Remember that the Fibonacci numbers are defined with the three equations

\[ F_1 = 1 \]
\[ F_2 = 1 \]
\[ F_n = F_{n-1} + F_{n-2} \]

For example, we have \( F_3 = 2 \), \( F_4 = 3 \), and \( F_5 = 5 \).

1. Let \( k \) be a positive integer. Prove that \( F_{3k} \) is always even.

2. Let \( k \) be a positive integer. Prove that \( F_{4k} \) is always a multiple of 3.

3. Suppose that \( G \) is a group, and for every element \( a \in G \), we have \( a = a^{-1} \). Prove that \( G \) must be abelian.

4. If \( G \) is a finite group of even order, show that there must be an element \( a \neq e \) such that \( a = a^{-1} \).

5. Suppose that \( G \) is a group in which \((ab)^2 = a^2b^2\) for every pair of elements \( a \) and \( b \) in \( G \). Prove that \( G \) must be abelian.

6. If \( A \) and \( B \) are subgroups of \( G \), show that \( A \cap B \) is a subgroup of \( G \).

7. Let \( G \) be a group in which \((ab)^3 = a^3b^3\) and \((ab)^5 = a^5b^5\) for all \( a, b \in G \). Show that \( G \) is abelian.

8. Suppose that \( G \) is a group in which for some fixed positive integer \( n \), we have the three equations

\[ (ab)^n = a^n b^n \]
\[ (ab)^{n+1} = a^{n+1} b^{n+1} \]
\[ (ab)^{n+2} = a^{n+2} b^{n+2} \]

for every pair of elements \( a \) and \( b \) in \( G \). Prove that \( G \) must be abelian.

9. Verify that \( Z(G) \), the center of \( G \), is a subgroup of \( G \).

10. If \( G \) is an abelian group and if \( H = \{ a \in G \mid a^2 = e \} \), show that \( H \) is a subgroup of \( G \).