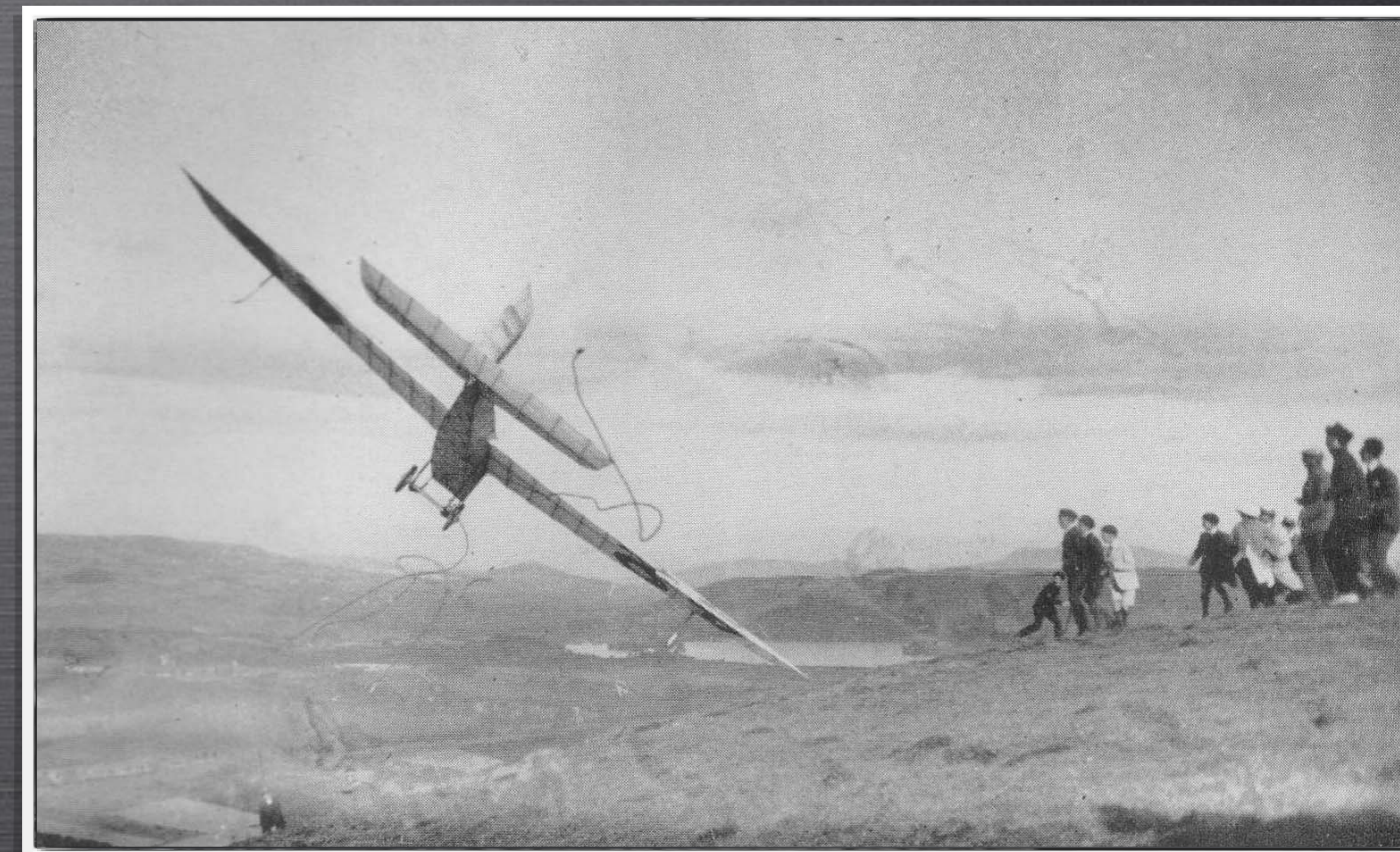
Seeing is believing: generalized added-variable plots



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  - interval excluding zero slope shows significance

Scatter plot with trend line
20
10
0
-10
-20
GDP per worker growth
600
700
800
900
1000
Child survival, previous generation

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  - condition on other variables X_2

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  - trend line slope *is* the OLS coefficient on x

Added-variable plot
1
0
-1
e(GDP per worker growth | X)
-45
-30
-15
0
15
30
e(Child survival, previous generation | X)

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    - at this value, x is entirely accounted for by X_2

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  - most research uses more complex estimation

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  - will take time - must be programmed for each estimator

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$$\mathbf{X} = [\mathbf{x}_1\mathbf{X}_2] \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ \mathbf{b}_2 \end{bmatrix}$$

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Added-variable plot: graph $\mathbf{e}_{\mathbf{x}_1}$ vs. $\mathbf{e}_{\mathbf{y}}$

trend line has slope b_1.

