Unit root tests for explosive behaviour

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September 2020

Introduction

The study of the dynamic properties of economic and financial variables is a key ingredient in econometric modelling.

One specific type of behaviour in which there has always been interest, particularly in times of crises or distress, is when the variable exhibits what appears to be explosive behaviour.

For example, there are instances in which prices increase well beyond the level that could be explained by their fundamentals (see e.g. Garber, 2000).

Introduction

Renewed interest in tests for explosive behaviour mainly because of novel theoretical findings by Phillips, Wu, and Yu (2011), PWY, and Phillips, Shi, and Yu (2015), PSY.

These authors provide a framework of analysis suitable for testing and date-stamping episodes where explosive behaviour might have occurred.

Implementation of these new testing strategies is possible thanks to:

- MATLAB; see https://sites.google.com/site/shupingshi/home/codes

- EViews add-in Rtadf (Caspi, 2017)

- R packages exuber (Vasilopoulos, Pavlidis, Spavound, and Martínez-García, 2020); psymonitor (Phillips, Shi, and Caspi, 2019)

Introduction

We present the community-contributed Stata command `radf` to test for explosive behaviour in time series.

This command implements the right-tail augmented Dickey and Fuller (1979) (ADF) unit root test, and its further developments based on supremum statistics derived from ADF-type regressions estimated using recursive windows (PWY) and recursive flexible windows (PSY).

Similar to the software listed above, `radf`

- supports date-stamping procedures to identify episodes of explosive behaviour; and

- implements the wild bootstrap proposed by Phillips and Shi (2020) to lessen the potential effects of unconditional hetero and multiplicity issues involved in recursive testing.

Introduction

In this presentation we:

- provide an overview of the tests supported by radf;

- describe radf;

- illustrate the use of radf with an empirical example;

- offer concluding remarks.

Tests for explosive behaviour: An overview

Following PSY, `radf` calculates three tests based on the ADF regression:

$$\Delta y_t = \alpha_{r_1, r_2} + \beta_{r_1, r_2} y_{t-1} + \sum_{i=1}^{k} \delta_{r_1, r_2}^{i} \triangle y_{t-i} + \varepsilon_t, \tag{1}$$

where Δ is the first difference operator, y_t is the time series of interest at time t, k is the number of lags of the dependent variable, and r_1 and r_2 denote the starting and ending points used for estimation, respectively.

With T as the total number of time periods in the sample, r_1 and r_2 are expressed as fractions of T such that $r_2 = r_1 + r_w$, where r_w is the window size of the regression, also expressed as a fraction of T. The number of observations to estimate (1) is $T_w = \lfloor Tr_w \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function which gives the integer part of the argument. The error term is ε_t.

Tests for explosive behaviour: An overview

Null hypothesis is $H_0 : \beta_{r_1,r_2} = 0$ (unit root).

Alternative hypothesis $H_1 : \beta_{r_1,r_2} > 0$ (explosive behaviour).

The ADF t-statistic required to test H_0 in (1) is denoted $\text{ADF}_{r_1}^{r_2}$.

In this setting, `radf` calculates the two statistics studied by PWY.

Tests for explosive behaviour: An overview

The first is the right-tailed ADF statistic based on the full range of observations, $r_1 = 0$ and $r_2 = 1$ (i.e., $r_w = 1$), denoted ADF_0^1:
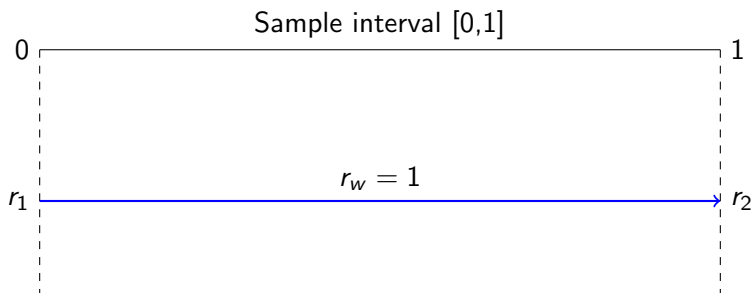
(a) Full sample

