MAT H1007
Homework 1
Due Friday, September 9

When submitting homework, please remember the following:
• Show all work leading to each solution.
• You must use a staple (not paper clip) if your answers are longer than a single page.
• Do not submit crossed-out or sloppy work.
• Do not submit ripped or torn pages.
• Be sure to submit your own work.

1. Consider the sequence defined by the formula \( A_N = \frac{2N + 3}{N + 11} \).
   (a) Find \( A_1 \).
   (b) Find \( A_9 \).

2. Consider the sequence defined by the recursive formula
   
   \[ \begin{align*}
   A_1 &= 1 \\
   A_2 &= 3 \\
   A_N &= A_{N-1} + 2A_{N-2}, \quad N \geq 3
   \end{align*} \]

   List \( A_3, A_4, A_5, \) and \( A_8 \).

3. Suppose that we have a linear sequence in which \( P_0 = 23 \) and \( P_1 = 24.1 \).
   (a) List \( P_2, P_3, P_4, \) and \( P_{23} \).
   (b) Use the formula for an arithmetic sum to add up \( P_0 + P_1 + \cdots + P_{23} \).

4. Suppose that we have an exponential sequence in which \( P_0 = 23 \) and \( P_1 = 24.1 \).
   (a) List \( P_2, P_3, P_4, \) and \( P_{23} \).
   (b) Use the formula for a geometric sum to add up \( P_0 + P_1 + \cdots + P_{23} \).

5. Suppose that we have a linear sequence in which \( P_0 = 24.1 \) and \( P_1 = 23 \).
   (a) List \( P_2, P_3, P_4, \) and \( P_{14} \).
   (b) Use the formula for an arithmetic sum to add up \( P_0 + P_1 + \cdots + P_{14} \).

6. Suppose that we have an exponential sequence in which \( P_0 = 24.1 \) and \( P_1 = 23 \).
   (a) List \( P_2, P_3, P_4, \) and \( P_{14} \).
   (b) Use the formula for a geometric sum to add up \( P_0 + P_1 + \cdots + P_{14} \).

7. The city of Sylvania currently has 401 LED streetlights. The city council has decided to install 3 additional LED streetlights at the start of each week for the next 52 weeks. Each LED streetlight costs $0.24 to operate for a week.
   (a) How many LED streetlights will Sylvania have at the end of 21 weeks?
   (b) What is the cost of operating the original 401 LED streetlights for 52 weeks?
   (c) What is the additional cost of the new LED streetlights at the end of 52 weeks?
8. This problem asks you to experiment with the logistic growth model
\[ p_n = rp_{n-1}(1 - p_{n-1}) \]
for various values of \( p_0 \) and \( r \). Do all of your work to at least 4 decimal places.
(a) Suppose that \( r = 0.5 \) and \( p_0 = 0.3 \). Compute \( p_1 \) up to \( p_{10} \).
(b) Suppose that \( r = 1.5 \) and \( p_0 = 0.3 \). Compute enough terms of the sequence for you to observe a pattern.
(c) Suppose that \( r = 2.5 \) and \( p_0 = 0.3 \). Compute enough terms of the sequence for you to observe a pattern.
(d) Suppose that \( r = 3.2 \) and \( p_0 = 0.3 \). Compute enough terms of the sequence for you to observe a pattern.
(e) Suppose that \( r = 3.5 \) and \( p_0 = 0.3 \). Compute enough terms of the sequence for you to observe a pattern.

9. Suppose that we consider the logistic equation with \( r = 3.4 \). What value of \( p_0 \) (other than 0) will produce a constant sequence in which \( p_0 = p_1 = p_2 \)?