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Essentially all estimators can be represented in the OLS form (asymptotically, everything is linear)

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Blundell & Bond dynamic panel estimation

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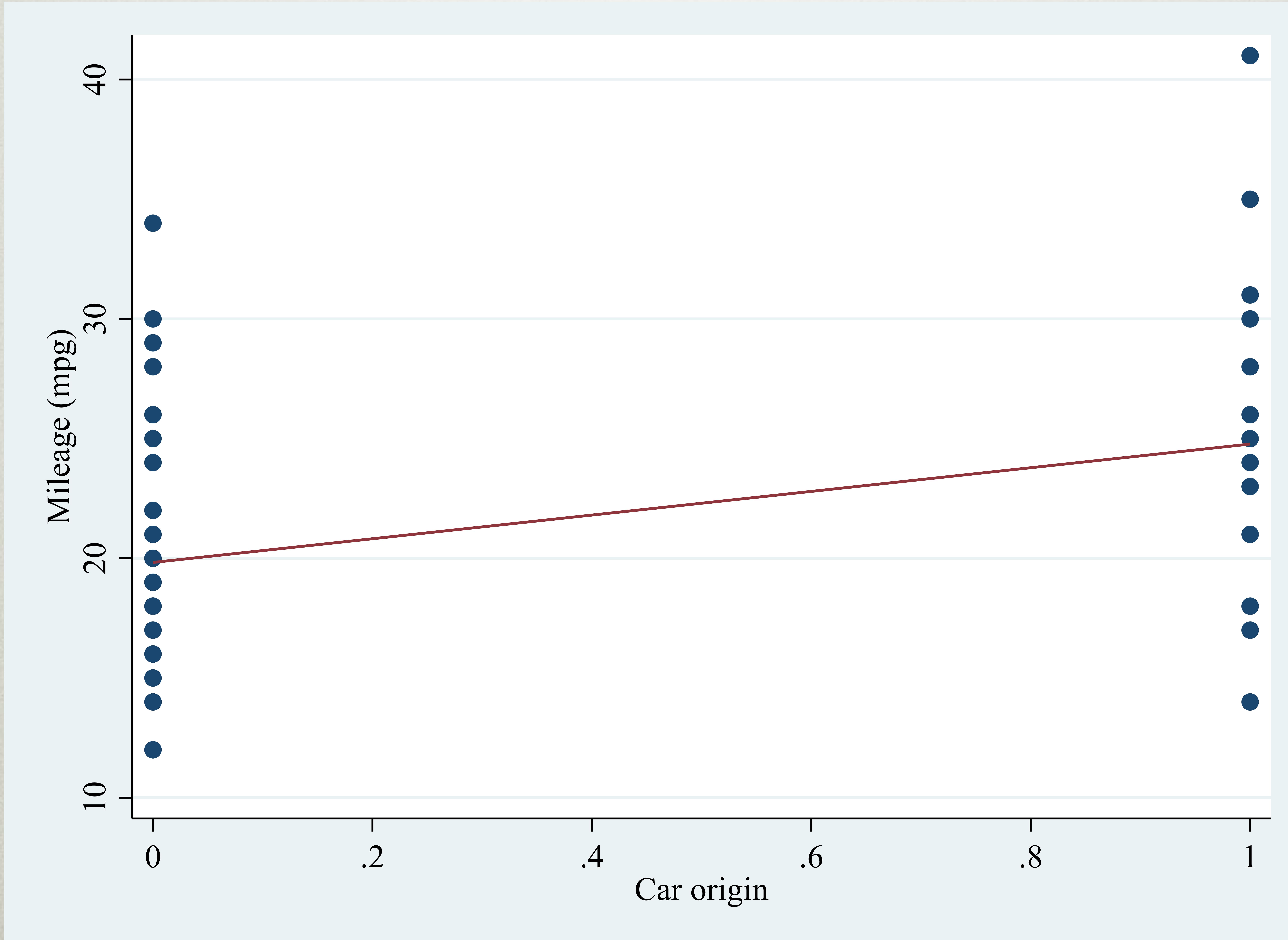
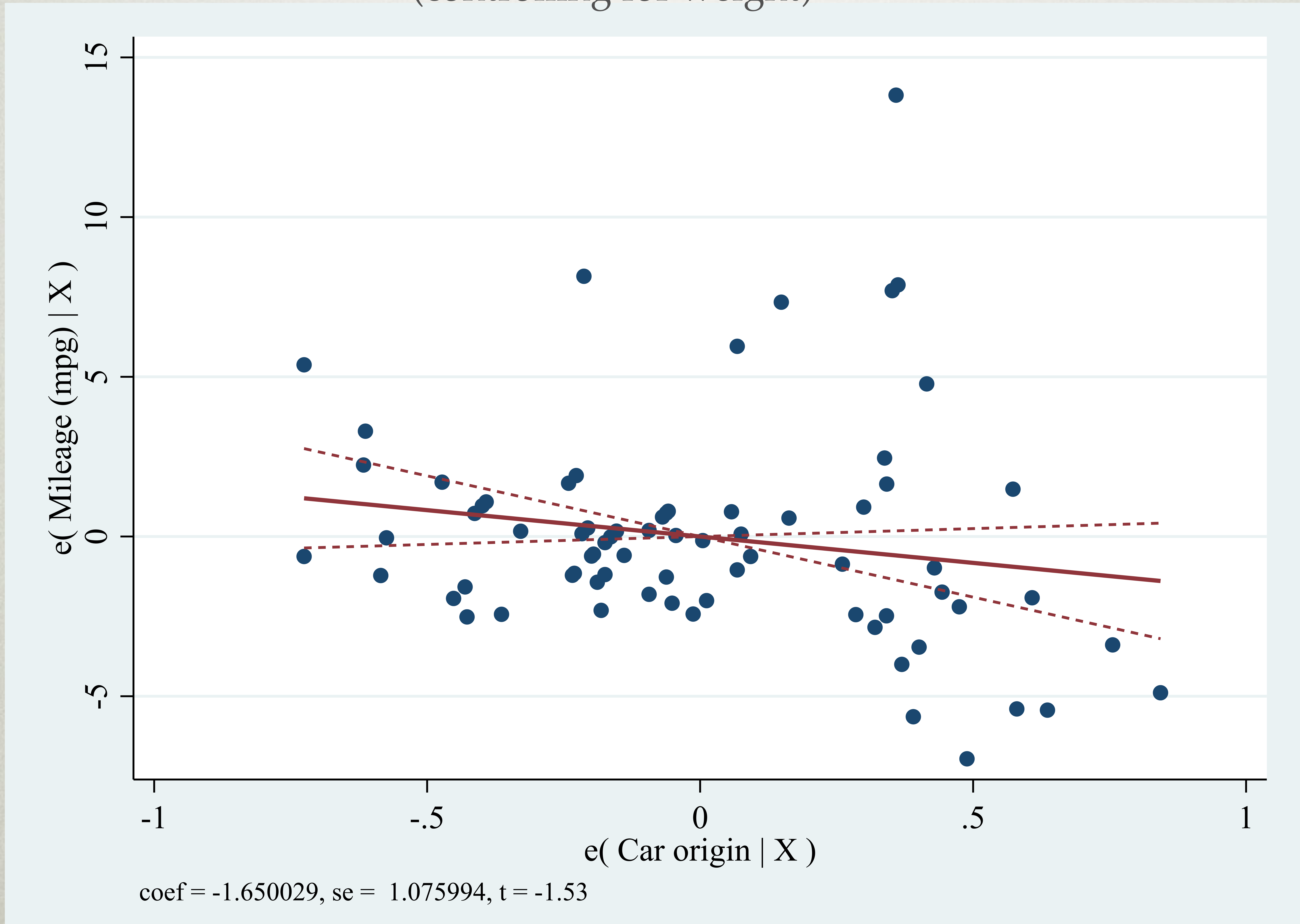
  - use binscatter `binsreg`, which can account for other correlates