Figure 1: Sample sequences and window widths supported by `radf`

Tests for explosive behaviour: An overview

The second statistic is based on the supremum t-statistic that results from a forward recursive estimation of (1):

$$\text{SADF}(r_0) = \sup_{r_2 \in [r_0, 1]} \text{ADF}_0^{r_2}. \tag{2}$$
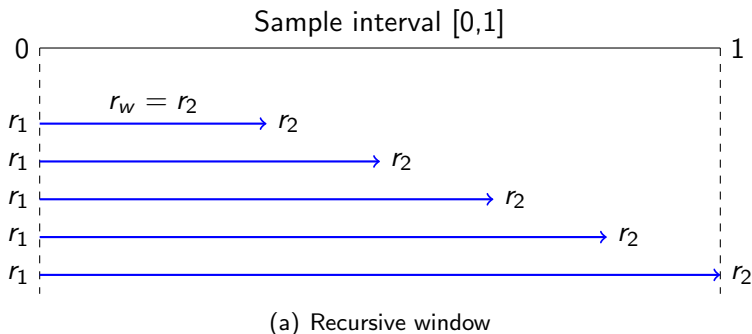

Sample interval [0,1]

(a) Recursive window

Figure 2: Sample sequences and window widths supported by radf
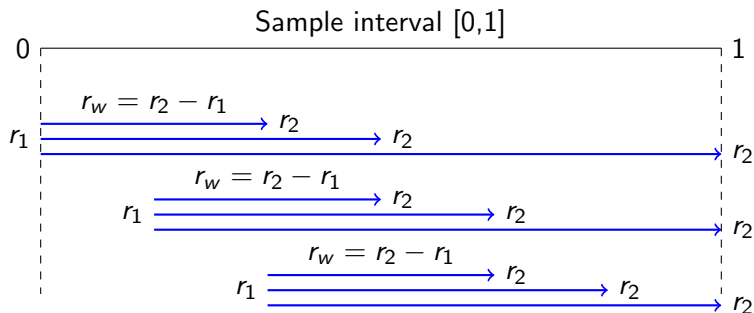
Tests for explosive behaviour: An overview

Phillips, Shi, and Yu (2015) find that when there are several bubbles, the recursive approach provides consistent estimates of the origination and ending dates of the first bubble, but not for subsequent ones.

To overcome this, PSY put forward the generalised supremum ADF (GSADF) test, which is the third statistic produced by `radf`.

As the name implies, the GSADF test involves a much more extensive set of regressions in which the first observation used for estimation varies from 0 to $r_2 - r_0$, while the last observation varies from r_0 to 1:

$$\text{GSADF}(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} \text{ADF}_{r_1}^{r_2}. \tag{3}$$

(a) Recursive flexible window

Figure 3: Sample sequences and window widths supported by `radf`

Tests for explosive behaviour: An overview

Inference for the right-tail ADF, SADF and GSADF statistics requires critical values computed using Monte Carlo simulations.

For computational convenience, `radf` takes advantage of a large set of critical values already available in the R Core Team (2020) package `exuber`; see Vasilopoulos et al. (2020) and Vasilopoulos et al. (2020).

Specifically, we incorporate the 90, 95 and 99% critical values provided by `exuber`, based on 2000 replications, seed equal to 123, initial window size given by $r_0 = 0.01 + 1.8/\sqrt{T}$, and $T = 6, 7, 8, \ldots, 600, 700, 800, \ldots, 2000$. For $600 < T \leq 2000$ the sample size is used to interpolate between the critical values. For $T > 2000$, the critical values are those for $T = 2000$.

Tests for explosive behaviour: An overview

In a further development, Phillips and Shi (2020) recommend a wild bootstrap to lessen the potential effects of unconditional hetero and to account for the multiplicity issue in recursive testing.

