1. Suppose that $H = \text{Span}\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$. Find a basis for $H$.

2. (Continued) Find a 4-by-4 matrix $A$ so that $\text{Col} A = H$.

3. (Continued) Find a 4-by-4 matrix $B$ so that $\text{Nul} B = H$.

4. Let $W_1 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : xy = 0 \right\}$. Is $W_1$ a subspace of $\mathbb{R}^2$? If so, prove your answer; if not, explain why not.

5. Let $W_2 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : x + y = 0 \right\}$. Is $W_2$ a subspace of $\mathbb{R}^2$? If so, prove your answer; if not, explain why not.

6. Recall that $P_2 = \{ at^2 + bt + c \}$, the vector space of all polynomials of degree no more than 2. Define a function $T_1 : P_2 \to \mathbb{R}^2$ by $T_1(p) = \begin{bmatrix} p(0) \\ p(1) \end{bmatrix}$. Show that the function $T_1$ is a linear transformation.

7. (Continued) Find a polynomial $f(t) \in P_2$ so that the kernel of $T_1$ is $\text{Span}\{f(t)\}$.

8. Now define a function $T_2 : P_2 \to \mathbb{R}^2$ with the formula $T_2(p) = \begin{bmatrix} p(0) \\ p'(0) \end{bmatrix}$. (As usual, $p'(0)$ means the derivative of $p(t)$ evaluated at $t = 0$.) Show that $T_2$ is a linear transformation.

9. (Continued) Find a polynomial $g(t) \in P_2$ so that the kernel of $T_2$ is $\text{Span}\{g(t)\}$.

10. Now define a function $T_3 : P_2 \to \mathbb{R}^2$ by $T_3(p) = \begin{bmatrix} p(0) \\ p'(0) \end{bmatrix}$. You may assume that $T_3$ is a linear transformation; the proof is similar to the proofs for the function $T_1$. Find polynomials $h_1(t), h_2(t) \in P_2$ so that $\{h_1(t), h_2(t)\}$ is a basis for the kernel of $T_3$. 

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.