Example: A rectangular printed page is to have margins 2 inches wide at the top and bottom and 1 inch margins along the sides. If the page is to have 35 in\(^2\) of printed text on it, what is the minimum area of the page itself?

Solution: Draw a picture:

Label constants and variables appropriately. Define variables in English.

Build objective function (which is to be optimized – here minimized).
Our objective function has two variables, so we seek an auxiliary relationship which allows us to eliminate a variable. What is that relationship?

Solve for $y$ in terms of $x$ to get $y =$ making our objective function depend on only one variable: $A(x) =$

Determine the restricted domain over which we are considering realistic, model consistent values for $x$. Restricted domain for $A$ is: Discover the critical points of $A$ within this domain. The derivative of $A$ is $A'(x) =$

On our domain, $A$ has no weird critical points, where $A'$ fails to exist. The derivative equals 0
when \( x = \)  and at that point we have \( y = \).

Notice that our second derivative is \( A''(x) = \frac{140}{x^3} \) which is always \( > 0 \) indicating that our critical point corresponds to a minimal area. For our critical value of \( x \), the area of the printed page with minimum area is given as \( A(x) = 76.4664 \).

Example: Where should Roger cut a piece of wire which is 16 inches long in order to form a circle and a square with the two pieces of wire and so as to have the total area enclosed by the two shapes to be maximal?

Solution: Draw a picture:
Label the constants and the variables:

How long is the section of wire destined to be the square? \( L_{\text{square}} = \) .
How long is the section of wire destined to be the circle? \( L_{\text{circle}} = \) .
What is the side length of the square? \( s = \) .
What is the radius of the circle? \( r = \) .

What area will the square enclose? \( A_{\text{square}} = \)
What area will the circle enclose? \( A_{\text{circle}} = \)
Our objective function therefore will be

\[ A(x) = \]

And our restricted domain for \( x \) is .
The derivative function is defined everywhere and the derivative is zero when
\[ 0 = A'(x) = \frac{-32}{\pi} + \left(\frac{2\pi + 8}{\pi}\right)x \quad \Rightarrow \quad x = \frac{16}{\pi + 4} \]

The second derivative is:
\[ A''(x) = \left(\frac{2\pi + 8}{\pi}\right) \]
which is always positive indicating our value for \( x \) provides us with a minimum value for \( A \).

That is NOT what we wanted. Let’s examine the behavior of the function at all extreme candidates. Remember because the function is continuous on a closed interval we are guaranteed both an absolute max and an absolute min.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \frac{16}{\pi + 4} )</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A(x) )</td>
<td>( 64/\pi )</td>
<td>8.96</td>
<td>16</td>
</tr>
</tbody>
</table>
Since $64/\pi = 20.37$, our interpretation is that the maximal area is enclosed when