1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4, 7, 8\}$, $B = \{3, 5, 6, 9\}$, and $C = \{1, 2, 4\}$. List the elements in the following sets (no explanation needed):

- $(a)$ $A \cup B$. 
- $(b)$ $B \cap C$. 
- $(c)$ $A' \cap B' \cap C'$. 
- $(d)$ $A \cup B' \cup C$. 
- $(e)$ $A \cap C$. 
- $(f)$ $A' \cup C$.

**Answer:** $(a)$ $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. $(b)$ $B \cap C = \emptyset$. $(c)$ $A' \cap B' \cap C' = \emptyset$. $(d)$ $A \cap B \cap C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. $(e)$ $A \cap C = \{1, 2, 4\}$. $(f)$ $A' \cup C = \{1, 2, 4, 5, 6, 9\}$.

2. An investment club consists of 200 women who have invested in stocks, bonds, or both. If 130 of the women have invested in stocks, and 105 in bonds, how many have invested in both?

**Answer:** Let $S$ be the set of women who have invested in stocks, and $B$ the set of women who have invested in bonds. We are told that $n(S \cup B) = 200$, $n(S) = 130$, and $n(B) = 105$. The Inclusion–Exclusion Principle tells us that $n(S \cup B) = n(S) + n(B) - n(S \cap B)$, and therefore $200 = 105 + 130 - n(S \cap B)$. The number of women who have invested in both stocks and bonds is 35.

3. Simplify each of the following expressions. No explanation needed.

- $(a)$ $(A')'$. 
- $(b)$ $A' \cup A$. 
- $(c)$ $A \cap A'$. 
- $(d)$ $A \cap \emptyset$. 
- $(e)$ $A \cup \emptyset$.

**Answer:** $(a)$ $(A')' = A$. $(b)$ $A' \cup A = U$. $(c)$ $A \cap A' = \emptyset$. $(d)$ $A \cap \emptyset = \emptyset$. $(e)$ $A \cup \emptyset = A$.

4. A survey of Boston College students revealed that 120 people in the survey own a pair of Reeboks, 130 people own a pair of Nikes, and 20 own both Reeboks and Nikes. How many students in the survey owned at least one pair of Reeboks or Nikes?

**Answer:** Let $R$ be the set of students who own Reeboks, and $N$ the set of students who own Nikes. We are told that $n(R) = 120$, $n(N) = 130$, and $n(R \cap N) = 20$. The Inclusion–Exclusion Principle tells us that $n(R \cup N) = n(R) + n(N) - n(R \cap N) = 120 + 130 - 20 = 230$, and therefore 230 students own at least one pair of Nikes, Reeboks, or both.

5. Draw some two-circle Venn diagrams with circles labeled $A$ and $B$ and darken the portion of the Venn diagrams corresponding to the following sets:

- $(a)$ $A \cup B'$. 
- $(b)$ $A' \cap B$. 
- $(c)$ $(A' \cup B)'$. 
- $(d)$ $A \cap B'$. 
- $(e)$ $(B \cap A)'$. 
- $(f)$ $(A \cap B) \cup (A' \cap B')$.

**Answer:** We have

$(a)$
(b) and (d) are identical because of de Morgan's Laws:

6. Draw some two-circle Venn diagrams with circles labeled A and B. Use this information to compute the number of elements of each basic region.

   (a) \(n(U) = 15, n(A) = 7, n(B) = 6, n(A \cap B) = 2\).
   (b) \(n(A) = 3, n(A \cup B) = 6, n(B) = 4, n(A' \cup B') = 11\).
   (c) \(n(A) = 9, n(B) = 11, n(A \cap B) = 5, n(A') = 14\).

   \textbf{Answer:} We have:
7. The Standard and Poor’s Index measures the average price of a collection of 500 stocks. The table at the end of this question gives the percentage change in the S&P for the first 5 business days of the year and the percentage change for the entire year, for years 1984–2011. Let

\[ U = \{1984, 1985, \ldots, 2010, 2011\} \]

\[ A = \{\text{all years in which the index declined during the first 5 business days}\} \]

\[ B = \{\text{all years in which the index increased during the entire year}\} \]

\[ C = \{\text{all years in which the index declined more than 2\% during the entire year}\} \]

List the elements of the following sets

(a) \( A \).

(b) \( B \).

(c) \( C \).

(d) \( A \cap B \).

(e) \( A \cup C \).

(f) \( B' \cap C \).

(g) \( (A \cap B) \cup C \).

(h) \( A \cap (B \cup C) \).

Answer: We have:


\[ B' \cap C = \{1990, 2000, 2001, 2002, 2008\} \]

**Percentage change in S&P:**

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