1. On last week’s homework, we used the bisection method to find a solution for the equation \( x^4 - 2x^3 - 4x^2 + 4x + 4 = 0 \) on the interval \([-1, 4]\).

   (a) Perform 4 iterations of Newton’s method to solve the same equation with \( p_0 = -1 \).
   (b) Perform 4 iterations of Newton’s method to solve the same equation with \( p_0 = 4 \).
   (c) Perform 4 iterations of the secant method with \( p_0 = -1 \) and \( p_1 = 4 \) to solve the same equation.
   (d) Perform 4 iterations of the secant method with \( p_0 = 4 \) and \( p_1 = -1 \) to solve the same equation.
   (e) Perform 4 iterations of the method of false position with \( p_0 = -1 \) and \( p_1 = 4 \) to solve the same equation.

2. Let \( f(x) = x \sin x \).

   (a) Show that \( f(x) \) has a double zero at \( x = 0 \).
   (b) Let \( p_0 = 1.5 \), and perform 3 iterations of Newton’s method to try to find the root.
   (c) Let \( \mu(x) = f(x)/f'(x) \). Perform 3 iterations of Newton’s method using the function \( \mu(x) \) to try to find the root. Is the convergence noticeably quicker than for \( f(x) \)?

3. The ordinary annuity equation is

\[
A = \frac{P}{i} (1 - (1 + i)^{-n}),
\]

where \( A \) is the amount of money to be borrowed, \( P \) is the amount of each payment, \( i \) is the interest rate per period, and there are \( n \) equally spaced payments. Suppose that a buyer needs a 30-year home mortgage of $135,000, with payments of at most $1,000 per month. (This means that there are 360 payments in all.) What is the maximal annual interest rate that the buyer can afford?

4. Suppose that \( f(x) \) has \( m \) continuous derivatives (in our usual notation, \( f \) is \( C^m \)). Modify the proof of Theorem 2.10 in the text to show that \( f \) has a zero of multiplicity \( m \) at \( p \) if and only if \( f(p) = f'(p) = f''(p) = \cdots = f^{(m-1)}(p) = 0 \) and \( f^{(m)}(p) \neq 0 \).

5. Given a function \( f(x) \) with continuous second derivative, let

\[
g(x) = x - \frac{f(x)}{f'(x)} - \frac{f''(x)}{2f'(x)} \left( \frac{f(x)}{f'(x)} \right)^2.
\]

   (a) Suppose that \( f(p) = 0 \). Show that \( g'(p) = g''(p) = 0 \). This means (you do not need to check this) that often the series \( p_n = g(p_{n-1}) \) will converge cubically.
   (b) Let \( f(x) = x^4 - 2x^3 - 4x^2 + 4x + 4 \). Iterate \( g(x) \) twice with a starting point of \( p_0 = -1 \). Is the result better than using the standard Newton’s method?