1. Let \( n \) be a positive integer. Prove using induction that
\[ \lim_{x \to 0^+} x(\log x)^n = 0. \]

The notation \( \lim_{x \to 0^+} \) means that \( x \) tends to 0 and is positive. The inequality \( x > 0 \) is required because \( \log x \) is only defined for positive \( x \). \textit{Hint:} Apply l'Hôpital’s rule, but make sure that you do it correctly.

2. Use an even–odd argument to show that \( \sqrt{13} \) is irrational. \textit{Hint:} This is a bit tricky, and requires a bit more thought than our previous irrationality proofs.

3. Find the smallest positive integer \( N \) so that \( n^3 \leq 2^n \) if \( n \geq N \), and prove your result using induction.

4. Let \( n > 2 \) be an integer. Show that
\[ F_n F_{n+1} - F_{n-1} F_{n+2} = (-1)^{n+1}. \]