Mathematics 210
Homework 3
Due Friday, September 26, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 7 & 4 & k \end{bmatrix} \). Find a value of \( k \) that makes the columns of \( A \) linearly dependent.

2. Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 7 & 4 & k \\ 11 & 3 & h \end{bmatrix} \). Find values of \( h \) and \( k \) that makes the columns of \( A \) linearly dependent.

3. (continued) Find values of \( h \) and \( k \) that makes the columns of \( A \) linearly independent.

4. Let \( A = \begin{bmatrix} 7 & 3 & 2 \\ 2 & 4 & 11 \\ 1 & 1 & 1 \end{bmatrix} \) and \( b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \). Is there a vector \( x \) so that \( Ax = b \)? If so, is that vector unique?

5. Let \( A = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} \), and let \( b = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \). Is there a vector \( x \) so that \( Ax = b \)? If so, is that vector unique?

6. Suppose that the set of vectors \( \{u, v, w\} \) is linearly dependent. Suppose that \( T \) is a linear transformation. Show that the set of vectors \( \{T(u), T(v), T(w)\} \) is linearly dependent.

7. Find a set of linearly independent vectors \( \{u, v, w\} \) and a linear transformation \( T \) so that the vectors \( \{T(u), T(v), T(w)\} \) are linearly dependent.

8. Suppose that \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) is a linear transformation defined by the two formulas \( T(e_1) = e_1 \) and \( T(e_2) = 2e_1 - 2e_2 \). What is the standard matrix of the linear transformation \( T \)?

9. Let \( A = \begin{bmatrix} 4 & 6 & 8 \\ 2 & 3 & 4 \end{bmatrix} \). Define a linear transformation \( A : \mathbb{R}^3 \to \mathbb{R}^2 \) with the formula \( T(x) = Ax \). Is \( T \) onto?

10. (continued) Is \( T \) one-to-one?