This bootstrap scheme can be implemented as an option with `radf`, which provides 90, 95 and 99% bootstrap critical values for the three tests described before.
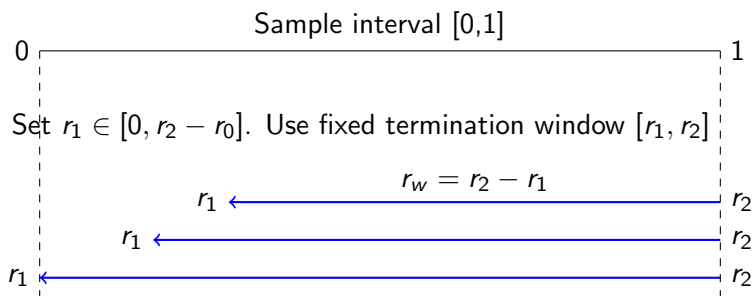
Tests for explosive behaviour: An overview

What happens if the unit root null hypothesis is rejected?

The testing strategies based on recursive/recursive flexible window estimation of (1) provide guidance to date-stamp in real time episodes of explosive behaviour.

Suppose one is interested in assessing whether any particular observation, say r_2, belongs to a phase of explosive behaviour. PSY recommend performing a supADF test on a sample sequence where the endpoint is fixed at r_2, and expands backwards to the starting point, r_1, which varies between 0 and $(r_2 - r_0)$. The backward SADF statistic is defined as:

$$\text{BSADF}_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \text{ADF}_{r_1}^{r_2}. \tag{4}$$
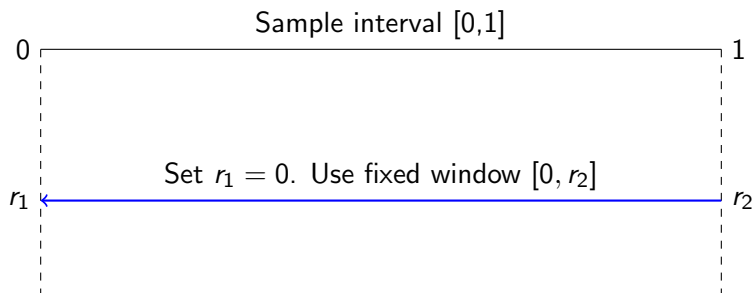
(a) The backward supADF test

Figure 4: Date-stamping in Phillips, Shi, and Yu (2015)

Tests for explosive behaviour: An overview

PSY indicate that this identification procedure is more general than the earlier suggestion in PWY which sets $r_1 = 0$ in (4), and therefore is more effective at identifying episodes of multiple bubbles

(a) The ADF test

Figure 5: Date-stamping in Phillips, Wu, and Yu (2011)

The radf command
Syntax

Before using the command radf, and similar to many other Stata time-series commands, it is necessary to tsset the data. Then:

radf *varname* [*if*] [*in*] [, prefix*(string)* maxlag*(integer)*
criterion*(string)*
window*(integer)* bs seed*(integer)* boot*(integer)* print graph]

Note that *varname* may not contain gaps, but may contain time-series operators. radf does not support the by: prefix.

The `radf` command
Options

The command `radf` supports the following options:

- `prefix` provides a 'stub' with which variables created in `radf` will be named if $T \leq 600$ in the specified sample. If this option is given, four Stata variables will be created for the appropriate range of dates: *prefix_SADF*, *prefix_BSADF*, *prefix_BSADF_95* and *prefix_Exceeding*. These variables record the SADF and BSADF statistics, with the third variable displaying the 95% critical values for the BSADF statistic, which vary over the estimation period. The fourth variable is an indicator, set to 1 in each period when the BSADF statistic exceeds its 95% critical value. The `prefix` option must be specified to enable the `graph` option.

