1. Show that there is no ring homomorphism $\phi : \mathbb{C} \to \mathbb{R} \oplus \mathbb{R}$.

2. Find a ring homomorphism $\phi : \mathbb{R} \oplus \mathbb{R} \to \mathbb{C}$.

3. Find the minimal polynomial in $\mathbb{Q}[x]$ for $\sqrt{2} + \sqrt[7]{7}$.

4. Suppose that $E$ is a field containing $q$ elements, and $E \subset F$. Suppose that $F$ is a field, with $[F : E] = n$. Show that $F$ contains $q^n$ elements.

5. Suppose that $E \subset F$, where $E$ and $F$ are fields, and suppose as well that $[E : F] = p$, where $p$ is a prime. Let $a$ be any element of $F \setminus E$. Show that $F = E(a)$.

6. Degrees do not always behave the way that we would hope. Find two numbers $a$ and $b$ which are algebraic over $\mathbb{Q}$ with $[\mathbb{Q}(a) : \mathbb{Q}] = 2$, $[\mathbb{Q}(b) : \mathbb{Q}] = 3$, but the degree of the minimal polynomial for $ab$ is less than 6.