Scatter plot of foreign dummy vs. mileage



Added-variable plot of foreign dummy vs. mileage

(controlling for weight)



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$E[\mathbf{g}(\boldsymbol{\beta}_0, \mathbf{y}, \mathbf{X})] = \mathbf{0}$ and $E[\mathbf{g}(\boldsymbol{\beta}, \mathbf{y}, \mathbf{X})] \neq \mathbf{0} \ \forall \ \boldsymbol{\beta} \neq \boldsymbol{\beta}_0$.

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If $\mathbf{X}$ is uncorrelated with $\mathbf{g}(\boldsymbol{\beta}, \mathbf{y}, \mathbf{X})$ then $\mathbf{Z} = \mathbf{X}$.

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$$\hat{\boldsymbol{\Omega}} \equiv \mathbf{Z}^\top \hat{\boldsymbol{\Sigma}}^{-1}\mathbf{Z} \text{ and } \hat{\mathbf{G}} \equiv \frac{\partial \mathbf{g}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^\top}\Big|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}}.$$

All the previous estimators are GMM

Estimator	$\mathbf{g}(\boldsymbol{\beta})$	$\mathbf{Z}$	$V[\mathbf{g}]$	$\tilde{\mathbf{X}}$	$\tilde{\mathbf{y}}$
OLS	$\mathbf{y}-\mathbf{X}\boldsymbol{\beta}$	$\mathbf{X}$	$\sigma^2\mathbf{I}$	$\mathbf{X}$	$\mathbf{y}$
GLS	$\mathbf{y}-\mathbf{X}\boldsymbol{\beta}$	$\mathbf{X}$	$\boldsymbol{\Sigma}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\mathbf{X}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\mathbf{y}$
2SLS	$\mathbf{y}-\mathbf{X}\boldsymbol{\beta}$	$\mathbf{Z}$	$\sigma^2\mathbf{I}$	$\mathbf{Z}(\mathbf{Z}^\top\mathbf{Z})^{-1}\mathbf{Z}^\top\mathbf{X}$	$\mathbf{Z}(\mathbf{Z}^\top\mathbf{Z})^{-1}\mathbf{Z}^\top\mathbf{y}$
3SLS	$\mathbf{y}-\mathbf{X}\boldsymbol{\beta}$	$\mathbf{Z}$	$\boldsymbol{\Sigma}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\mathbf{Z}(\mathbf{Z}^\top\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{Z})^{-1}\mathbf{Z}^\top\mathbf{X}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\mathbf{Z}(\mathbf{Z}^\top\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{Z})^{-1}\mathbf{Z}^\top\mathbf{y}$
MLE exp	$\mathbf{t}(\mathbf{y})-\boldsymbol{\mu}(\mathbf{X},\boldsymbol{\beta})$	$\hat{\boldsymbol{\mathcal{M}}}$	$\boldsymbol{\Sigma}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\hat{\boldsymbol{\mathcal{M}}}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}(\mathbf{t}(\mathbf{y})+\hat{\boldsymbol{\mathcal{M}}}\hat{\boldsymbol{\beta}}-\hat{\boldsymbol{\mu}})$
NLS	$\mathbf{y}-\boldsymbol{\mu}(\mathbf{X},\boldsymbol{\beta})$	$\hat{\boldsymbol{\mathcal{M}}}$	$\sigma^2\mathbf{I}$	$\hat{\boldsymbol{\mathcal{M}}}$	$\mathbf{y}+\hat{\boldsymbol{\mathcal{M}}}\hat{\boldsymbol{\beta}}-\hat{\boldsymbol{\mu}}$
NLS $\boldsymbol{\Sigma}$	$\mathbf{y}-\boldsymbol{\mu}(\mathbf{X},\boldsymbol{\beta})$	$\hat{\boldsymbol{\mathcal{M}}}$	$\boldsymbol{\Sigma}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\hat{\boldsymbol{\mathcal{M}}}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}(\mathbf{y}+\hat{\boldsymbol{\mathcal{M}}}\hat{\boldsymbol{\beta}}-\hat{\boldsymbol{\mu}})$
lin GMM	$\mathbf{y}-\mathbf{X}\boldsymbol{\beta}$	$\mathbf{Z}$	$\boldsymbol{\Sigma}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\mathbf{Z}(\mathbf{Z}^\top\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{Z})^{-1}\mathbf{Z}^\top\mathbf{X}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\mathbf{Z}(\mathbf{Z}^\top\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{Z})^{-1}\mathbf{Z}^\top\mathbf{y}$
NL GMM	$\mathbf{g}(\boldsymbol{\beta},\mathbf{y},\mathbf{X})$	$\mathbf{Z}$	$\boldsymbol{\Sigma}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\mathbf{Z}(\mathbf{Z}^\top\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{Z})^{-1}\mathbf{Z}^\top\hat{\mathbf{G}}$	$\hat{\boldsymbol{\Sigma}}^{-1/2}\mathbf{Z}\hat{\boldsymbol{\Omega}}^{-1}\mathbf{Z}^\top\left(\hat{\mathbf{G}}\hat{\boldsymbol{\beta}}+\hat{\mathbf{g}}\right)$

Among other GMM estimators, this adds AV plots for nonlinear instrumental variables

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