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The `radf` command
Options

- `maxlag` sets the number of lags in the test regression. If not specified, `radf` sets the number of lags following Schwert (1989) using $maxlag = \text{int}\left(4\left(\frac{T}{100}\right)^{0.25}\right)$. If `maxlag` is given and the window width is set by the data-dependent procedure, they may conflict. In this case, `radf` reduces `maxlag` so that each ADF regression has positive degrees of freedom.

- `criterion` is used to specify `AIC`, `SIC`, `GTS05` or `GTS10` as alternatives to the default value of `FIX` specified by the user.

- `window` is used to select a window width; the default value is $r_0 = 0.01 + 1.8/\sqrt{T}$. However, as the critical values have been developed for the default value, a warning is provided if the window width is set by the user, showing the default and the selected widths.

The `radf` command
Options

- `bs` computes right-tail Monte Carlo critical values for 90, 95 and 99 percentiles based on the wild bootstrap advocated by Phillips and Shi (2020), using 199 replications. This option does not permit the user to set a different number of replications, which may be done with the `boot()` option. Also notice that the bootstrap critical values cannot be replicated unless `bs` is used along with option `seed`.

- `boot` sets the number of replications to perform the wild bootstrap advocated by Phillips and Shi (2020).

- `seed` sets the initial seed for random number generation.

The `radf` command
Options

- `print` specifies that detailed results are to be printed, showing the ADF statistics and lag lengths for each of the regressions being estimated.

- `graph` specifies that the time series of the SADF and BSADF statistics, which can be saved as variables with the `prefix()` option, should be graphed along with their 90% and 95% critical values. The graphs will be saved with names specified by the `prefix()` option as *prefix*_SADF.gph and *prefix*_BSADF.gph. The `graph` option is not available if more than 600 observations are included in the specified sample, and requires the use of the `prefix()` option.

Empirical application

Housing is the most important asset in the portfolio of many individuals, and so it is important to follow property prices to see if they may be viewed as reaching levels that compromise affordability.

We use data from the International House Price Database of the Dallas Fed; quarterly prices on 23 countries starting in 1975q1 (see Mack and Martínez-García, 2011).

Following (Vasilopoulos et al., 2020, section 5.2), we downloaded the data from the R console and created a Stata version of the data which was placed with the Boston College Economics Stata datasets.

Empirical application

We being by loading the dataset using the community-contributed command `bcuse`, and declaring it as a time series:

```
. bcuse hprices
. tsset yq
time variable: yq, 1975q1 to 2015q1
delta: 1 quarter
```

We test whether the price index of the United Kingdom, named `uk`, contains a unit root, against the alternative that it is an explosive process.

Using the default specifications for `criterion` and `window` and setting a `maxlag` of $p = 1$, the results of applying the command `radf` over the full range of observations are:

Empirical application

```
. radf uk, maxlag(1)

Right-tail ADF statistics for uk with first observations 1975q3 - 2009q2

Number of obs = 159  lag selection[FIX]  maxlag = 1  window = 24 periods
```

	Test	Tab90	Tab95	Tab99
ADF	-0.0907	-0.3562	0.0200	0.8674
SADF	2.9384	1.0697	1.3693	1.9305
GSADF	4.0630	1.8267	2.1139	2.6669

```
Test: ADF, SADF (PWY,2011), GSADF (PSY,2015)
Tab : right-tail tabulated critical values for 90, 95, 99 confidence levels
      from Vasilopoulos, Pavlidis, Spavound and Martínez-García (2020)
```

Empirical application

Identical results are obtained using the R package `exuber` and the Eviews add-in `Rtadf`.

Computation time using laptop with Intel(R) Core(TM) i7-7500U CPU @ 2.70GHz 2.90GHz and installed memory (RAM) of 16 GB:

- `radf` runs in 0.159 seconds;

- R package `exuber` takes only 0.021 seconds

- Eviews add-in `Rtadf` takes 0.106, 0.536 and 21.756 seconds to obtain the ADF, SADF and GSADF statistics, respectively.[1]

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[1]Caspi (2017) reports running time improvements of more than 90% using a MATLAB parallel computer option.

