1. Let $n$ be a positive integer. Prove using induction and integration by parts that
\[ \int_0^1 (-\log x)^n \, dx = n!. \]
This is an improper integral, so you will need to explain how you evaluated the lower limit when applying integration by parts.

2. Let $m$ and $n$ be positive integers, with $m \leq n$. Prove that
\[ \sum_{k=m}^{n} \binom{k}{m} = \binom{n+1}{m+1}. \]

3. Let $a$ and $b$ be positive numbers, with $b > 1$. Prove that
\[ F_a F_{b-1} + F_{a+1} F_b = F_{a+b}. \]