -.0062364   .0043078    -1.45   0.148    -.0146812     .0022083
           3 |   .0080582   .0059128     1.36   0.173     -.003533     .0196495
             |
       chme11 |
   Portugais |   .0135398   .0039052     3.47   0.001     .0058841     .0211955
Autres UE-15 |   .0262902   .0178432     1.47   0.141     -.008689     .0612694
   Non UE-15 |   .0418941   .0086454     4.85   0.000     .0249461     .0588421
             |
       _cons |   .0476424   .0050467     9.44   0.000     .0377491     .0575358
------------------------------------------------------------------------------
```

Results: Atkinson(0.5)

```
.         svy: regress rif1 ib9.(rot) i.(chme11) ib6.(chme09)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs     =      5,987
Number of PSUs     =     5,987                  Population size   = 515,119.97
                                                Design df         =      5,983
                                                F(  10,   5974)   =       5.90
                                                Prob > F          =     0.0000
                                                R-squared         =     0.0108

------------------------------------------------------------------------------
             |             Linearized
        rif1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .0141385   .0178513     0.79   0.428    -.0208565    .0491335
           2 |  -.0061202   .0043087    -1.42   0.156    -.0145667    .0023264
           3 |   .0082485   .0059444     1.39   0.165    -.0034047    .0199016
             |
       chme11 |
   Portugais |   .0159073   .0039841     3.99   0.000      .008097    .0237177
Autres UE-15 |   .0269482   .0181449     1.49   0.138    -.0086224    .0625188
   Non UE-15 |   .0442646   .0085935     5.15   0.000     .0274182     .061111
             |
       chme09 |
     [16-24] |   .0032088   .0138918     0.23   0.817    -.0240242    .0304418
     [25-34] |  -.0125041   .0078015    -1.60   0.109    -.0277978    .0027897
     [35-49] |  -.0050532    .007715    -0.65   0.512    -.0201773    .0100708
     [50-64] |   .0021521   .0129697     0.17   0.868    -.0232733    .0275774
             |
        _cons |   .0499364   .0094564     5.28   0.000     .0313985    .0684744
------------------------------------------------------------------------------
```

Results: Atkinson(2)

```
.         svy: regress rif3 ib9.(rot) i.(chme11) ib6.(chme09)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs     =      5,987
Number of PSUs     =     5,987                  Population size   = 515,119.97
                                                Design df         =      5,983
                                                F(  10,   5974)   =       2.68
                                                Prob > F          =     0.0029
                                                R-squared         =     0.0173

------------------------------------------------------------------------------
             |             Linearized
        rif3 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .0197437    .033453     0.59   0.555    -.0458363    .0853236
           2 |  -.0112723   .0133852    -0.84   0.400    -.0375122    .0149676
           3 |   .0245481   .0159634     1.54   0.124    -.0067459    .0558422
             |
      chme11 |
  Portugais  |   .0076757   .0114413     0.67   0.502    -.0147533    .0301047
Autres UE-15 |   .0634587   .0338566     1.87   0.061    -.0029124    .1298298
  Non UE-15  |   .1332594   .0373124     3.57   0.000     .0601136    .2064052
             |
      chme09 |
    [16-24]  |    .046107   .0539939     0.85   0.393    -.0597405    .1519544
    [25-34]  |  -.0031152   .0206771    -0.15   0.880    -.0436499    .0374194
    [35-49]  |   .0021401   .0159684     0.13   0.893    -.0291637    .0334439
    [50-64]  |   .0165315   .0241836     0.68   0.494    -.0308771      .06394
             |
       _cons |   .1764981   .0180461     9.78   0.000     .1411213    .2118749
------------------------------------------------------------------------------
```

Results: Poverty rate

```
.         svy: regress rifh ib9.(rot) i.(chme11) ib6.(chme09)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs     =      5,987
Number of PSUs     =     5,987                  Population size   = 515,119.97
                                                Design df         =      5,983
                                                F(  10,   5974)   =       2.89
                                                Prob > F          =     0.0013
                                                R-squared         =     0.0142

------------------------------------------------------------------------------
             |             Linearized
        rifh |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .0114206   .0234601     0.49   0.626    -.0345697    .0574109
           2 |   .0040279   .0209573     0.19   0.848     -.037056    .0451117
           3 |   .0380812   .0219422     1.74   0.083    -.0049334    .0810959
             |
      chme11 |
  Portugais  |  -.0770121   .0238951    -3.22   0.001    -.1238552    -.030169
Autres UE-15 |   .0329958    .019137     1.72   0.085    -.0045195    .0705112
  Non UE-15  |   .0465431   .0511668     0.91   0.363    -.0537624    .1468486
             |
      chme09 |
    [16-24]  |   .0411687   .0877057     0.47   0.639     -.130766    .2131034
    [25-34]  |   .0480566   .0257379     1.87   0.062     -.002399    .0985123
    [35-49]  |   .0379237   .0192831     1.97   0.049     .0001218    .0757256
    [50-64]  |   .0562376   .0190103     2.96   0.003     .0189707    .0935046
             |
       _cons |   .1004148   .0159395     6.30   0.000     .0691676     .131662
------------------------------------------------------------------------------
```

Results: Poverty rate (fixed poverty line)

```
.         svy: regress rifhbis1 ib9.(rot) i.(chme11) ib6.(chme09)
(running regress on estimation sample)

Survey: Linear regression

Number of strata   =         4                  Number of obs     =      5,987
Number of PSUs     =     5,987                  Population size   = 515,119.97
                                                 Design df         =      5,983
                                                 F(  10,   5974)   =      16.31
                                                 Prob > F          =     0.0000
                                                 R-squared         =     0.1074

------------------------------------------------------------------------------
             |             Linearized
    rifhbis1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         rot |
           1 |   .0178995   .0213374     0.84   0.402    -.0239295    .0597284
           2 |   .0059625   .0192948     0.31   0.757    -.0318624    .0437873
           3 |    .038296   .0218492     1.75   0.080    -.0045364    .0811283
             |
      chme11 |
  Portugais  |   .2040203   .0231579     8.81   0.000     .1586225    .2494181
Autres UE-15 |    .052723   .0189874     2.78   0.006     .0155009    .0899452
  Non UE-15  |   .3275548   .0547087     5.99   0.000     .2203059    .4348036
             |
      chme09 |
     [16-24] |   .1271475   .0982018     1.29   0.195    -.0653634    .3196585
     [25-34] |    .047582   .0231179     2.06   0.040     .0022626    .0929015
     [35-49] |   .0561238    .015962     3.52   0.000     .0248325    .0874151
     [50-64] |   .0382346    .014837     2.58   0.010     .0091487    .0673204
             |
       _cons |   .0181208   .0122119     1.48   0.138    -.0058189    .0420606
------------------------------------------------------------------------------
```