Empirical application

Speed gains achieved by the R package `exuber` probably obey to the fact that Vasilopoulos et al. (2020) use partitioned inversion techniques which avoid the need to invert sequences of matrices when computing the OLS estimators.

`radf` is based on Stata's `Mata` language.

All in all, we consider that the speed of `radf` is satisfactory for applied work and also if the user requires the computation of CVs for different values of T, r_0 and lags p.
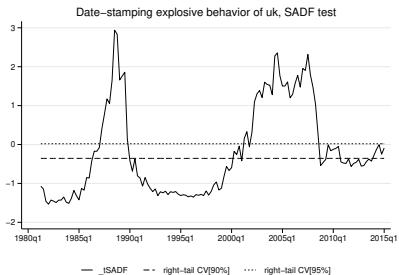
Empirical application

As the null is rejected, one might proceed to plot the sequences of
t-statistics and CVs to identify the episodes of explosive behaviour. To this
end, we run the previous command using the prefix and graph options:
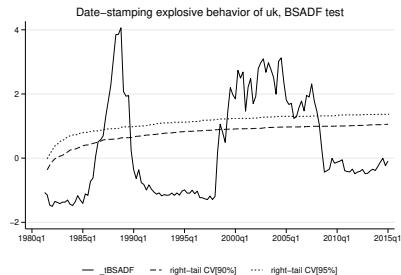
```
. radf uk, maxl(1) prefix(_t) graph
```

The resulting plots are presented in Figure 6.

Regarding the duration of the episodes, PSY focus on those that last more
than $\ln(T)$ units of time, which in this illustration is $\ln(161) \approx 5$ quarters.
This refinement, however, is left at the discretion of the user.

Empirical application

(a) Recursive estimation

(b) Recursive flexible estimation

Figure 6: Date-stamping analysis for the UK house price series

Empirical application

We now illustrate `radf` using the options `boot(499)` and `seed(123)`:

```
. radf uk, maxlag(1) boot(499) seed(123)
Right-tail ADF statistics for uk with first observations 1975q3 - 2009q2
Number of obs = 159  lag selection[FIX]  maxlag = 1  window = 24 periods
```

	Test	RTMC90	RTMC95	RTMC99	Tab90	Tab95	Tab99
ADF	−0.0907	0.3207	0.6189	1.0457	−0.3562	0.0200	0.8674
SADF	2.9384	2.5879	2.9414	3.4737	1.0697	1.3693	1.9305
GSADF	4.0630	3.5604	3.9052	4.8017	1.8267	2.1139	2.6669

```
Test: ADF, SADF (PWY,2011), GSADF (PSY,2015)
RTMC: right-tail Monte Carlo critical values for 90, 95, 99 percentiles
      based on wild bootstrap with 499 replications
Tab : right-tail tabulated critical values for 90, 95, 99 confidence levels
      from Vasilopoulos, Pavlidis, Spavound and Martínez-García (2020)
```

In all cases the simulated bootstrap CVs, columns RTMC90, RTMC95 and RTMC99, are larger than the corresponding tabulated CVs from Vasilopoulos et al. (2020), columns Tab90, Tab95 and Tab99.

Using the simulated bootstrap critical values, the unit-root null is rejected at the 10% and 5% levels by the SADF and GSADF tests, respectively.

Concluding remarks

We present the Stata command `radf` which calculates the right-tail ADF test statistics for explosive behaviour proposed by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015). The command offers options to:

- select the number of lags in the test regressions either by the user, or endogenously using a data-dependent procedure.

- generate/graph the sequences of recursive and flexible recursive statistics which allow identification of episodes of explosive behaviour.

- implement the wild bootstrap advocated by Phillips and Shi (2020), which aims to lessen the potential effects of unconditional hetero and to account for the multiplicity issue in recursive testing.

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